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STABILITY AND HANDLING CRITERIA OF ARTICULATED VEHICLES
Part I
Technical Report

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September 1973
Final Report

PREPARED FOR:
U.S. DEPARTMENT OF TRANSPORTATION
NATIONAL HIGHWAY TRAFFIC SAFETY ADMINISTRATION
WASHINGTON, D.C. 20590

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16. Abstract

An analytical and experimental investigation on the stability and handling of articulated vehicles was conducted to develop stability criteria for tractor single-, double-, and triple-semitrailer vehicles during commonly encountered road and speed maneuvers subject to environmental influences. To this end, an experimentally validated digital simulation model, AVDS3, was further developed and validated. It was utilized to generate physical stability limits for articulated vehicles. During this investigation, the direct method of Lyapunov was employed to generate mathematical stability limits of cornering articulated vehicles. In addition, an effort was made to apply the concept of finite time stability to this problem.

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PREFACE

This is the final report on IIT Research Institute Project No. J6281 entitled, "Stability and Handling Criteria of Articulated Vehicles." The work was performed for the Department of Transportation, National Highway Traffic Safety Administration, under contract No. DOT-HS-105-2-392.

The work, completed over a twelve month period between June 1972 to June 1973, was monitored by Mr. Craig Keifer of the National Highway Traffic Safety Administration. IITRI personnel contributing to the technical content of this project include Mr. S. Desai, Dr. R. L. Eshleman, Dr. A. F. D'Souza, Mr. R. Joyce, Mr. J. McDonald, Mr. D.L. Kettman, and Mr. T. Scopelite. Special acknowledgement is due to the following consultants retained on the project: Dr. E.C. Mikulcik of the University of Calgary, Dr. H. Sachs of Wayne University, Dr. J. Walker of Northwestern University, and Dr. P. Julich of Louisiana State University.

Respectfully submitted,
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STABILITY AND HANDLING CRITERIA OF ARTICULATED VEHICLES

1. INTRODUCTION

The advent of large complex road trains on public highways, such as the tractor triple-semitrailer shown in Figure 1, has necessitated prediction of the dynamic behavior of such vehicles during commonly encountered road maneuvers and subject to environmental influences. Vehicle stability and handling criteria based on an experimentally verified mathematical model provide the framework for pretest safety reviews of existing and new vehicle designs. Such criteria also aid the generation of effective and enforceable standards that ensure articulated vehicle safety on the highway.

This final report covers an analytical and experimental investigation of stability and handling criteria of articulated vehicles from a response (physical stability) and mathamatical viewpoint. IIT Research Institute (IITRI) conducted this program for the Department of Transportation (DOT) under contract No. DOT-HS-105-2-392. The work was monitored by the National Highway Traffic Safety Administration (NHTSA), an agency of DOT.

In this program, stability and handling criteria were developed for tractor single-,double-, and triple-semitrailer vehicles supporting varied loads with a practical range of physical parameters; e.g., fifth wheel location, center of gravity location, trailer length, etc. Mathematical models were used to investigate stability limits featuring the use of the Articulated Vehicle Dynamic Simulation (AVDS) computer program, Lyapunov's direct method, and the finite time stability concept. Stability limits were verified experimentally

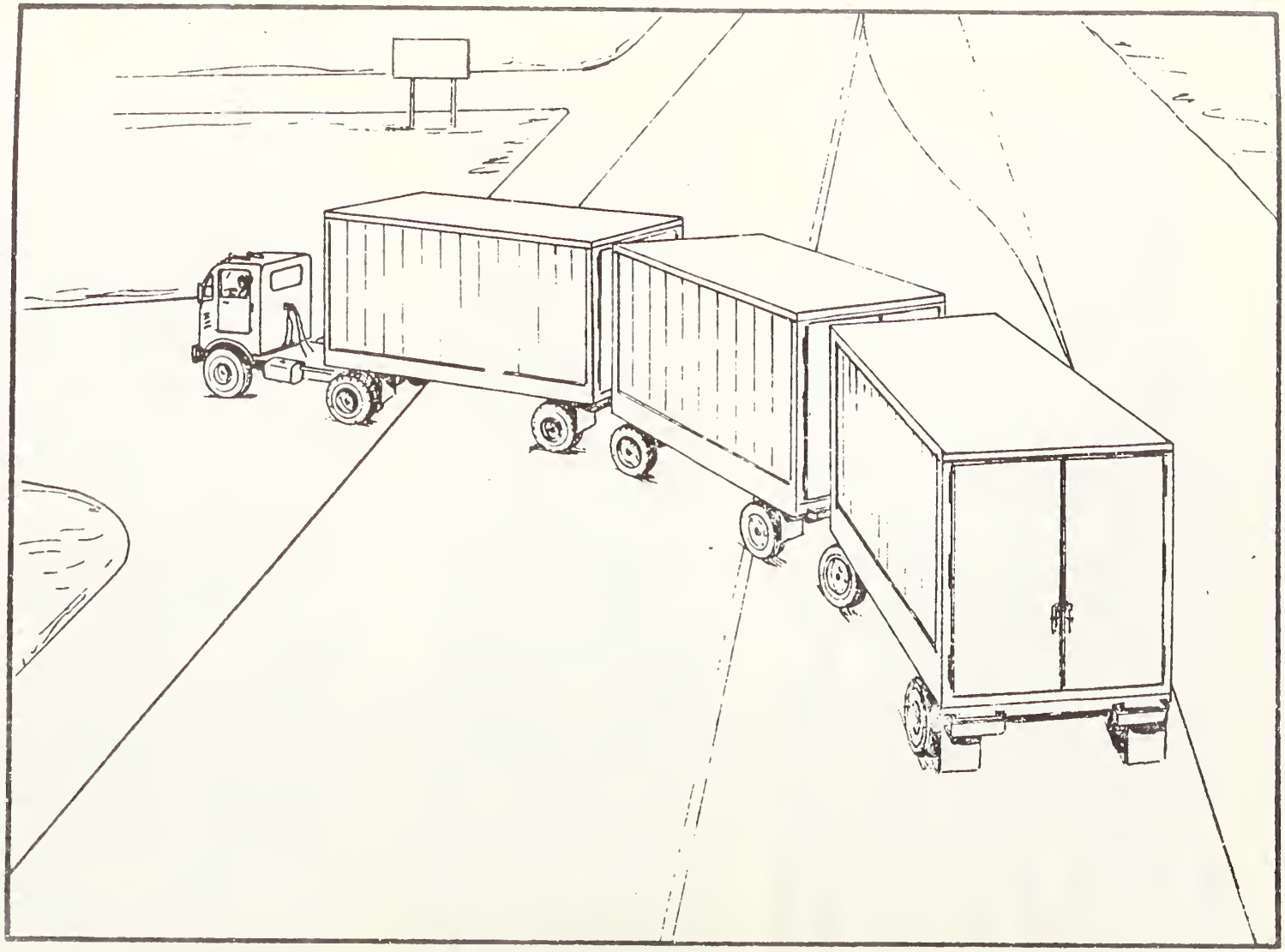


Figure 1. Articulated vehicle system.

for tractor single- and double-semitrailer vehicles. Special emphasis was placed on the assessment of vehicle maneuverability and stability in its normal road environment. The AVDS3 model simulates the dynamic response and evaluates the stability of single-, double-, and triple-trailer articulated vehicles of varied geometry and load distribution subject to:

- Tractive and braking forces which can be distributed as desired.
- Nonlinear cornering forces described by an empirical model developed by IITRI which is constrained by a tire cornering-tractive force relationship as indicated by the "friction ellipse" concept.
- Steady and transient aerodynamic forces.
- Fifth wheel friction and damping.
- Prescribed road maneuvers including grades and banks.

The scope of this program includes a literature review of recent developments in articulated vehicle technology and of the application of the Lyapunov direct method to the determination of stability of dynamic systems. Mathematical models were reviewed for their applicability to stability analyses and the ability to calculate vehicle nominal motions while performing a prescribed trajectory using the AVDS3 model was developed. From the mathematical models, system perturbations equations were developed which describe the response of the system to perturbed variables. Mathematical stability analyses using Lyapunov's direct method, which considers stability during infinite time, and finite time stability concepts were investigated for their applicability to the problem. Physical stability criteria considering the response of the vehicle were developed. An experimental verification program involving tractor single- and double-semitrailer systems was conducted to validate mathematically and physically inferred stability limits. Some parameter variations were conducted to determine the effect of road surface condition on vehicle stability for cornering, lane changing and evasive maneuvers.

variations were conducted to determine the effect of road surface condition on vehicle stability for cornering, lane changing and evasive maneuvers.

The trajectory approach, in which the articulated vehicle is required to maintain stability while traveling through a prescribed maneuver involving braking and/or acceleration, was selected because it yields results directly applicable to the needs of DOT/NHTSA, i.e., it relates vehicle performance to road environment. Figure 2 shows the IITRI approach to vehicle handling and stability problems.

A prescribed maneuver commonly needed on the highway, such as lane changing or cornering, serves as input data. The mathematical model reflects physical characteristics of the vehicle, the road and the environment. The mathematical model yields the driver control requirements, information on nonprescribed vehicle response, and information on its stability for the prescribed maneuver. The AVDS computer program (developed by IITRI for NHTSA) was used to determine physical stability (based on available cornering and tractive/braking forces, tire and yaw angles) and to provide nominal motions for mathematical stability analyses. Techniques for the mathematical determination of articulated vehicle stability were investigated and developed for limited applications using the second or direct method of Lyapunov.

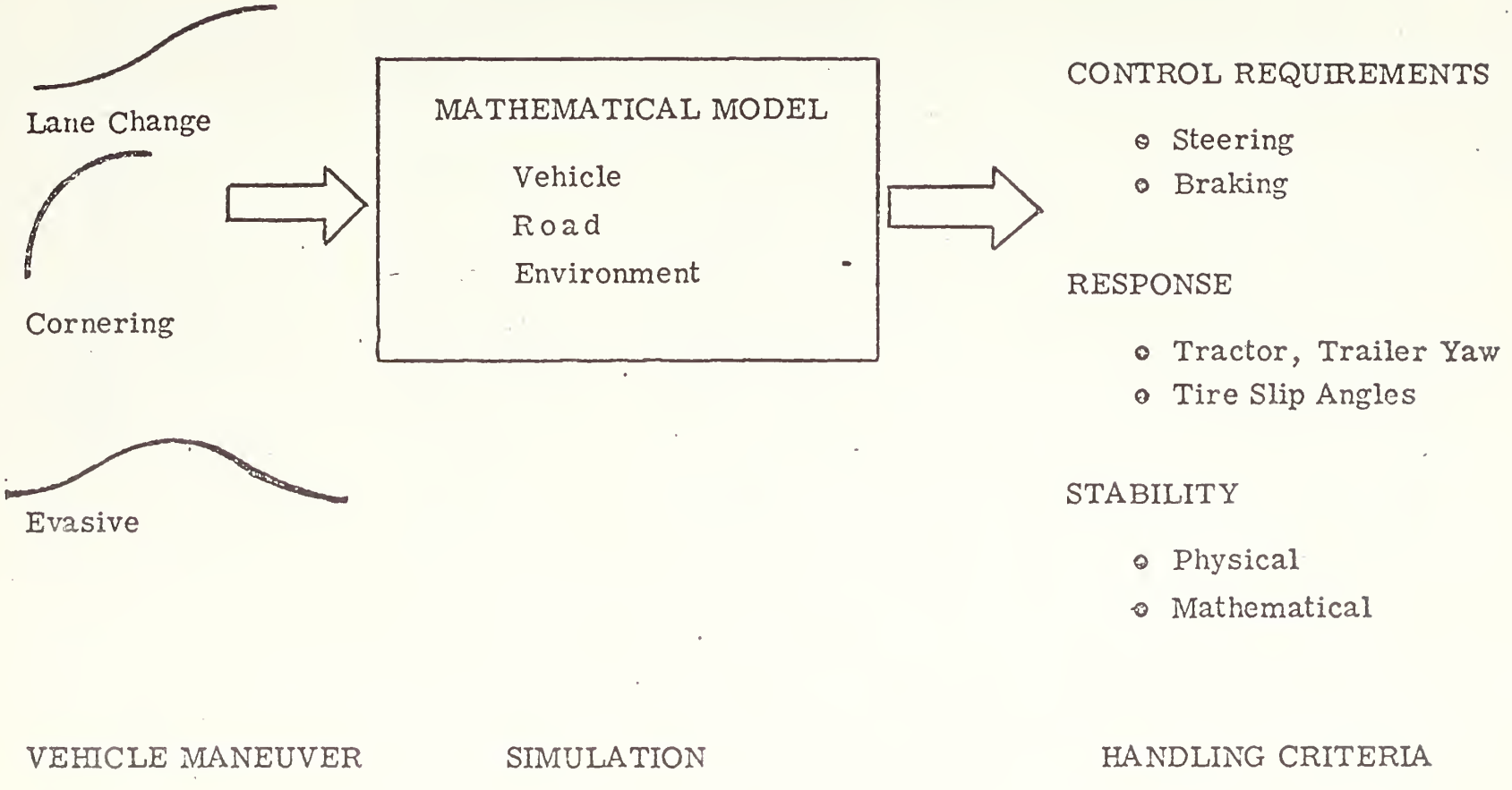


Figure 2. Articulated vehicle handling methodology.

2. LITERATURE REVIEW

A literature survey and review were conducted on technical material relevant to the development of stability and handling criteria for articulated vehicles. Technology related to articulated vehicle handling was surveyed and reviewed by Eshleman[1*] concurrent with the publication of this report and a synopsis of significant work is given herein. For a more detailed analysis Reference 1 should be consulted. The literature involving the application of the Lyapunov direct method to the development of articulated vehicle stability limits is reviewed in detail in this report. In addition, a complete bibliography on articulated vehicle dynamics is given in Appendix E.

2.1 Historical Background

The state of the technology shows that the fundamental dynamics of multibody highway vehicles has been developed. The characterization of tractor tire cornering and tractive behavior is in its infancy. Mathematical prediction techniques for articulated vehicle handling and stability phenomena using nonlinear models are being developed. There are few recorded experimental studies involving directional stability and handling of articulated vehicles.

Table 1 shows an evaluation of the major work in articulated vehicles in synoptic form. Discussion of modern works is ordered according to when a fundamental understanding of the phenomena that governs the dynamic behavior of these vehicles was developed.

---

*References are listed in Section 10.

Table 1

MAJOR TECHNOLOGICAL ADVANCES IN TRUCK AND ARTICULATED VEHICLE DYNAMICS

Date	Contributor	Contribution	Ref. No.
1963	F. D. Hales Motor Industry Research Assoc.	Simple simulation model for articulated vehicle lateral stability; four degree of freedom linear model; lateral, forward and yaw motion of tractor, and articulation angle between tractor trailer; lateral tire force linear function of slip angles.	2
1963	F. Jindra General Motors Corp.	Simple directional stability of three degree of freedom longitudinal, lateral, and relative yaw linear model; constant speeds, steady flat turn.	3
1963, 1964	J. R. Ellis Advanced School of Automotive Engr. Cranfield, England	Analysis of tractor-semitrailer with four degrees of freedom lateral-forward motion tractor and trailer yaw angles; first use of nonlinear tire model including braking and side force; digital computer solutions for instability based on response.	4,5
1965	F. D. Hales Motor Industry Research Assoc.	Effect of locked wheels on the stability of tractor-semitrailer vehicles; linear model; experimental work.	6
1965	F. Jindra General Motors Corp.	Extension of earlier work on lateral handling; constant speed, flat turn; influence of design parameters on lateral stability; linear tire model, four degree of freedom linear system model.	7
1965	F. Jindra General Motors Corp.	Analog computer study of tractor semitrailer and full-trailer combination handling at constant velocity on a flat road; five degree of freedom linear vehicle model; linear tire model.	8
1966	F. D. Hales, et al. Motor Industry Research Assoc.	Extensive field testing on the lateral behavior of articulated vehicles during braking; antijackknife device evaluation; trailer swing instability; jackknifing instability.	9
1967	I. Schmid Battelle Institute	A study on the directional stability of tractor single and double semitrailer and truck-trailer systems. Routh criteria are used to study stability of a linear model. Special attention is given to braking.	10

Table 1 (Contd)

Date	Contributor	Contribution	Ref. No.
1968	R. E. Nelson and J. W. Fitch American Brakeblok; Western Highway Inst.	Braking and stability tests of single, double, and triple trailer articulated vehicles; test results; brake system evaluation and modifications.	11
1968	R. P. Joyce and T. M. Scopelite IIT Research Inst.	Full scale experimental evaluation of an anti-jackknife device on a tractor-semitrailer vehicle.	12
1968	T. A. Byrdsong NASA-Langley Research Center	Effect of braking on tire cornering force; experimental data for wet and dry road surfaces.	13
1968	H. Dugoff Stevens Inst. Tech.	Theoretical analysis of lateral dynamic stability of articulated vehicles; scale model aerodynamic tests; influence of aerodynamic effects on stability.	14
1968	E. C. Mikulcik Univ. Calgary	First complete nonlinear model of a tractor-semitrailer with three translational and three rotational degrees of freedom; steering-braking input; comparison of linear and nonlinear models; parameter variation studies.	15
1969	K. E. Holmes and R. D. Stone Road Research Lab. Ministry of Transport Crawthorne, England	Tire braking and cornering forces on dry and wet surfaces; experimental data; theoretical models.	16
1969	E. E. Hahn and T. M. Scopelite IIT Research Inst.	Articulated Vehicle dynamic simulation; linear simulation model; operator response evaluation.	17
1970	T. M. Scopelite IIT Research Inst.	Articulated vehicle dynamic simulation (AVDS I); nonlinear digital simulation model for tractor-semitrailer handling with variable trajectory; fifth wheel damping; nonlinear tire model.	18
1970	P. M. Leucht Univ. of Michigan	Examined the effect of brake-torque distribution on tractor semitrailer directional dynamics.	19

Table (Concl)

Date	Contributor	Contribution	Ref. No.
1971	R. Limpert Highway Safety Research Inst. Univ. Michigan	Tractor-semitrailer braking predictions in terms of available tire/roadway friction; effects of load transfer; tandem axles; braking distribution studies.	20
1971	A. K. Krauter and D. L. Bartel Cornell Univ.	Design and analysis of a tractor-semitrailer using a digital simulation model; optimization technique used for design; wheel rotation degrees of freedom; braking and steering input.	21
1971	R. W. Murphy, R. Limpert, and L. Segel Univ. of Mighigan HSRI	The range of braking performance currently exhibited by buses, trucks, and tractor-trailers and to establish the maximum braking performance capabilities of these vehicles based upon full utilization of the technology related to brake system design was investigated.	22
1972	R. L. Eshleman and S. Desai IIT Research Inst.	Articulated vehicle dynamic simulation (AVDS II); nonlinear digital simulation model for single and double bottom tractor-trailers; trajectory analysis including hills, bends and curves; multiple degrees of freedom per body; variable braking input; tire model (modified Ellis model); steady and transient aerodynamic loads; full scale verification tests.	23
1972	V. T. Nicolas and T. R. Comstock Univ. of Cincinnati	A computer simulation model for determining the dynamic behavior of tractor semitrailer vehicles when anti-skid braking systems was employed. Mathematical models representing the brake system, anti-skid controller, tire mechanics, and three-dimensional vehicle dynamics were developed. Results of simulating the braking performance of a vehicle employing a simple on-off wheel controller during straight line and fixed steering angle stops are presented.	24
1972	E. A. Susemihl Cornell Univ.	This Ph.D. work was concerned with the problem of indirect sensing (identification of the initiation of instability before it can be directly observed) of tractor jackknifing. Forms of corrective action are also considered.	25

2.2 Stability Methods

The major effort expended on the literature survey was directed toward obtaining information on the application of Lyapunov's direct method to nonlinear, autonomous and non-automous mechanical systems (i.e., differential equations that are explicit functions of time). A large number of methods have been developed in the past for the generation of Lyapunov functions. Many of the methods are, however, not general and restricted to very specific system configurations. These special methods are briefly mentioned in the following paragraph and several methods that appeared promising for our application are given.

2.2.1 Special Methods

Among the several methods that do not apply to our problem are the following: 1. the method pioneered by Lucre' and continued largely by LaSalle and Lefschetz[26,27]; 2. the method proposed by Popev and its extensions[27,28]; 3. circle criterion developed independently by Sandberg[29] and Zames[30]. Almost all of these techniques are restricted to feedback control systems with a single nonlinearity and an autonomous, linear transfer function.

2.2.2 Methods Based on the Use of the Hamiltonian as a Lyapunov Function

The stability of artificial satellites has led to a large number of studies of the stability of the steady state motion of rigid bodies. In these studies, the Hamiltonian in the perturbations is used as the Lyapunov function. The applications are discussed by Pringle[31], Meirovitch[32,33] and others.

The Hamiltonian involves tedious algebraic manipulations but is likely to yield deep insight into the physical causes of

instability since the Hamiltonian in the perturbations is related to the total energy of the perturbations. Regarding the method based on the Hamiltonian, it is desirable to refer the motion of the articulated vehicle to a system of axes which is neither an inertial space nor a set of body axes. The coordinates employed in this case are called quasi-coordinates and the equations of motion differ from the commonly used Lagrange and Hamilton equations. These techniques were studied with the objective of employing the Hamiltonian in the quasi-coordinates as the Lyapunov function.

2.2.3 Zubov Power Series Method[34,35]

This method appears to be quite general. It only disadvantage is that it may lead to some difficult convergence problems. Fortunately, the Zubov method of generating Lyapunov functions is amenable to machine computation.

Margolis and Vogt[36], Rodden[37], and Julich[38], have employed Zubov's method with digital computer algorithms. Rodden's computational procedure employs geometric concepts and a search for the tangency of two hypersurfaces. Julich formulates the problem as the minimization of V function subject to the constraint that $\dot{V} = 0$. Julich's procedure is computationally simpler than that of Rodden. Julich's computational scheme was studied for stability in the large investigations.

Margolis and Vogt[36] in a partly tutorial paper outline the fundamental principles of Zubov's procedure for constructing Lyapunov functions for a system of differential equations and O'Shea[39] discusses the procedure for a system of difference equations. In the Zubov method, the Lyapunov function is obtained from the solution of a linear partial differential equation. However, in most cases of practical interest, an analytical solution of the equation is not possible.

The solution for the Lyapunov function can in many cases be obtained in the form of a Taylor series expansion about the origin. Rodden[37] has presented a procedure in which a digital computer can be used to estimate the region of asymptotic stability. His computational procedure employs the method of tangency and geometric considerations, namely, the determination of tangency of two hypersurfaces. Another procedure of using a digital computer to estimate the region of asymptotic stability in the Zubov's method has been presented by Julich[38].

In Julich's formulation, the domain of asymptotic stability is obtained from the solution of a constrained minimization problem as suggested by Margolis and Vogt[36]. Julich[38] employs the Davidson method[40] which is a powerful computer technique for obtaining the minimum of a function. Julich claims that his approach is preferable to that of Rodden[37]. This claim is also corroborated by the results obtained by Geis[41].

2.2.4 Gradient Method of Generating Lyapunov Functions[42,43,44]

This method also is quite general. Its disadvantage is that it presents a great many alternatives in choosing a Lyapunov function. It is further limited by the inability to determine the sign definiteness of general nonquadratic functions.

2.2.5 Quadratic Form Lyapunov Function

In this method, the use of constant-coefficient quadratic form Lyapunov functions is extended to cover time-varying nonlinear problems. The result is a new method of generating Lyapunov functions that often extends constant-coefficient results to more complicated cases. A method that falls in this class is given by Newman[45].

A survey paper on stability theory that also contains an extensive bibliography is that of Brockett[46].

2.3.6 Finite Time Stability

A stability theory, called finite time stability has been developed by Weiss and Infante[47,48], Dorato[49], Gunderson[50], Heinen and Wu[51], and others. The notion of finite time stability considered to have emerged around 1953 and is credited to Kamenkov and Lebedev[52,53]. Finite time stability theory parallels, to a certain extent, the Lyapunov stability theory but differs from it in a number of significant respects and is motivated by the following advantages. In the Lyapunov stability theory, time tends to infinity, whereas in many cases of practical interest such as the articulated vehicle, there is concern with the behavior over a fixed interval of time, e.g., will a given system exhibit a response to given stimuli which is contained within certain specified bounds during the fixed time interval? Lyapunov stability theory involves the solution of the nominal motion and performance of complicated transformations in order to obtain the equations of perturbed motion. In finite time stability, the stability of a system trajectory can be investigated without the necessity of performing complicated transformations.

3. MATHEMATICAL MODELING

The equations of motion of the articulated vehicle constitute the mathematical model. They must be realistic enough to include the major portion of the primary causes and effects and yet must not be more mathematically complicated than needed. The major drawback of a complicated mathematical model that includes all secondary effects is that computational costs soar and the desired results are hidden.

3.1 Coordinate Systems

The choice of the coordinate system is important from the point of view of obtaining the equations of motion in as simple a form as possible. Among the various alternatives, the following were used in this program : (1) tractor body axes for translation and Euler angles for rotation; and (2) fixed axes for translation and Euler angles for rotation. The equations of motion used in the AVDS 3 simulation model to generate nominal motion solutions and physical stability criteria are contained in Appendix A. The equations of motion used in the stability analyses, given in Appendix B, are formulated in tractor body coordinates.

The equations of motion including aerodynamic effects were written in space Cartesian and body Cartesian coordinates for the tractor semitrailer vehicle allowing for forces and moments at the vehicle and suspension attachment points. The coordinate system that best facilitates the analyses was selected for the tractor double- and triple-semitrailer vehicles.

The equations of motion for space Cartesian coordinates utilize three degrees of translational motion and three degrees of rotational motion to describe each vehicle unit. A set of 12, 24, and 36 order nonlinear differential equations of motion are obtained for the single-, double-, and triple-articulated vehicles, respectively.

Kinematic and kinetic constraint equations will be developed to reflect the attachment of the vehicle units. Kinematic constraints involve matching positions and relative angular displacements at each vehicle connection, while kinetic constraints involve forces and moments at these connections. In addition, the translational and rotational velocities and accelerations are required later. Therefore, at each vehicle attachment point the following constraint conditions are obtained:

$$\left.\begin{array}{l}\text{Forces, } x_i \text{ directions}\\ \text{Moments, } x_i \text{ directions}\\ \text{Translational positions, } x_i\\ \text{Rotational positions, } \theta_i\end{array}\right\} \quad i = 1,2,3$$

Other equations of value in later elimination of excess variables include:

$$\left.\begin{array}{l}\text{Translational velocities, } \dot{x}_i\\ \text{Translational accelerations, } \ddot{x}_i\\ \text{Angular velocities, } \dot{\theta}_i\end{array}\right\} \quad i = 1,2,3$$

3.2 Vehicle Modeling

At this point the equations of motion and constraint conditions were examined for means of obtaining the least complicated meaningful mathematical model. The development of the vehicle motion perturbation equations requires much algebraic manipulation; a model overburdened with unneccessary complexity poses obstacles to problem insight. Therefore, the results of the first series of articulated vehicle stability tests were used to justify simplifications in the model. The use of semi-static load transfer to model the effects of roll and pitch motions was examined. The nonlinearities involved in vehicle yaw motions and the tires were retained because they are fundamental to a realistic model. Assumptions on the constraint conditions, e.g., the dolly hitch to the semitrailer where only lateral and forward forces are transmitted, were made prior to the elimination of the excess variables.

The assumptions involved in the use of semistatic load transfer were experimentally verified during the first series of vehicle tests. It was found that pitch angles were not measurable except when braking was employed, and for heavy braking pitch, angles of the tractor were larger with magnitudes varying from 1.5 to 5 deg depending upon the type maneuver. It should be noted that both roll and pitch angles are about an order of magnitude smaller than vehicle yaw angles. These results are discussed in Section 5. Essentially the values obtained in this study confirm the analytical results of Mikulcik[15].

Constraint equations are used to extract excess nonidependent variables by elimination of the vehicle coupling forces and moments. From this manipulation, which is extremely involved but has been completed for the tractor single- and double-semitrailer articulated vehicles by Mikulcik[15] and IITRI[23] respectively, eight, 13, and 18 independent second-order nonlinear differential equations are obtained for the singles, doubles and triples if roll and pitch variables are retained. When load transfer is used and the vehicle vertical motions are uncoupled, four, six and eight independent second-order partial differential equations are obtained for singles, doubles and triples respectively.

3.3 Tires

Tire side forces and braking/tractive forces are introduced to the differential equations through the suspension system. In order to permit more refined tire modeling, multisuspension attachment points were introduced in the tractor and trailer models. This feature allows modeling of single and tandem axles. In addition, the mathematical model was expanded to include individual tire characterization. Tractive/braking forces are related to side forces using the friction ellipse concept.

The new experimental tire data (10.00-20 Super Hi-Miler) describing the side force and aligning torque as a function of slip angle for discrete normal loads was released by the Goodyear Tire and Rubber Company. In the process of obtaining this data, two papers on tire behavior, unreferenced to date, were uncovered. Livingston and Brown[63] give their analytical and experimental findings on the physics of the slipping wheel. In the first paper[63], they describe their work on side force and aligning torque as a function of slip angle. The side force and aligning torque is derived on the basis of an assumed pressure distribution (normal load) across the tire contact patch. In addition, a method is given for analyzing the pressure distribution from experimental data. In the second paper[54], the interaction between side force and tractive force is examined and modeled with experimental validation. The aforementioned work and experimental data was used along with the work of Freudenstein[64] to select a tire model for the stability and handling analyses.

In the process of simulation of a vehicle performance (Section 6) it was found that results superior (compared to experiment) to those previously computed were obtained. It was noted that the normal load on a tire decreases as the tire-road friction coefficient increases and that the angle at which the maximum tire side force occurs decreases. The increase of effective side force at lower normal loads made it mandatory that the model consider each tire individually. A detailed report of our tire modeling effort is given in Appendix A.

3.4 Trajectory Modeling

The trajectory approach used in this study requires the mathematical characterization of standard vehicle maneuvers performed on the highway for the determination of stability and handling criteria. When experimental validation is being performed, the actual test data is used in the model rather than

mathematical forms. The road maneuvers specified for the tractor center of gravity used in this study are lane change, cornering, and evasive (double lane change). The tractor center of gravity velocity during the maneuver is specified as a function of time. The velocity and trajectory functions are then differentiated for values of acceleration.

A seventh-order polynomial function is used to prescribe the lane change maneuver trajectory (length L x width S is described in Appendix A). Tractor position, velocity and acceleration are all described mathematically.

In an evasive maneuver, a lane change is immediately followed by a reverse lane change. As such, the evasive maneuver trajectory is also specified by the polynomial function for the lane change maneuver. The length and the width values for both lane changes are specified.

The cornering maneuver trajectory is specified in three distinct parts. The approach to the curve and exit from the curve are specified as partial lane change maneuvers and are prescribed with the polynomial function for lane change trajectory. The tractor center of gravity path around the curve is specified as circular and a math function for the circle is used to prescribe it.

A typical maneuver trajectory is shown in Figure 3. The segments AB and CD are the partial lane changes respectively; BC is the circular path representing motion around the curve; Xc, Yc are the center coordinates of BC; and AB and CD are segments of the complete trajectories AA' and DD' respectively. The radius of curvature (ρ) at B on AA' (also at C on DD') is equal to the radius (R) of BC. The radius of curvature at a point on the curve is given mathematically in Appendix A.

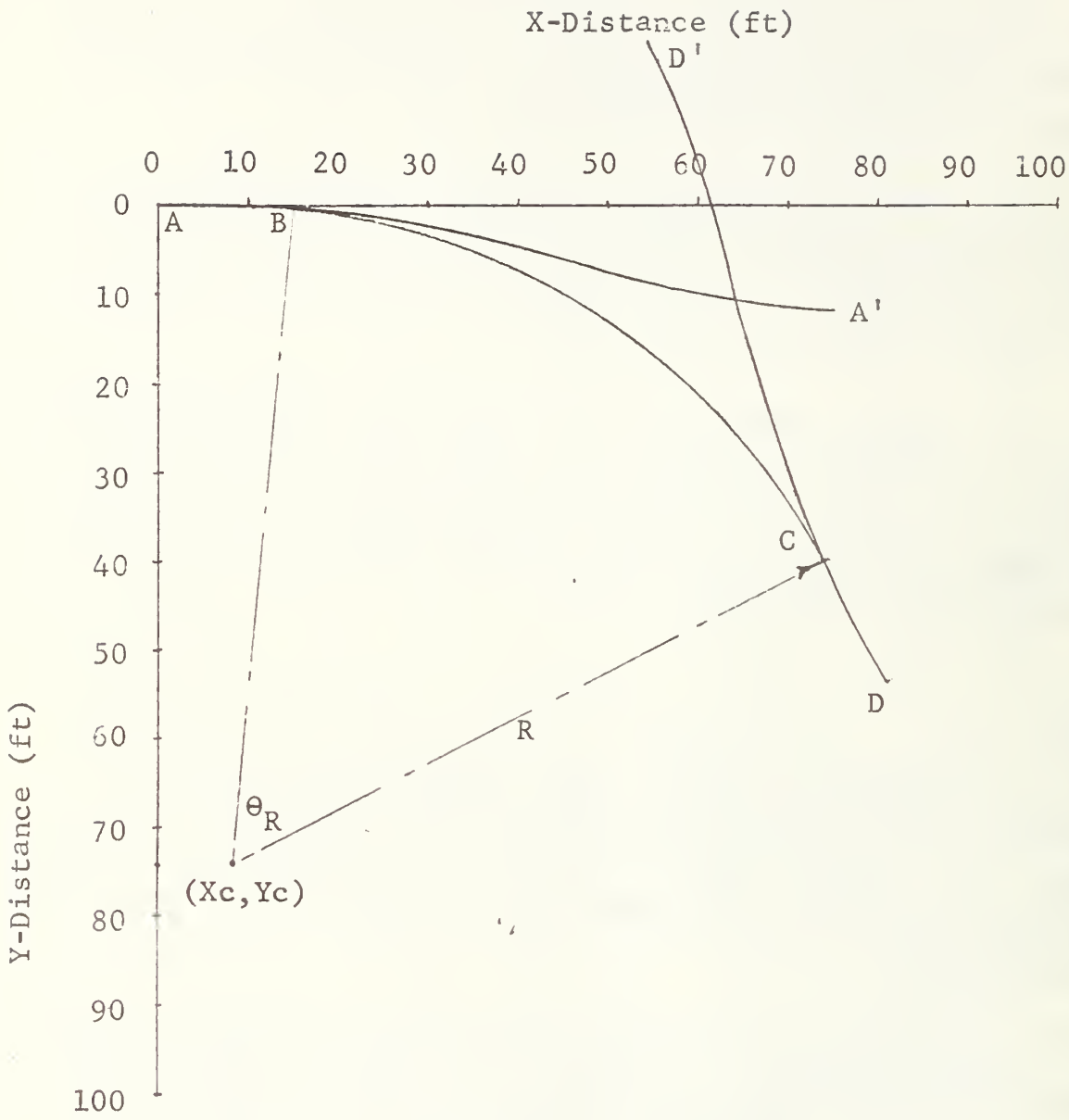


Figure 3. Cornering maneuver (74.2 ft radius) at 20 mph.

Finally the velocity and acceleration of the tractor center of gravity is given in Appendix A. This formulation was used in experimental verification when direct acceleration velocity and yaw measurements were used to obtain prescribed vehicle motions.

4. STABILITY CRITERIA DEVELOPMENT

The stability of motion of articulated vehicles was studied both from mathematical and physical viewpoints using the mathematical models described in Section 3. Mathematical stability was assessed by perturbing the response variables of the vehicle and examining whether or not the perturbed motion decayed to the original nominal motion. In practice, perturbations are caused by imperfections in driver, road and vehicle response. Physical stability was assessed by examining the response of the vehicle as it performed a prescribed maneuver. If cornering or tractive force was lost at a wheel set or violent yaw angles (violation of space constraints) were obtained, the vehicle was deemed to be unstable.

Numerical simulations for articulated vehicle response (nominal motions) when performing nominal cornering, evasive, and lane changing maneuvers were performed by the AVDS 3 model. These simulations were used as the basis of both mathematical and physical stability techniques. Mathematical stability analyses of articulated vehicles were successfully conducted for autonomous (constant speed cornering) motions using Lyapunov's direct method. It was found that the concept of Lyapunov stability is too restrictive for nonautonomous motions (time-varying response). Whereas the Lyapunov method implies stability over infinite time, the articulated vehicle can be instantaneously unstable in the sense of mathematics and yet recover at a new nominal motion. It was found that a modified Lyapunov stability concept called "finite time stability" is physically more applicable to articulated vehicle stability analysis than the Lyapunov direct method. Several finite time stability techniques (Appendix C) were unsuccessfully applied to this problem. In each case, results were very conservative; i.e., stable motions were judged to be unstable. At this time, it appears that the implementation of mathematical stability concepts to articulated vehicles performing time-varying motions is beyond the state of the art.

4.1 Mathematical Stability

4.1.1 Nominal Motions

Mathematical stability techniques utilized herein employ response solutions to vehicles performing prescribed maneuvers called nominal motions. The motion involved when articulated vehicles perform a prescribed nominal maneuver was obtained for two classes of problems. The first is a stationary maneuver where the nominal motion is steady state and not a function of time. Specifically, the motion is a circular trajectory at a constant forward velocity and the state variables (tractor and trailer yaw rates and tractor accelerations) and control inputs braking and steering) are constants. Two cases involving unsteady motions were also considered: (1) the case in which forward velocity, steering and braking force are held fixed and braking force is changed impulsively, hence the perturbed motion is described by perturbations in tractor and trailer yaw and yaw rate, and tractor forward velocity about their nominal constant values; and (2) the case in which steering is held fixed and braking force is changed impulsively, hence the perturbed motion is described by perturbations in tractor and trailer yaw and yaw rate, and tractor forward and lateral velocity about their nominal constant values.

The second class of problem is described by a nonstationary maneuver where the nominal motion is unsteady state and hence is time varying. During this nominal motion, the state variables and the control inputs (braking and steering) vary with time.

Classes of nominal trajectories including cornering and lane changing with and without traction/acceleration were selected based on practical road geometries. Nominal motion solutions were obtained from AVDS3, which was generated in this program and provides the facility to simulate the response of triple-trailer articulated vehicles. The AVDS model is formulated and described in Appendix A and its implementation is described in the Articulated Vehicle Dynamic Simulation User's Manual, Part 2 of this report.

4.1.2 Perturbation Equations

The perturbation equations are a perturbed form of the vehicle equations of motion. To conduct the mathematical stability analyses of this program, the equations of motion were formed in tractor body coordinates. This problem formulation is described in Appendix B. To further facilitate these analyses, the equations of motion were expressed in state variable form,

$$\dot{\underline{q}} = \underline{f}\ (\underline{q}, \delta, \underline{D}) \tag{1}$$

where the five-dimensional state variable vector $\underline{q}(t)$ is defined by

$$q = (\psi_1 - \psi_2,\ \psi_1,\ \dot{\psi}_1 - \dot{\psi}_2,\ v_{y_1}, v_{x_1}) \tag{2}$$

where ψ_1 = tractor yaw

ψ_2 = tractor yaw

v_{y_1} = tractor lateral velocity

v_{x_1} = tractor forward velocity

and the steering angle δ and the braking forces $\underline{D}$ are the control inputs. Three types of nominal maneuvers are considered, namely, circular trajectory at constant forward velocity, lane change with and without braking, and cornering with and without braking. A prescribed maneuver whose stability is to be investigated is selected and the nominal motion $q^*(t)$ and the steering angle $\delta^*(t)$ demanded by the maneuver and the distribution of braking force $D^*(t)$ at each wheel are computed by employing AVDS3.

The motion predicted by the mathematical model may not be identical with that of a practical maneuver because the latter is subjected to disturbances caused by variations in the aerodynamic loading, inexact realization of the control inputs $\delta^*(t)$ and $\underline{D}^*(t)$ and similar causes. A nominal motion that is unstable is not realizable in practice because if it is slightly perturbed by disturbances, it diverges from the nominal motion (Figure 4). The question to be answered is how does the motion $q(t)$ which starts near a nominal motion $q^*(t)$ behave with respect to q^*? Does it remain very near $\bar{q}^*$ as time increases which would imply that q^* is unstable? In order to answer this question, a particular maneuver $q^*(\delta^*, D^*, t, q_o, t_o)$ starting at time t_o with initial conditions $\bar{q}_o$ and control inputs δ^* and D^* is considered. This nominal motion is obtained by numerical integration of Equation (1) by employing the AVDS3 program.

Consider the effect of perturbations Δq_o on the initial state and $\Delta\delta$ and ΔD on the steering and braking inputs. Let $\underline{q}(\delta^* + \Delta\delta, D^* + \Delta D, t; q_o, + \Delta q_o, t_o)$ be the resultant perturbed motion. The vector $\underline{x}(t)$ of perturbed state variables is defined by

$$\underline{x}(t) = \underline{q}(t) - \underline{q}^*(t)$$

or

$$\underline{q}(t) = \underline{q}^*(t) + \underline{x}(t) \qquad (3)$$

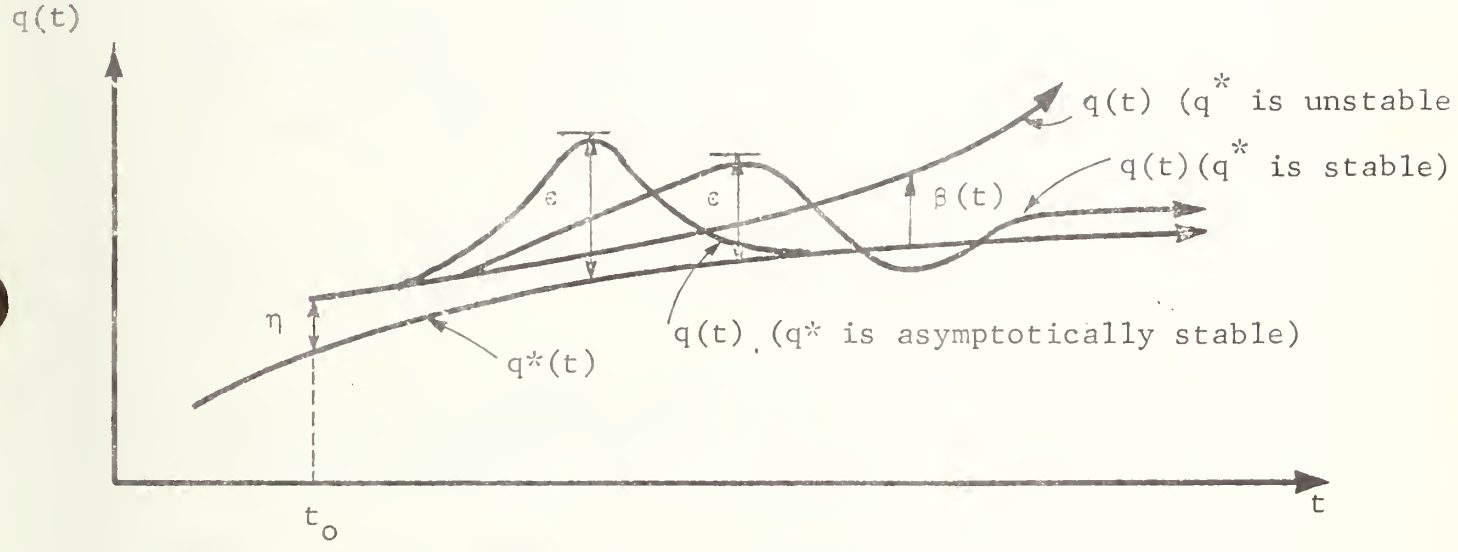


Figure 4. The concept of stability.

Now, $\underline{q}(t)$ from Equation (3) is substituted into Equation (1) and the resulting equation is expanded in Taylor series about the nominal motion $\underline{q}^*$ to obtain

$$\dot{\underline{q}}^* + \dot{\underline{x}} = \underline{f}(\underline{q}^*, \delta^*, \underline{D}^*) + \left.\frac{\partial \underline{f}}{\partial \underline{q}}\right|_{\substack{\underline{q}^* \\ \underline{u}}} \underline{x} + \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\substack{\underline{q}^* \\ \underline{u}}} \Delta\underline{u} + \underline{h}(\underline{x}, \Delta\underline{u}, t) \tag{4}$$

where $\underline{u} = \begin{bmatrix} \delta \\ \underline{D} \end{bmatrix}$ and $\Delta\underline{u} = \begin{bmatrix} \Delta\delta \\ \Delta\underline{D} \end{bmatrix}$ and $\underline{h}(\underline{x}, \Delta\underline{u}, t)$ contain terms that are higher than the first degree in $\underline{x}$ and $\Delta\underline{u}$ resulting from the Taylor series expansion about $\underline{q}^*$. Since the nominal motion satisfies the equation

$$\dot{\underline{q}}^* = \underline{f}(\underline{q}^*, \delta^*, \underline{D}) \tag{5}$$

it follows from Equations (4) and (5) that the differential equation for the perturbations in state variables becomes

$$\dot{\underline{x}} = A(t)\ \underline{x} + B(t)\ \Delta\underline{u} + \underline{h}(\underline{x}, \Delta\underline{u}, t) \tag{6}$$

where $A(t)$ and $B(t)$ denote the Jacobian matrices defined by

$$A(t) = \left.\frac{\partial \underline{f}}{\partial \underline{q}}\right|_{\substack{\underline{q}^* \\ \underline{u}^*}} \quad \text{and} \quad B(t) = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\substack{\underline{q}^* \\ \underline{u}^*}} \tag{7}$$

4.1.3 Stability Analyses

The original problem of determining the stability of a specified maneuver $q^*(\delta^*, D^*, t; \underline{q}_o, t_o)$ is now equivalent to the problem of determining the stability of the null solution, i,e., trivial solution of Equation (7). Consider a special class of maneuvers where the translation of the vehicle is described by a circular arc at a constant forward velocity. For this class of maneuvers, all the state variables of the nominal motion defined by Equation (2) have constant values, and in particular $\dot{\psi}_1 - \dot{\psi}_2$ is zero. Also the control inputs δ^* and D^* are constant. Such steady state nominal motions can be represented by isolated quilibrium points (also called singular points) in the state space $\underline{q}(t)$ as shown in Figure 5.

The steady state nominal motion q^* whose stability is to be investigated is denoted by $\underline{q}_e^*$ to indicate that it is an equilibrium point in the state space rather than a time-varying trajectory $\underline{q}^*(t)$. The transformation of Equation (3), namely,

$$\underline{x}(t) = \underline{q}(t) - \underline{q}_e^* \tag{8}$$

is now merely a transformation of coordinates such that the origin of the state space of $\underline{x}(t)$ is equivalent to the equilibrium point $\underline{q}_e^*$ of the state space of $q(t)$ as shown in Figure 5. In this case, Equation (6) becomes autonomous, i.e., its right-hand side is not an explicit function of time, and is described by

$$\dot{\underline{x}} = A\,\underline{x} + B\Delta\underline{u} + \underline{h}(\underline{x}, \Delta\underline{u}). \tag{9}$$

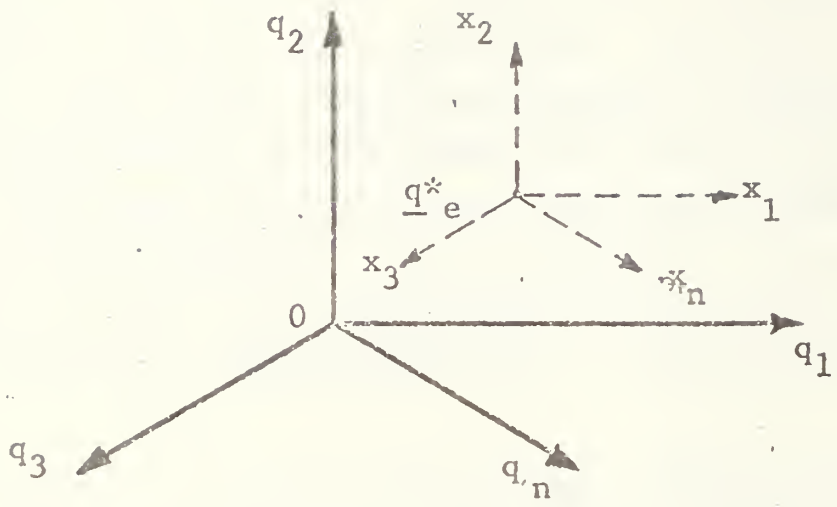


Figure 5. State space representation.

Stability in the sense of Lyapunov. When considering the stability of a maneuver in the sense of Lyapunov, the control inputs δ^* and D^* of the nominal motion are not perturbed. This step is necessary since the concept of stability in the sense of Lyapunov does not admit perturbations in the inputs. The nominal motion is perturbed only by nonzero initial conditions which may be caused by impulsive changes in the control inputs. Letting $\Delta\underline{u} = \underline{0}$ in Equation (6), the perturbation vector is described by the equation

$$\dot{\underline{x}} = A(t)\ x + \underline{h}(\underline{x},t) \tag{10}$$

and in the autonomous case,

$$\dot{\underline{x}} = Ax + \underline{h}(\underline{x}) \tag{11}$$

is obtained. The definitions of stability in the sense of Lyapunov are contained in Appendix C.

Lyapunov stability in the small. In the investigation of stability in the small, it is assumed that the perturbations are sufficiently small and the higher order terms in the Taylor series expansion, i.e., $\underline{h}(\underline{x},t)$ in Equation (10) or (11) is investigated. The approach yields stability information in the small, i.e., the perturbations should be sufficiently small but there is no indication of the magnitude of perturbations that could be considered as small. Hence, when asymptotic stability exists, the size of the region of asymptotic stability is not known. To determine the size of this region, the stability in the large of the nonlinear Equation (10) or (11) is to be investigated. The theorems that were employed for the stability investigation in the small are stated in Appendix C.

For an autonomous system (cornering vehicle), the perturbation equation,

$$\dot{\underline{x}} = A(t)\ x + \underline{h}(\underline{x}) \tag{12}$$

was locally linearized. If the eigenvalues have only negative real parts, the nominal motion, q_e^*, is asymptotically stable in the small. This theorem was employed for the stability investigation in the small for maneuvers where the translation of the vehicle is described by a circular arc at a constant forward velocity. The conditions for the matrix A to have eigenvalues with negative real parts was determined by employing Routh's criterion. A computer program was written for this purpose.

The stability in the small of nonautonomous maneuvers was investigated by employing as theorem which relates the stability in the small for a nominal maneuver $q^*(t)$ to the stability of the null solution of the perturbation equation,

$$\dot{\underline{x}} = A(t)\underline{x} \tag{13}$$

which is the linear approximation of the nonlinear perturbation Equation (10). The problem becomes one of establishing uniform asymptotic stability for the time-varying linear system. One approach is to employ a suitable Lyapunov function for which there is no structural procedure to find such a function.

Another approach, which has been sometimes employed in the existing literature is the "freezing time" method. At each instant of time $t = t_1, t_2, \ldots, t_k$, the time-varying parameters are fixed at their current values and the matrix $A(t_i)$ is treated as a constant matrix for the interval t_i to t_{i+1}. The conditions for the constant matrix $A(t_i)$, with $t_i = t_1, t_2, \ldots, t_k$, to have eigenvalues with negative real parts can then be determined by employing Routh's criterion.

The following question then arises. If all eigenvalues of the matrix $A(t_i)$ have negative real parts for $t_i = t_1, t_2, \ldots, t_k$, does it mean that the origin of the linear system is uniformly asymptotically stable? The answer is not always in the affirmative as demonstrated by two counter-examples in Appendix C which show asymptotically stable results for unstable systems.

The examples reveal that the eigenvalues of A(t) do not carry a great deal of information regarding stability. Yet, the freezing time method has been successfully employed in the aerospace industry for the design of autopilots for aircraft and missiles. It appears that if the elements of A(t) are periodic in time or if the eigenvalues are near the imaginary axis, as in the examples, then the stability results based on the eigenvalues of A(t) may be invalid.

The freezing time method was used in this investigation to study the stability of cornering maneuvers. The matrix $A(t_i)$ was obtained for $t_i = t_1, \ldots, t_k$ by applying the AVDS 3 simulation results and Routh's criterion was used to check the location of the eigenvalues. The results indicate that the maneuvers that are possible by AVDS 3 simulation are stable, i.e., the eigenvalues of A(t) have negative real parts.

Definitions and theorems are provided in Appendix C which discuss the use of suitable Lyapunov functions to investigate the stability of the nonautonomous, linearized system. In general these methods did not yield good results because of the requirement for numerical differentiation.

Finite time stability. Articulated vehicles execute the commonly encountered maneuvers such as cornering and lane change in a short and finite interval of time. However, the classical definition of stability in the sense of Lyapunov involves an infinite interval of time. Stability in this sense

is then appropriate for maneuvers that are executed over a relatively long interval of time, as for example, a maneuver where the translation of the vehicle is described by a long circular arc; it may not be meaningful for lane change and cornering maneuvers. Also, the classical definition does not admit persisting perturbations in the inputs, whereas the actual steering and braking inputs by the driver are nearby to the nominal inputs. Hence, it may be preferable to employ a more practical concept of stability called "finite time stability". A motion is called finite time stable if it lies within prescribed bounds over a specified finite interval of time and is unstable if it does not.

Kamenkov[52] and Lebedev[53] were among the pioneers in introducting the concept of finite time stability in 1953 and 1954, respectively. Their papers appeared in untranslated versions of Prikladnaya Matematica i Mekhanika; however, some description of their work is given by Hahn[55]. The ideas were later extended by Chzhan-Sy-In[56,57] and Dorato[49] for linear systems. Weiss and Infante[47,48,58] generalized the concepts and proved several new theorems. They used Lyapunov-like functions whose properties differ significantly from those of classical Lyapunov functions. In particular, there is no requirement of sign definiteness on such functions or their time derivatives. Definitions of finite time stability are given in Appendix C. The perturbations in the state variables and inputs about a nominal motion $\underline{q}^*(t)$ were obtained earlier and are given in Equation (6).

Finite time stability in the small. In the investigation of finite time stability in the small, it is assumed that the perturbations about the nominal motion and in the nominal inputs are sufficiently small and the higher order terms in the Taylor series expansion, i.e., $\underline{h}(\underline{x}, \Delta\underline{u}, t)$ in Equation (6) are dropped. The stability of the linear approximation of the

perturbation equations,

$$\dot{\underline{x}} = A(t)\underline{x} + B(t)\Delta\underline{u} \qquad (14)$$

is investigated. In case the nominal inputs are not perturbed, Δu is set equal to zero in Equation (14). Several methods involving quadratic V-functions described in Appendix C were employed to investigate finite time stability in the small of nonautonomous maneuvers. These methods all showed very conservative results not useful to the establishement of articulated vehicle stability theory.

4.1.4 Lyapunov Stability in the Large of Autonomous Maneuvers

Stability in the large was investigated for steady state motions. A Lyapunov function and the equations of perturbed motion were used to obtain the region of asymptotic stability. This region yields the magnitude of the excursions that are permissible in the variables (such as side velocity, yaw angles and their rates) to guarantee asymptotic stability of the nominal motion. The goal in this analysis is to interpret the size of the region of stability with respect to the side force versus slip angle relationship as shown in Figure 6.

The region of stability for a maneuver represented by point A of Figure 6 would be larger than that represented by point B. Point C would represent asymptotic stability in the small but the region of stability could be so small that for practical purposes it may be considered as unstable.

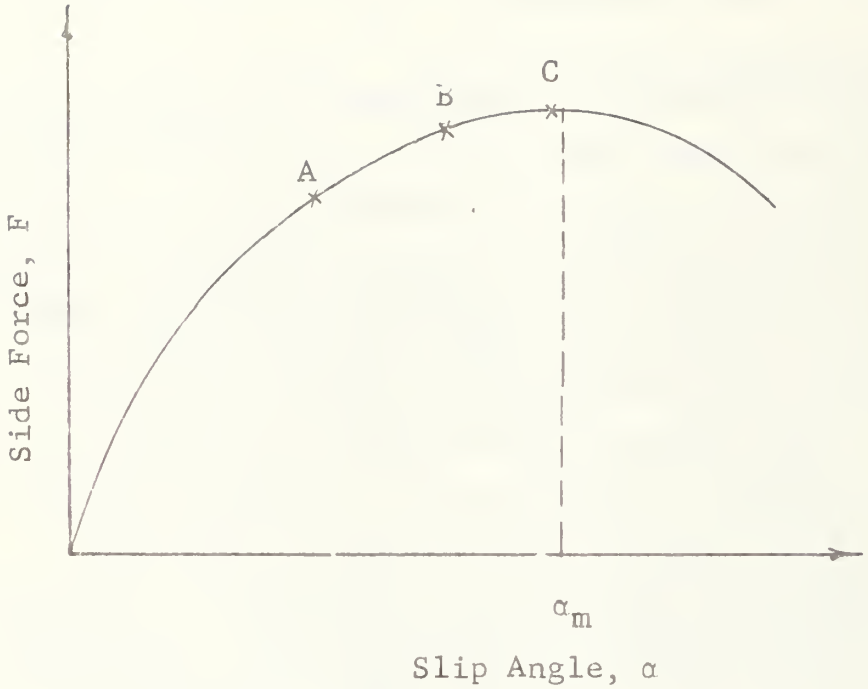


Figure 6. Tire side force as a function of slip angle.

Lyapunov stability in the small of autonomous maneuvers was discussed in an earlier section by assuming that the perturbations in the state variables are sufficiently small during the nominal motion and that the nominal inputs are not perturbed.

For Lyapunov stability in the large of autonomous maneuvers, the nominal inputs are not perturbed, but it is not assumed that the perturbations in the state variables are small. The perturbations are described by the nonlinear autonomous Equation (11).

The objective is to determine the size of the region of asymptotic stability, also known as the domain of attraction, by application of the second method of Lyapunov. A large number of methods have been investigated for the generation of Lyapunov functions. Most of the methods are not very general and are restricted to very special system configuration such as a single nonlinearity of a special kind. The method of Lucre', which is discussed in detail by Lefschetz[27], is applicable to a very restricted class of feedback systems with a single nonlinearity in the loop. In the variable gradient method developed by Schultz and Gibson[42,59], the gradient of V is assumed, $\dot{V}$ is obtained and constrained to be at least negative semidefinite, and the V-function is obtained by line integration.

The variable gradient method presents a great number of alternatives in choosing a V-function and it is not known a priori which alternative would yield the largest domain of attraction. A serious disadvantage is that the V-function generated is not usually a quadratic form. Sylvester's theorem[60] is applicable to the determination of sign definiteness of quadratic functions, but for general nonquadratic functions such techniques are not available.

The stability of artificial satellites has led to a number of studies of the stability of steady motion of the mechanical system of rigid bodies which may also include some flexible parts. Pringle[31], Meirovitch[32,33] and others have studied this problem. Firstly, an expression is obtained for the Hamiltonian function in generalized coordinates and generalized momenta coordinates. For a steady state nominal motion, the coordinates have constant values. The Hamiltonian is perturbed about this steady state motion and the Hamiltonian in the perturbations is used as a Lyapunov function. Stability conditions are obtained by restricting the Hamiltonian in the perturbations to be a positive definite function and its time rate of change to be a negative semidefinite function. In the case of articulated vehicles, the Hamiltonian and its time rate of change are not quadratic functions. The use of the Hamiltonian as a Lyapunov function does not yield any useful results because of the absence of a criterion to determine the sign definiteness of general nonquadratic functions.

The foregoing techniques and other methods reported in the literature for the construction of Lyapunov functions were examined in order to determine their suitability for the study of stability of maneuvers of articulated vehicles. The conclusion is that the methods have various shortcomings for complex problems. For engineering problems, closed-form expressions defining the stability boundary can be rarely obtained. What is needed is an effective computational procedure for estimating the domain of attraction. For the study of stability in the large of maneuvers of articulated vehicles, estimation of the domain of attraction is the only suitable technique other than simulation. The Zubov method[34] and the optimal quadratic estimation method[61] are the two methods which are amenable to machine computation for estimating the domain of attraction. These are discussed in detail in Appendix C.

All methods of analytically obtaining a Lyapunov function are not useful for the articulated vehicle study. The only systematic numerical method available for the generation of Lyapunov functions was discussed in detail. The approach recommended for estimating the domain of asymptotic stability of autonomous maneuvers of articulated vehicles is the quadratic estimation method which is outlined in Appendic C. This method was not pursued in this study because of limited applicability, cornering at a constant velocity, and associated complex computational problems.

4.2 PHYSICAL STABILITY

Physical stability criteria were developed through the examination of articulated vehicle response. The AVDS 3 simulation model was modified to obtain stability limits based on bounded articulated vehicle response. The limits thus developed imply an ideal driver and vehicle performance. The criteria for vehicle stability is based on available vehicle control (braking and side) forces. This stability concept is implemented through the use of the friction ellipse concept at the individual tires at each instant during the maneuver.

4.2.1 The AVDS Model

The AVDS 3 model features a transient solution of the nonlinear equations of motion of the single-, double-, and triple-tractor trailers with the following independent degrees of freedom:

tractor forward motion $\dot{V}_{x_1}$

tractor lateral motion $\dot{V}_{y_1}$

tractor yaw angle ψ_1

first semitrailer yaw angle ψ_2

first dolly yaw angle ψ_3

second semitrailer yaw angle ψ_4

second dolly yaw angle ψ_5

third semitrailer yaw angle ψ_6

The model emphasizes the dynamic response of the vehicle in its normal road environment. It simulates the dynamic response and evaluates the stability of single-, double-, and triple-trailer articulated vehicles of varied geometry and load distribution subject to the following:

- Tractive and braking forces which can be distributed as desired.
- Nonlinear cornering forces described by an empirical model developed by IITRI which is constrained by a tire cornering-tractive force relationship as indicated by the friction ellipse concept.
- Steady and transient aerodynamic forces.
- Fifth wheel friction and damping.
- Prescribed road maneuvers including grades and banks.

The computational strategy used is an inverse response calculation whereby the forward and lateral motion of the tractor is specified at small time increments and the required values of steering angle and braking forces are computed.

4.2.2 Physical Stability Criteria

Stability limits were computed and experimentally verified based on the safe handling operation of articulated vehicles. Limits described in Sections 6 and 7 are based on physical stability criteria.

Figures 7 and 8 show the two modes of instability common to articulated vehicles. Jackknifing occurs when the tractor swings around the kingpin because of a loss of side force at the rear wheels. Trailer swing, the second mode of instability, is characterized by a large yaw angle; it occurs as a result of a partial loss of side force at the trailer wheels.

In any case, the sudden loss or lack of sufficient tire side force is the cause of articulated vehicle instability. This condition may occur during partial skidding when braking is applied or because of a total slip of the wheel caused by tire/road interaction. No universally accepted figure of merit on allowable yaw angles for articulated vehicles has been established to date; however, large "controlled" responses in the form of trailer swing are deemed undesirable on the highway. For tractor double-semitrailer vehicle handling ability there are many modes of "controlled" responses or instability due to the length and complexity of this vehicle.

In establishing stability and handling criteria the difference between vehicle instability and safe operation must be emphasized. A vehicle can be stably operational on the highway and cause an accident with a large trailer swing. Allowable trailer swing depends to a large extent on the length and width of the vehicle, and the maneuver geing performed. Therefore, no allowable values on trailer swing have been established herein. To establish such limits, a considerable parameter study will have to be conducted to determine allowable trailer excursions in given road situations.

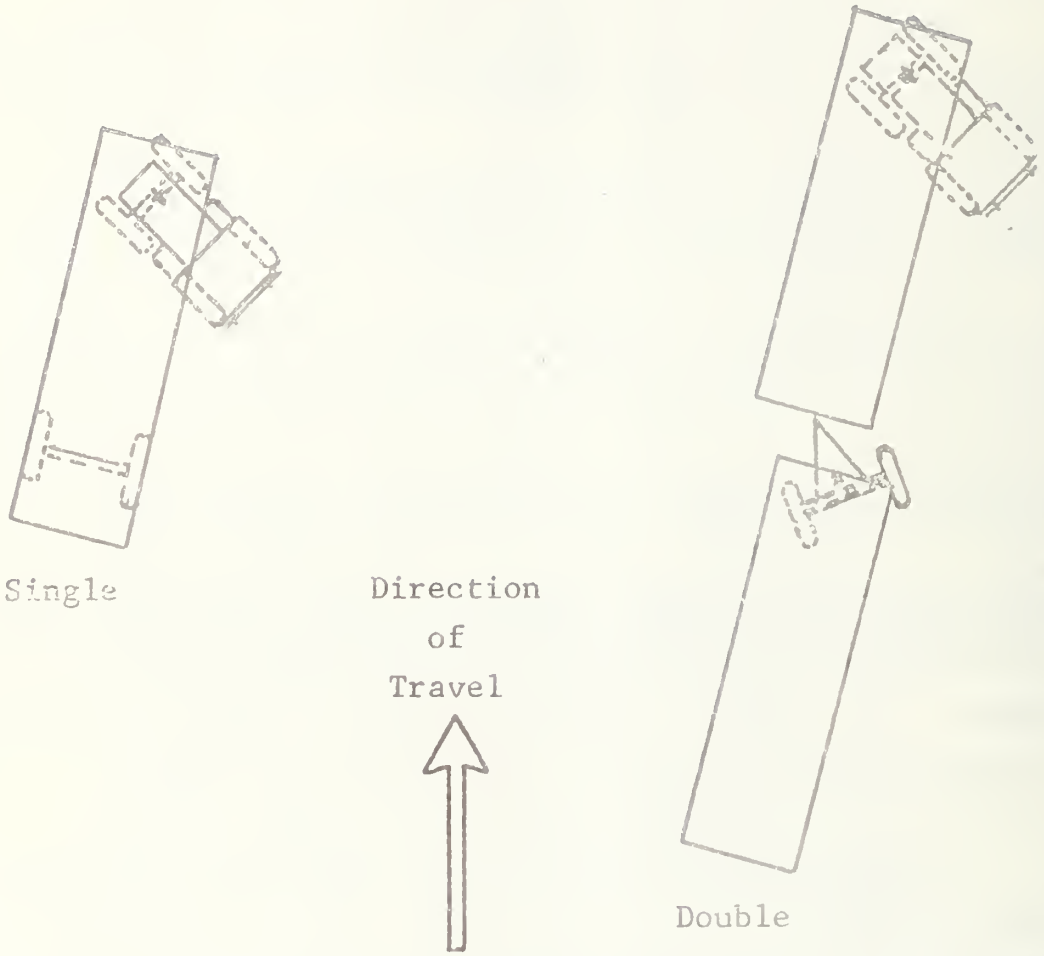


Figure 7. Articulated vehicle jackknifing.

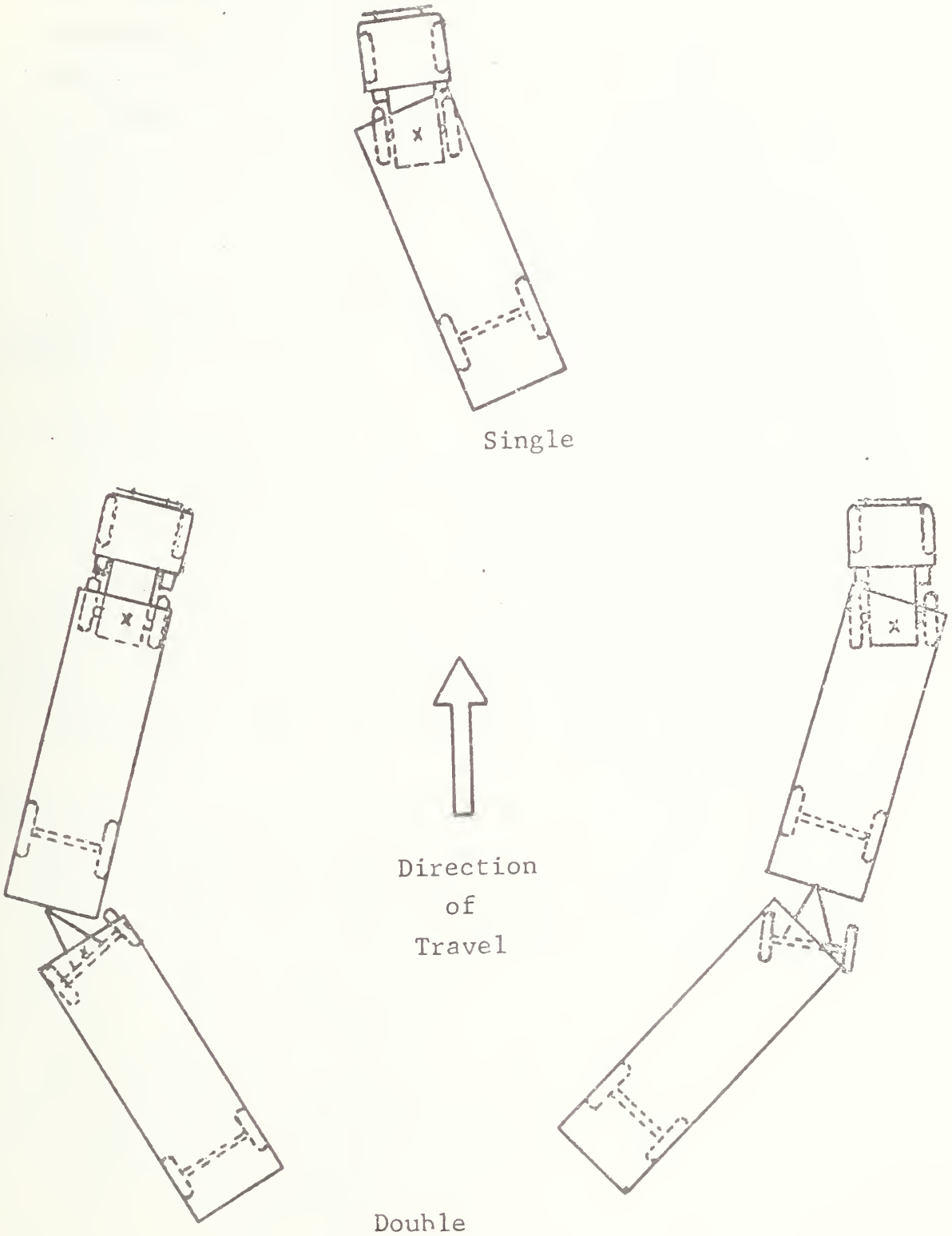


Figure 8. Articulated vehicle trailer swing.

A nonallowable tire skid constitutes a meaningful conservative criterion for articulated vehicle stability. If vehicle parameters, speed, braking, environmental influences and/or maneuvering combine to create uncontrollable vehicle side slip jackknifing or unusually large trailer response, then the vehicle is unstable. For the purposes of this study, the loss of tire side or tractive/braking force has been chosen as the stability criterion. Stability limits based on this premise are contained in Section 7. In order to assess the stability of a vehicle performing a prescribed maneuver at a given speed with or without braking on a prescribed road surface, a time transient simulation using the AVDS3 model is performed. At each time step in the simulation, all tire cornering and braking/tractive forces are checked to determine whether the friction ellipse force apportionment is violated. If it is violated, the time step is recalculated with allowable side force to determine whether or not the side force required to keep the vehicle stable will be reapportioned according to the reserve potential of the opposite tire. If the opposite tire is not capable of absorbing more side force, the yaw response will abruptly increase. This condition is judged to constitute instability.

The essence of articulated vehicle stability and handling lies in the efficient energy dissipation at the tires, with the interaction of side force generation and braking force being a major crucial unknown.

5. EXPERIMENTAL VERIFICATION

Full-scale experimental work was conducted to assess the validity of mathematical modeling assumptions, to validate the AVDS3 model for computation of the dynamic response of tractor double-semitrailers, and to develop stability and handling limits for articulated vehicles. Two series of tests were conducted involving three vehicle configurations and two drivers. The trajectory method of testing was used in which the driver performs a prescribed maneuver as fast as safely possible.

5.1 Test Strategy

The test strategy adopted in this program involves the use of selected full-scale test to develop computer simulation capabilities. Cornering, lane changing and evasive maneuvers were selected and simulated with AVDS3 prior to testing to insure safety within the constraints of the track space. In addition, levels of approximate measured response values were obtained for use in setting instrumentation levels. On a given maneuver, a goal was set for the driver. This goal involved the trajectory shape and speed with or without braking for a limit maneuver. The driver then attempted to perform this maneuver at the highest possible safe speed (without instability). This trajectory was marked and recorded along with all other response measurements. The analog results of these tests were used to compare vehicle response to AVDS3 model simulations and the peak results were compared to stability limits generated analytically. All tests were intended to be limit maneuvers where tire skid was the criterion for test acceptability. The goal of the driver was to run tests at maximum safe speed with minimum tire skid.

5.2 Test Equipment and Facilities

Tests were conducted on the skid pad at Bendix Automotive Development Center in late October 1972 and early April 1973. In the first series of tests, a Diamond Reo tractor was used with a 45 ft trailer van with tandum axles. The wheels of one axle were removed for single-axle tests and the van was loaded with gravel boxes half loaded and fully loaded for the tests. The second series of tests was conducted with a Diamond Reo tractor, two 27 ft trailer vans and a dolly. Both vans were single-axle configurations. In the first series, mixed tires were utilized. In the second series, Goodyear Super Hi-Miler tires were used on the entire vehicle except for some initial experiments with mixed Firestone and Goodyear tires (Figure 9). The physical data for the test vehicles is given in Table 2.

Since the experimental verification tests were used to verify and develop the mathematical model, the vehicle was fully instrumented. Table 3 shows the measured variables and transducers used in the first series of tests. Data were recorded for both modeling and stability limit verification with these transducers. Table 4 shows the transducer arrangement for the second test series which was used mainly to verify AVDS3 for double articulated vehicles and to establish stability limits. All data were recorded on a magnetic tape recorder which was mounted in the tractor bunk space. A voice channel was recorded on all test records. A detailed description of the instrumentation and test equipment is given in Appendix D.

5.3 Driver Performance

In both series of tests more data were obtained on driver performance through recorded steering angles. Different drivers were used in the two series of tests and the steering characteristics of each are shown in Figure 10 compared to the AVDS3 synthesized steering for lane change maneuvers.

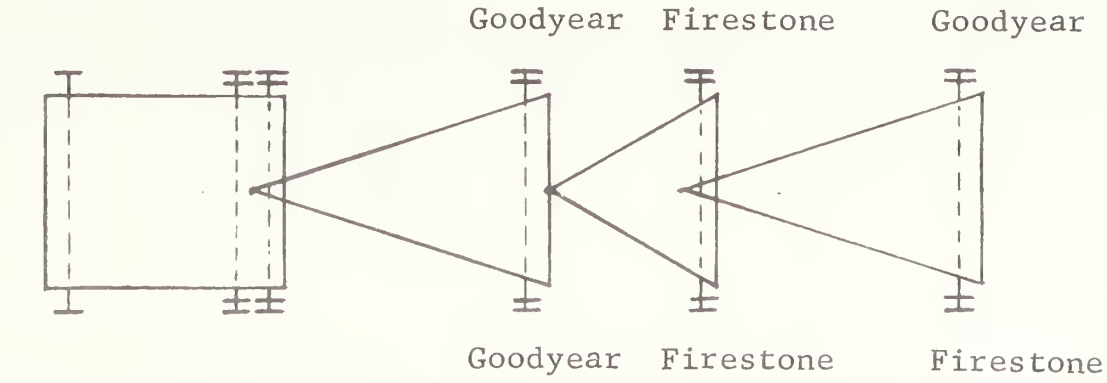


Figure 9. Mixed tire configuration for tractor double-semitrailer systems.

Table 2

ARTICULATED VEHICLE PHYSICAL CHARACTERISTICS

Parameter	Tractor 1	Tractor 2	Trailer 1	Trailer 2	Dolly
Weight (lb)	14155	17190	11175	7560	2600
Inertia-Yaw (slug-ft^2)	10400	16650	53900	25000	1500
Length between axles (ft)	12.25	14.24	-	-	-
Length between rear axle and c.g. (ft)	7.78	7.24	7.71	7.29	0.80
Length between c.g. and fifth wheel (ft)	6.61	6.62	-	-	0.55
Height of c.g. above axle (ft)	0.65	0.77	5.67	4.65	0.83
Height of c.g. above fifth wheel (ft)	-0.69	-0.43	-	-	-0.05
Location of trailer c.g. from kingpin	-	-	25.60	12.79	-
Trailer Length (ft)	-	-	45	27	-
Length between c.g. and trailer-dolly pin connection (ft)	-	-	-	10.54	5.53
Height of c.g. above trailer-dolly pin connection (ft)	-	-	-	3.52	-0.29
Suspension stiffness (lb/ft)	40000 (front) 120000 (rear)	40000 (front) 120000 (rear)	60000	60000	60000

Table 3

ARTICULATED VEHICLE EXPERIMENTAL VERIFICATION
MEASUREMENTS - TEST SERIES NO 1

Measured Variable	Transducer	No. of Channels	Test* Type
Steering	Rotary potentionmeter	1	1,2
Brake signal	Brake electrical signal monitor	1	1,2
Tractor forward and lateral acceleration	Inertial reference system - accelerometers	2	1,2
Tractor yaw, roll and pitch angles	Inertial reference system - gyro stabilized platform	3	1,2
Tractor position	Pulsed trajectory marker	1	1,2
Tractor velocity	Fifth wheel	1	1,2
Tractor-trailer yaw angle	Rotary potentionmeter	1	1
Trailer forwarded lateral acceleration	Biaxial accelerometer	2	1
Trailer yaw, and roll and pitch angles	Free gyro	2	1,2

*Test type: 1. Modeling
2. Stability limit verification

Table 4

ARTICULATED VEHICLE EXPERIMENTAL VERIFICATION

MEASUREMENTS - TEST SERIES NO. 2

MEASURED VARIABLE	TRANSDUCER	NO. OF CHANNELS
Steering	Rotary potentiometer	1
Brake signal	Brake electrical signal monitor	1
Tractor forward and lateral acceleration	Inertial reference system - accelerom-eters	2
Tractor yaw, roll and pitch angles	Inertial reference system - gyro stabilized platform	3
Tractor position	Pulsed trajectory marker	1
Tractor velocity	Fifth wheel	1
Tractor-trailer yaw angle	Rotary potentiometer	1
Trailer forward and lateral acceleration	Biaxial accelerometer	2
Dolly yaw	Free gyro	1
Dolly-Trailer yaw angle	Rotary potentiometer	1

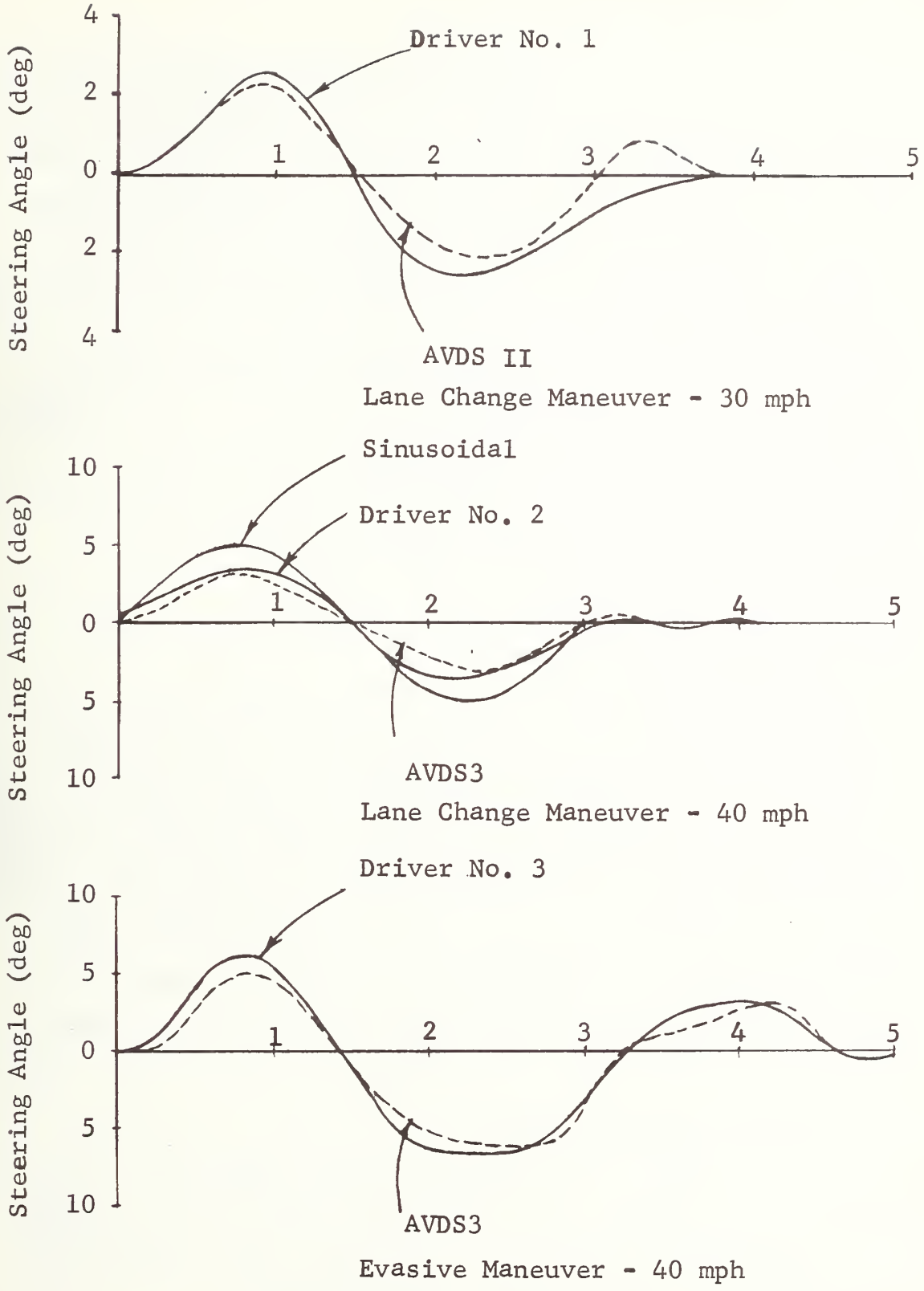


Figure 10. Driver characteristics.

In addition the driver characteristic shown earlier by Eshleman and Desai is repeated in Figure 10. It is apparent from tests results that among these three professional drivers, an efficient mode of steering is followed with the steering approximating a sine wave. Two major diversions from the sine wave are noted, first being the finite slope of the wave at maneuver initiation. The actual driver and simulation (optimum) steering starts at zero slope and eases into the sinusordal shape curve. The second diversion (both driver and simulation) is that the second steering loop is greater than the first. This is caused by the fact that the vehicle is starting the second portion of the maneuver with a different set of dynamic initial conditions. At the beginning of the maneuver the vehicle had no yaw or lateral acceleration; however, at the change of direction of the lane change it does have initial accelerations in the opposite direction to overcome.

Examination of the driver characteristics shown in Figure 10 (others are contained in Section 6) reveals that driver 1 followed the computer simulation closely except at the end of the maneuver where he gradually steers to reduce tractor-trailer yaw, whereas the computer reduces the time frame. This driver was not instructed to perform a limit maneuver. The results for drivers 2 and 3 show close agreement with computer simulation throughout the maneuver. It was found that the professional driver has control of his vehicle at the stability limit where tire slip angles are high.

5.4 Discussion of Test Results

The test data for the experimental verification tests are summarized in Tables 5 and 6. Peak values of the vehicle response variables are listed for cornering, lane changing and evasive maneuvers. The response variables are:

- Tractor forward acceleration.
- Tractor lateral acceleration.
- Tractor yaw, pitch and yaw angles.
- Steering.
- Tractor-trailer yaw angles.
- Dolly yaw angles.
- Dolly-trailer yaw angles.

The test results shown in Table 5 are grouped according to maneuver.

- 75 ft radius cornering
- 200 ft radius cornering
- Lane changing at speeds varied between 32 and 45 mph.

Within these groups, the individual tests reflect varied load, road surface, and axle combinations. In the cornering tests, the tractor yaw angles are not given because they were measured with respect to an initial reference axis and show yawing due to the maneuver geometry plus the actual vehicle yaw. The tractor-trailer relative yaw angle provides a more significant measure of the vehicle's yawing motion. In all cases, peak forward accelerations are given when the vehicle employs braking during the maneuver.

In the 75 ft radius cornering maneuvers of test series No. 1, a peak of 0.46 g lateral acceleration was obtained for an unloaded tandem axle configuration. Roll and steering angles had peak values of 4.9 and 12.5 deg respectively. These values agree well with the stability limits generated in Section 7. In general the cornering limits decreased only slightly for wet surface.

Table 5

SUMMARY OF EXPERIMENTAL RESULTS FROM ARTICULATED

VEHICLE LIMIT MANEUVERS (peak values)

Maneuver Description		Axle/Load Conf.*	Road Cond.	Tractor							Tractor-Trailer Yaw Angle (deg)	Experiment No.
				Nominal Speed (mph)	Lat. Acc. (g)	For. Acc. (g)	Steer. Angle (deg)	Yaw Angle (deg)	Roll Angle (deg)	Pitch Angle (deg)		
Corner 75 ft Radius (Nominal)		T/U	Dry	22.2	0.46	---	9.1	---	4.9	---	20.8	14
		S/U	Dry	20.0	0.35	---	11.0 (±1.5)	---	2.9	---	24.0	15
		S/U	Wet	17.5	0.28	---	10.3	---	2.6	---	26.2	21
		T/20	Wet	21.5	0.37	---	12.4 (±2.0)	---	3.3	---	27.9	23
		T/10	Wet	20.0	0.36	---	11.5 (±1.5)	---	3.8	---	25.9	37
Corner 200 ft Radius (Nominal)		S/U	Dry	36.2	0.47	---	4.1	---	3.2	---	9.0	16
	B**	S/U	Dry	36.2	0.43	0.53	3.4	---	2.5	1.2	9.5	17
		T/20	Dry	32.4	0.35	---	5.6	---	2.8	---	8.1	24
	B	T/20	Dry	33.0	0.38	0.41	5.3	---	2.5	---	10.3	25
	B	T/10	Dry/Wet	37.5	0.42	0.23	5.6	---	3.7	0.7	9.3	35
	B	T/10	Dry/Wet	37.5	0.36	0.40	4.0	---	1.8	1.1	8.9	36
14'x145'		T/U	Dry	37.7	0.34	---	3.9	10.8	1.7	---	5.2	9
12'x208'		T/U	Dry	44.2	0.32	---	3.4	7.9	1.6	---	4.1	10
14'x221'	B	T/U	Dry	45.2	0.28	0.51	2.5	9.5	1.5	1.0	4.7	12
12'x	B	T/U	Dry	45.0	0.29	0.49	2.9	9.9	1.6	1.1	4.8	13
12'x120'		S/U	Wet	37.1	0.28	---	2.9	9.2	1.6	---	4.0	19
12'x133'		T/20	Dry	39.4	0.25	---	3.9	9.8	1.9	---	3.6	28
12'x160'	B	T/20	Dry	38.1	0.23	0.4	4.0	9.6	1.5	1.1	5.6	30

* S = single axle
T = tandem axle
U = unloaded trailer
10 = 10,000 lb load
20 = 20,000 lb load

** B = braking

Table 6

EXPERIMENTAL RESULTS FOR TEST SERIES NO. 2

Measured Variable	Channel	Units	1	2	3	4	5	6	7	8	9	10	11	12	13
Tractor Forward Acceleration	1	ft/sec^2			-0.353	-0.522		+0.099 -0.323		-0.420		-0.40		-0.32	
Tractor Lateral Acceleration	2	ft/sec^2	+0.191 -0.254	+0.242 -0.235	-0.026 +0.026	+0.078 -0.076	+0.202 -0.268	+0.100 -0.184	+0.265 -0.308	+0.206 -0.245	+0.378		+0.29 -0.30	+0.25 -0.18	+0.31 -0.29
Tractor Yaw	3	deg	-9.34	-10.10 +9.38		-3.09	-10.0	-10.21	-9.28 +8.66	-8.97 +10.93	-19.9		-10.14 +1.99	-8.76	-11.42
Tractor Roll	4	deg	+1.43 -1.53	+1.80 -1.35		+0.313	+2.23 -1.57	+1.88 -1.15	+2.12 -2.16	+2.16 -1.50	+3.23		+2.48 -2.48	+1.72 -1.52	+2.84 -2.16
Dolly Yaw	5	deg	-8.08	-8.86 +8.06	-0.805	+1.03 -1.98	-8.95	-8.47	-8.32 +7.44	-8.00 +9.03	-19.96		-9.74 +3.09	-8.08	-11.64 +1.20
Steering	6	deg	-4.71 +4.86	-6.43 +5.43	+1.14 -3.10	+2.23 +0.216	-5.69 +4.68	-3.88 +4.32	-5.23 +5.08	-4.90 +4.43	-15.41		-5.73 +5.84	-4.90 +3.99	-6.15 +5.62
Tractor-Trailer Yaw	7	deg	-2.66 +3.22	-3.63 +4.19		-1.31 +0.51	-3.12 +2.64	-4.22 +2.61	-2.26 +2.98	-2.70 +3.66	-12.32		-2.51 +3.56	-3.08 +3.08	-2.72 +3.59
Dolly-Trailer Yaw*	8	deg	-2.87 +1.91	-3.56 +3.07		-0.914 +1.60	-2.96 +1.60	+0.614 -2.88	+0.50 -1.34	-3.29 +2.63	-11.56	-0.44	-3.48 +3.62	-3.04 +1.10	-4.06 +3.66
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.215	+0.204			+0.273	+0.246	+0.286	+0.275	+0.389		+0.30	+0.26	+0.35
Trailer Lateral Acceleration, 2**	10	ft/sec^2	+0.230 -0.199	+0.207 -0.246			+0.311 -0.287	+0.297 -0.154	+0.284 -0.377	-0.237	+0.381		+0.35 -0.41	+0.26 -0.11	+0.42 -0.41
Speed	11	mph	33.10	42.86	45.71	40.91	39.09	38.18	39.09	39.09	19.54	36.9	36.4	37.3	38.7
Load E=empty F=full H=half		---	E	E	E	E	E	E	E	E	E	F	F	F	F
Braking		---	No	No	Yes	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
Nominal Speed		mph	40	40	40	40	40	40	40	40	-	40	40	40	40
Surface W=wet D=dry		---	W	W	W	D	D	D	D	D	D	D	D	D	D
Maneuver***		---	LC	E	SLB	SLB	LC	LC	E	E	C	SLB	LC	LC	E

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 6 (contd)

Measured Variable	Channel	Units	14	15	16	17	18	19	20	21	22	23	24	25	26
Tractor Forward Acceleration	1	ft/sec^2		-0.33				-0.375		-0.450	-0.424			-0.350	
Tractor Lateral Acceleration	2	ft/sec^2	+0.33 -0.40	+0.28 _0.23	+0.40	+0.262	+0.357		+0.312 -0.284	+0.215 +0.096	+0.282	+0.293 -0.370			+0.299 -0.359
Tractor Yaw	3	deg	-10.28 9.34	-10.21 +10.41	-19.07	-16.29	-19.87	-1.55	-10.83	-9.28	-9.18	-9.79 +8.14			-11.83 +1.76
Tractor Roll	4	deg	+2.57 -3.59	+1.98 -1.87	+8.40 -5.68	+2.88	+4.16		+2.52 -2.00	+1.56 -4.00	+1.52	+2.20 -3.52			+2.24 -3.68
Dolly Yaw	5	deg	-10.30 +10.77	-9.42 +8.55	-17.34	-14.81	-18.71	+0.55 -0.94	-11.71 1.65	-7.70	-7.15	-9.98 +8.65		-0.634	-11.72 +4.12
Steering	6	deg	-6.33 6.44	-4.75 +5.09	-14.84	-13.85	-14.7	+1.80	-5.41 +5.67	-3.64 +2.54	-3.97 +3.64	-5.45 +5.70		+4.79	-5.99 +7.26
Tractor-Trailer Yaw	7	deg	-2.46 +4.31	-3.66 +4.27	-16.42	-15.32	-12.85	-0.697	-2.46 +3.71	-3.71 +2.68	-2.92 +4.48	-2.18 +4.11		-3.86	-3.68 +5.03
Dolly-Trailer Yaw*	8	deg	-3.66 +4.85	-3.47 +2.85	-14.87	-15.50	-15.38	-0.645	-4.00 5.42	-2.58 +1.68	-2.71 +1.49	-3.67 +5.07		+1.65 -0.506	-3.88 +7.13
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.31 -0.44	+0.30 -0.20	+0.27	+0.238	+0.316	+0.047	+0.276 -0.331	+0.211 -0.076	+0.165	+0.271 -0.373			+0.287 -0.327
Trailer Lateral Acceleration, 2 **	10	ft/sec^2	+0.35 -0.49	+0.31 -0.16	+0.30	+0.239	+0.342	+0.051	+0.393 -0.471	+0.203 -0.032	+0.194	+0.330 -0.439			+0.320 -0.367
Speed	11	mph	39.5	36.9	20.7	16.82	19.77	38.63	23.18	37.27	37.27	39.54		36.40	34.15
Load E = empty F = full H = half		---	F	F	F	F	F	F	F	F	F	F	F	H	H
Braking		---	No	Yes	No	No	No	Yes	No	Yes	Yes	No	Yes	Yes	No
Nominal Speed		mph	40	40	-	-	-	40	40	40	40	40	40	40	40
Surface W = wet D = dry		---	D	D	D	W	D	D	D	D	D	D	D	W	W
Maneuver***		---	E	E	C	C	C	SLB	LC	LC	LC	E	E	SLB	LC

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 6 (contd)

Measured Variable	Channel	Units	27	28	29	30	31	32	33	34	35	36	37	38	39
Tractor Forward Acceleration	1	ft/sec^2	-0.363		-0.502						-0.381		-0.452	-0.438	
Tractor Lateral Acceleration	2	ft/sec^2	+0.275 -0.136	+0.402 -0.355	+0.355 -0.202	+0.241	+0.417	+0.372 -0.366		+0.280 -0.368	+0.323 -0.244	+0.372 -0.422	+0.310 -0.297	+0.064	+0.387 -0.484
Tractor Yaw	3	deg	-0.882 +6.95	-13.18 +9.45	-12.87 +15.67	-15.88	-22.06	-12.06 +2.37		-11.78 +1.33	-9.63	-11.67 +10.96	-10.83	-7.94	-13.41 +1.74
Tractor Roll	4	deg	+1.72 -0.80	+2.88 -3.32	+2.68 -2.60	+2.94	+4.08	+2.80 -3.16		+3.20 -2.84	+1.91 -1.27	+2.68 -3.72	+2.44 -2.88		+2.70 -3.45
Dolly Yaw	5	deg	-16.0 +1.11	-13.23 +9.98	-13.70	-14.89	-19.87	-12.81 +4.48	-2.97	-13.13 +3.93	-9.44 +5.38	-12.73 +12.73	-12.28	-3.48	-14.23 +4.24
Steering	6	deg	-4.97 +3.74	-7.88 +7.88	-7.62 +6.79	-17.15	-17.36	-7.77 +7.95	+1.45 -0.327	-8.24 +7.26	-6.17 +4.64	-8.07 +8.21	-7.22 +6.61	+5.54	-7.41 +8.97
Tractor-Trailer Yaw	7	deg	-3.14 +6.43	-4.39 +5.50	-4.13 +5.92	-15.43	-15.43	-4.02 +4.72	-0.975 -0.663	-3.82 +5.03	-3.04 +3.63	-3.82 +5.54	-4.37 +5.23	-5.93	-6.55 +6.63
Dolly-Trailer Yaw*	8	deg	-10.22 +10.61	-5.28 +6.71	-6.46 +10.55	-15.71	-15.39	-6.21 +7.43	-2.58 +2.63	-6.80 +8.56	-5.21	-6.75 +7.88	-6.48 +8.43	-2.54	-6.34 +9.29
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.222 -0.156	+0.320 -0.356	+0.340 -0.201	+0.252	+0.404	+0.348 -0.465		+0.342 -0.412	+0.294	+0.339 -0.570	+0.318 -0.315		+0.234 -0.443
Trailer Lateral Acceleration, 2 **	10	ft/sec^2	+0.141 -0.239	+0.342 -0.410	+0.335 -0.220	+0.246	+0.420	+0.427 -0.517		+0.483 -0.531		+0.414 -0.552	+0.393 -0.299		+0.406 -0.525
Speed	11	mph	35.05	34.60	35.50	18.87	21.78	37.75	40.00	36.89	37.72	36.81	35.90	39.55	37.30
Load E = empty F = full H = half		---	H	H	H	H	H	H	H	H	H	H	H	E	E
Braking		---	Yes	No	Yes	No	No	No	Yes	No	Yes	No	Yes	Yes	No
Nominal Speed		mph	40	40	40	-	-	40	40	40	40	40	40	40	40
Surface W = wet D = dry		---	W	W	W	W	D	D	D	D	D	D	D	D	D
Maneuver***		---	LC	E	E	C	C	LC	SLB	LC	LC	E	E	SLB	LC

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 6 (contd)

Measured Variable	Channel	Units	40	41	42	43	44	45	46	47	48	49	50	51	52
Tractor Forward Acceleration	1	ft/sec^2	-0.478		-0.429 -0.548		-0.421		-0.394		-0.396		-0.487		-0.445
Tractor Lateral Acceleration	2	ft/sec^2	+0.314 -0.148	+0.400 -0.441	+0.347 -0.293	+0.247		+0.368 -0.268	+0.274 -0.118	+0.396 -0.385	+0.338 -0.278	+0.278		+0.333 -0.323	+0.310 -0.305
Tractor Yaw	3	deg	-9.48 +0.93	-13.62 +9.22	-10.04 +8.70	-18.15	-0.722 +0.619	-10.31	-14.43 +6.80	-11.86 +9.18	-10.21 +16.08	-20.62		-11.0	-7.47 +2.80
Tractor Roll	4	deg	+1.95 -1.27	+3.00 -3.92	+2.28 -2.36	+2.52		+2.36 -1.72	+1.48 -0.64	+2.24 -2.80	+2.04 -2.32	+3.40		+2.68 -2.44	+1.84 -0.40
Dolly Yaw	5	deg	-9.51	-14.93 +8.80	-10.22 +12.67	-15.84	+0.396 -0.317	-10.53	-24.79	-12.67 +7.76	-11.01 +17.11	-16.27		-9.43	-6.60
Steering	6	deg	-5.66 +3.32	-9.04 +7.41	-6.72 +5.06	-14.83	+0.731 -0.292	-6.05 +4.87	-4.48 +10.58	-6.84 +5.02	-5.76 +5.41	-15.47		-6.99 +6.35	-5.12 +2.61
Tractor-Trailer Yaw	7	deg	-5.57 +6.26	-7.18 +6.07	-5.77 +4.30	-14.31	-1.40 +0.53	-5.55 +2.46	-7.58 +8.69	-5.94 +4.06	-4.83 +8.69	-14.75	-1.07	-4.10 +5.12	-3.42 +7.12
Dolly-Trailer Yaw*	8	deg	-4.81 +7.30	-7.57 +8.33	-5.25 +7.02	-14.90	+0.711 -0.610	-4.72 +6.40	-18.29 +9.65	-6.05 +5.84	-5.44 +13.21	-14.40		+1.08 -0.538	+0.806
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.173 -0.322	+0.399 -0.476	+0.313	+0.220		+0.323 -0.272	+0.203	+0.334 -0.323	+0.312 -0.385	+0.287		+0.312 -0.294	+0.276
Trailer Lateral Acceleration, 2**	10	ft/sec^2		+0.487 -0.548	+0.359 -0.188	+0.250		+0.333 -0.299	+0.260 -0.192	+0.307 -0.371	+0.333 -0.414	+0.265	-0.534	-0.192	-0.495
Speed	11	mph	38.20	38.66	36.85	17.08	39.55	38.22	37.75	36.85	37.30	19.77	40.91	37.72	39.54
Load E = empty F = full H = half		---	E	E	E	E	E	E	E	E	E	E	E	E	E
Braking		---	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Nominal Speed		mph	40	40	40	-	40	40	40	40	40	-	40	40	40
Surface W = wet D = dry		---	D	D	D	W	W	W	W	W	W	W	D	D	D
Maneuver***		---	LC	E	E	C	SLB	LC	LC	E	E	C	SLB	LC	LC

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 6 (concl)

Measured Variable	Channel	Units	53	54	55	56	57	58	59
Tractor Forward Acceleration	1	ft/sec^2		-0.452		-0.276		-0.304	
Tractor Lateral Acceleration	2	ft/sec^2	+0.398 -0.415	+0.430 -0.368	+0.396 -0.387	+0.303 -0.178	+0.409 -0.378	+0.417 -0.327	+0.381
Tractor Yaw	3	deg	-12.77 +11.21	-11.63 +10.90	-11.96	-8.82 +9.45	-12.37 +7.73	-10.21 +15.88	-23.77
Tractor Roll	4	deg	+2.96 -3.16	+3.20 -2.92	+2.68 -2.20	+2.44 -0.76	+2.76 +2.80	+2.76 -2.00	+4.04
Dolly Yaw	5	deg	-10.45 +9.35	-9.27 +7.86	-10.06	-8.02 +1.73	-10.30 +6.60	-8.10 +7.62	-16.03
Steering	6	deg	-8.27 +7.04	-8.71 +6.74	-6.70 +6.53	-6.75 +2.48	-7.80 +6.70	-7.28 +5.09	-17.71
Tractor-Trailer Yaw	7	deg	-5.27 +5.71	-5.56 +6.10	-4.63 +5.42	-3.94 +10.28	-4.68 +5.77	-3.94 +10.85	-15.83
Dolly-Trailer Yaw*	8	deg	+1.25 -1.12	+1.03 -0.672	+0.844 -0.444	+0.933 -0.835	+1.07 -0.710	+0.835 -0.800	+1.97 -5.82
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.348 -0.407	+0.345 -0.330	+0.316 -0.276	+0.265 -0.290	+0.309 -0.276	+0.280 -0.454	+0.334
Trailer Lateral Acceleration, 2**	10	ft/sec^2	-0.367	-0.474	-0.192	-0.286	-0.192	+0.256 -0.265	
Speed	11	mph	37.27	40.91	40.45	39.09	39.54	41.36	32.72
Load E = empty F = full H = half		---	E	E	E	E	E	E	E
Braking		---	No	Yes	No	Yes	No	Yes	No
Nominal Speed			40	40	40	40	40	40	-
Surface W = wet D = dry		---	D	D	W	W	W	W	W
Maneuver***		---	E	E	LC	LC	E	E	C

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

The 200 ft radius cornering maneuvers show high lateral and forward acceleration. Test No. 17 shows a nominal speed of 36.2 mph with a lateral acceleration of 0.43 g and a forward deceleration of 0.53 g due to braking. Steering angles show values around 5 deg which indicate a probable reserve in steering. In these cornering experiments on wet and dry surfaces, roll angles peak at about 3.5 deg and pitch angles peak at about 1.0 deg. These values are similar in magnitude to those predicted by Mikulcik[15] in his analytical work.

The lane change maneuvers were performed at speeds ranging from 38 to 45 mph with and without braking. Lane changing peak lateral and forward accelerations of 0.34 to 0.51 were recorded. Steering angles of 4.0 deg indicate that steering was not critical in these maneuvers. Yaw, roll, and pitch angle peaks were 11.8, 1.9 and 1.1 deg respectively in this series of experiments and selected response records are shown in Section 6 where analytical-experimental validation is presented.

The second series of experimental validation tests are summarized in Table 6 and are grouped according to testing order while performing the following maneuvers.

- 82 ft radius cornering.
- Lane changing at 40 mph with and without braking.
- Evasive maneuvers at 40 mph with and without braking.

Forty-nine experiments were conducted on a tractor double-semitrailer unit and 10 experiments were conducted on a tractor semitrailer. In the test matrix, the following parameters were varied.

- Tires (mixed Firestone and Goodyear and all Goodyear were used).
- Load (empty, half, and full).
- Surface (dry and wet).
- Maneuvers (lane change (LLC), evasive (EM), and corner (CCW).
- Braking (straight line (SLB), and braking with maneuvers).

loaded configuration. In this series of experiments, deceleration due to braking ranged from 0.4 to 0.5 g and peak lateral accelerations as high as 0.48 g were obtained. Steering demands as high as 10 deg were observed.

Test No. 39, a lane change maneuver conducted at 37 mph, showed the peak lateral acceleration of 0.48 g with a 13.4 deg tractor yaw and a dolly yaw of 14 deg. This experiment showed trailer lateral accelerations of 0.4 to 0.5 g.

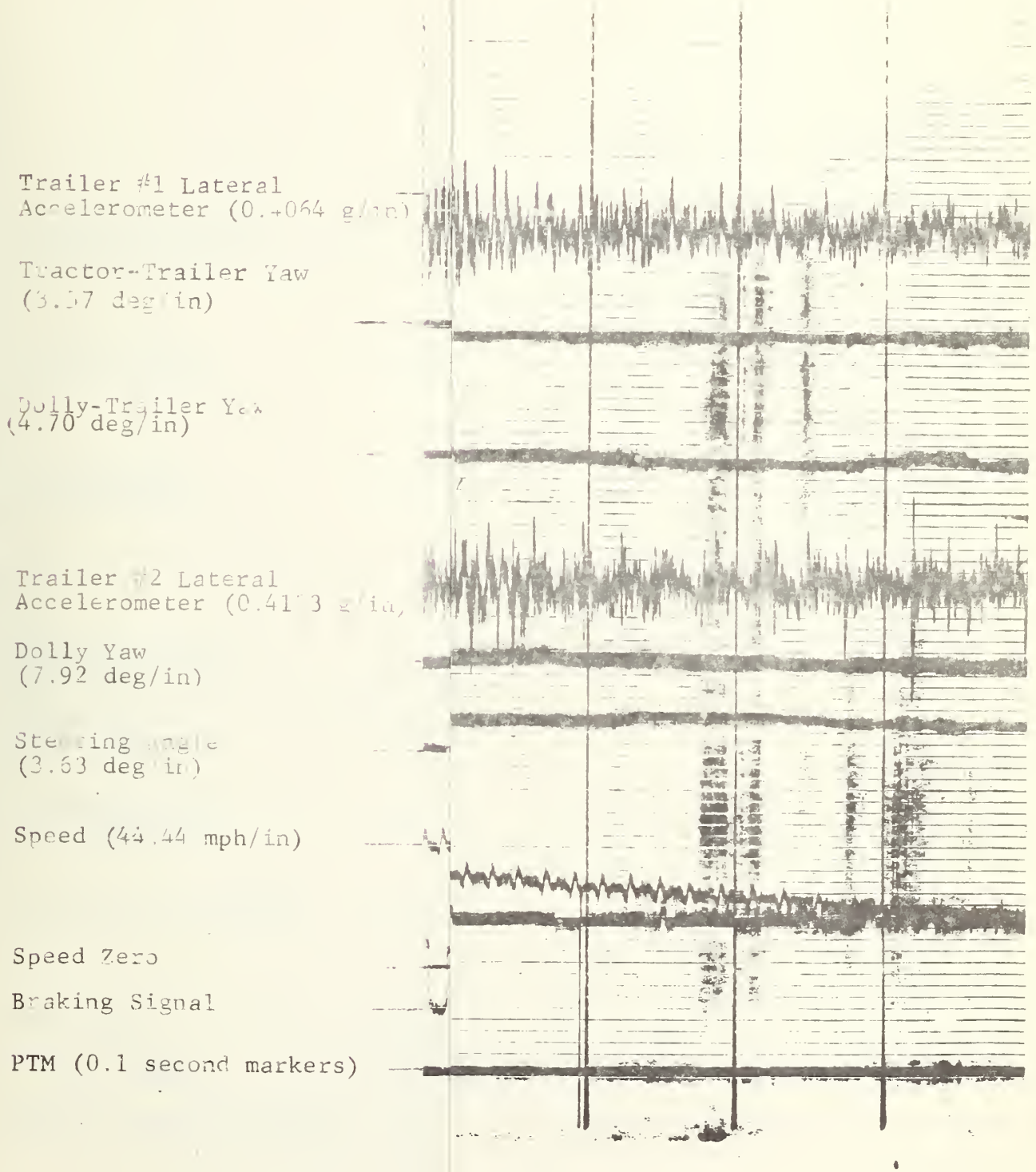
The final nine experiments, 50 to 59, were conducted with an empty single 27 ft trailer van. Peak lateral and forward accelerations are of the order of 0.4 g. In general, values of tractor yaw, tractor-trailer yaw and tractor roll do not appreciably change from those obtained with the double articulated vehicle which bears out earlier conclusions that the criticality of maneuver with a double unit lies in its required rattlespace and maneuverability time.

Results indicate that handling of tractor double-semitrailer vehicles is tedious on wet surfaces. Movies of the experiments show that control of the vehicle is lost at the dolly during severe braking resulting in jackknifing. In violent maneuvers with empty trailers it was found that large unallowable (in the sense of road operation) trailer swing was obtained. A typical record of these experiments, Figures 11 and 12, shows the order of 1/2 g peak lateral accelerations for lane change and evasive maneuvers. Figure 11 shows response results for a double articulated vehicle performing a lane change maneuver at 40 mph without braking. Figure 12 shows similar results for a double trailer vehicle unit in an evasive maneuver with braking. Specific experimental results are discussed more fully in Section 6, where comparison with simulations is shown.

Tests 1 through 16 were conducted with the combination of Firestone and Goodyear tires shown in Figure 9. A complete series of maneuvers was conducted with and without braking with an empty and fully loaded (according to industry) trailer. In the cornering maneuvers, it was found that the double 27 ft trailer unit could successfully follow an 82 ft radius corner at 20 mph on a dry surface, whether loaded or unloaded, and at 17 mph on a wet surface. Peak lateral accelerations of 0.4 g were recorded. In this series of tests, lane change and evasive maneuvers conducted with and without braking showed peak lateral and forward accelerations of 0.4 and 0.52 g respectively; however, the largest magnitude of the combination lateral and forward accelerations was obtained in test No. 8, an evasive maneuver. In this test forward and lateral accelerations of 0.42 and 0.25 g respectively were obtained. Tractor yaw and dolly yaw (absolute values) are about 9 deg, while relative yaw between the tractor trailer and dolly trailer is 3 to 4 deg.

Tractor roll angles typically are from 2 to 3 deg in evasive and lane change maneuvers and 8 deg in cornering maneuvers. Steering angles are peak at 6 deg for evasive and cornering maneuvers and 15 deg in cornering. The large steering requirements for cornering bear out the driver's observation that, in cornering, steering is critical.

The remaining tests were performed on Goodyear tires and the results for double unit tests are given in tests 17 through 49. It is interesting to note that results for cornering on a dry surface are similar to those obtained with the combination tires and that cornering with a fully loaded front trailer and empty rear trailer yielded higher limits (19 mph wet and 22 mph dry), whereas a totally empty unit on a wet surface had a limit of 17 mph which is similar to that of the



record of a double articulated vehicle lane change maneuver.

Tests 1 through 16 were conducted with the combination of Firestone and Goodyear tires shown in Figure 9. A complete

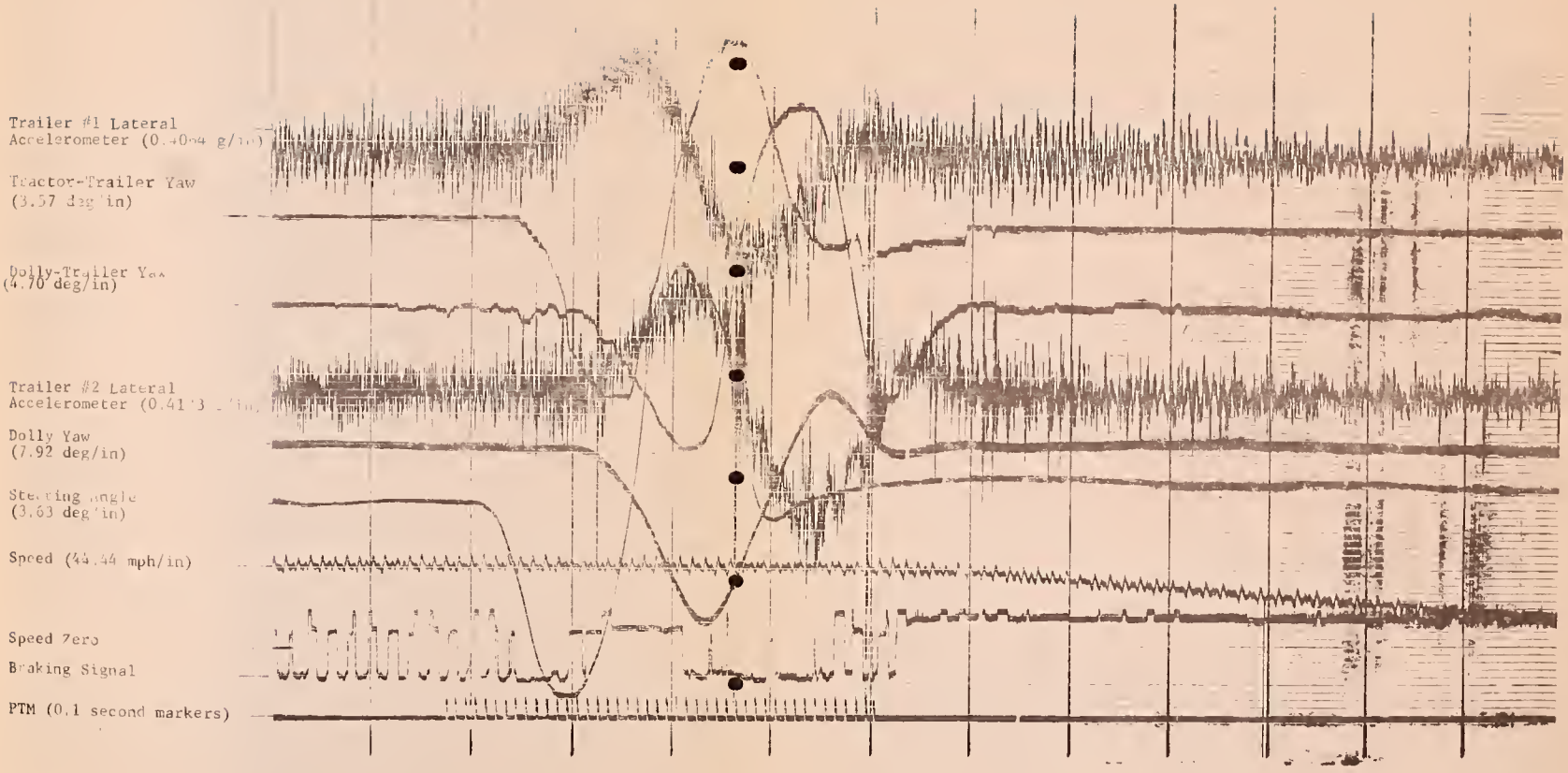


Figure 11. Experimental record of a double articulated vehicle performing a lane change maneuver.

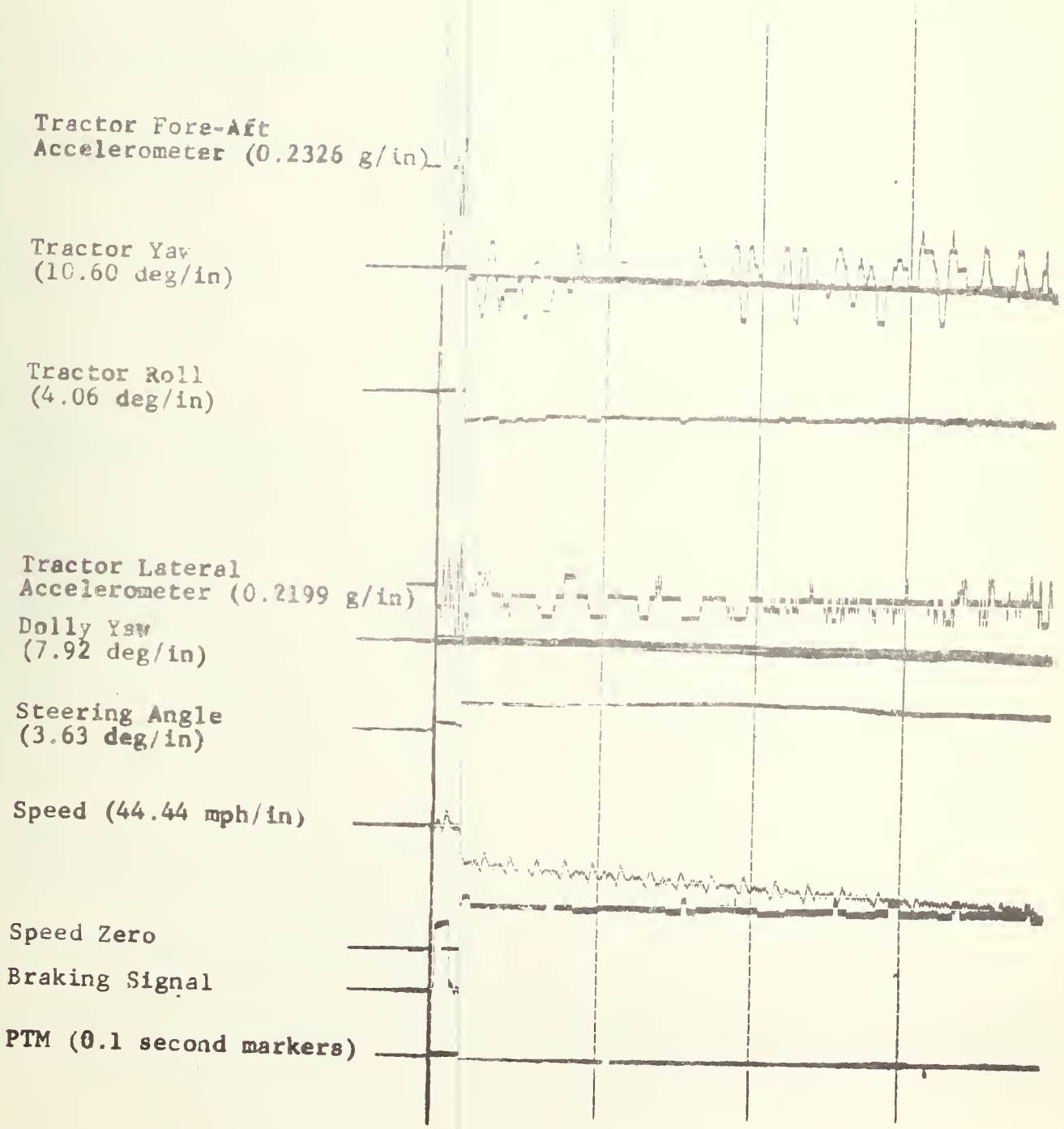


Figure 11. (concl)

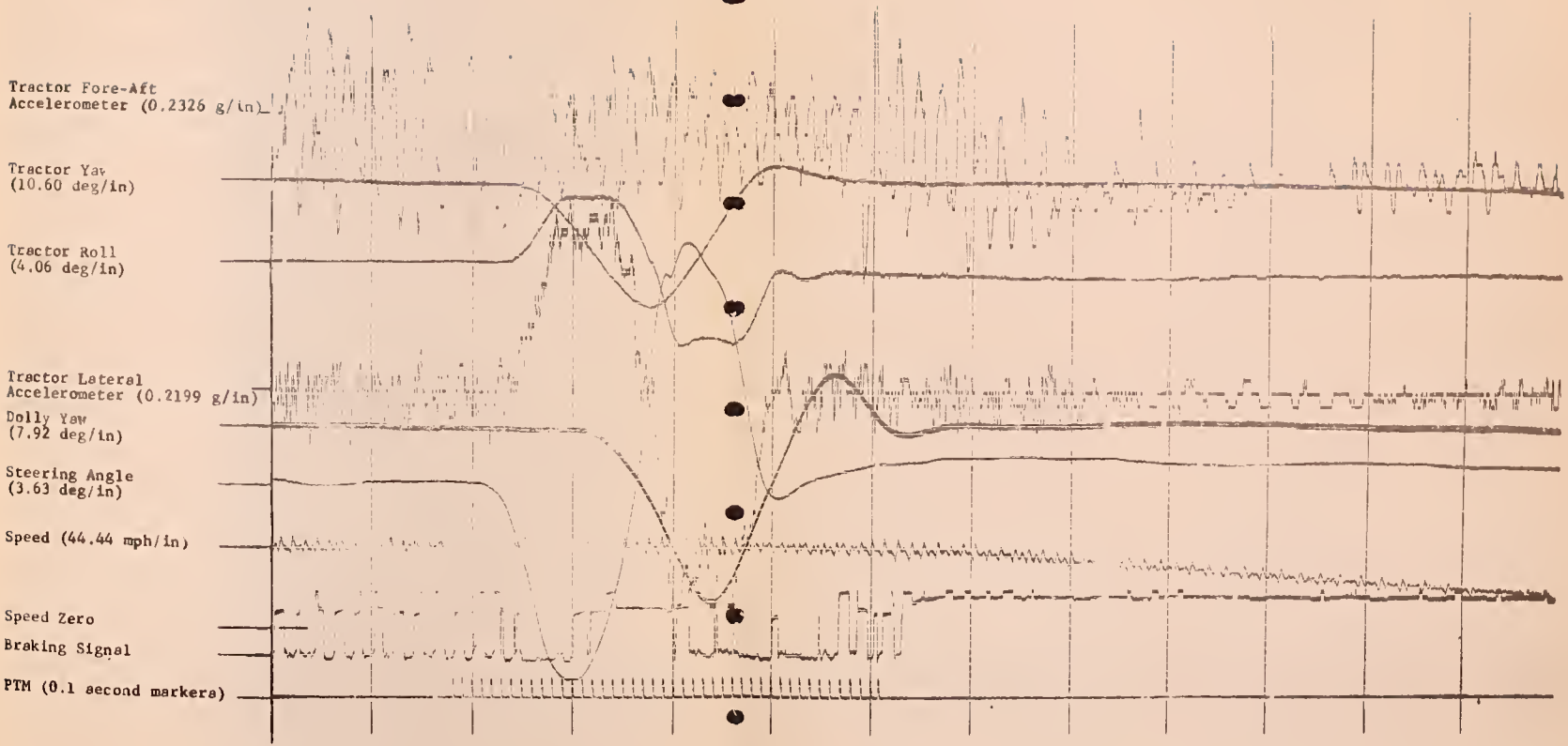
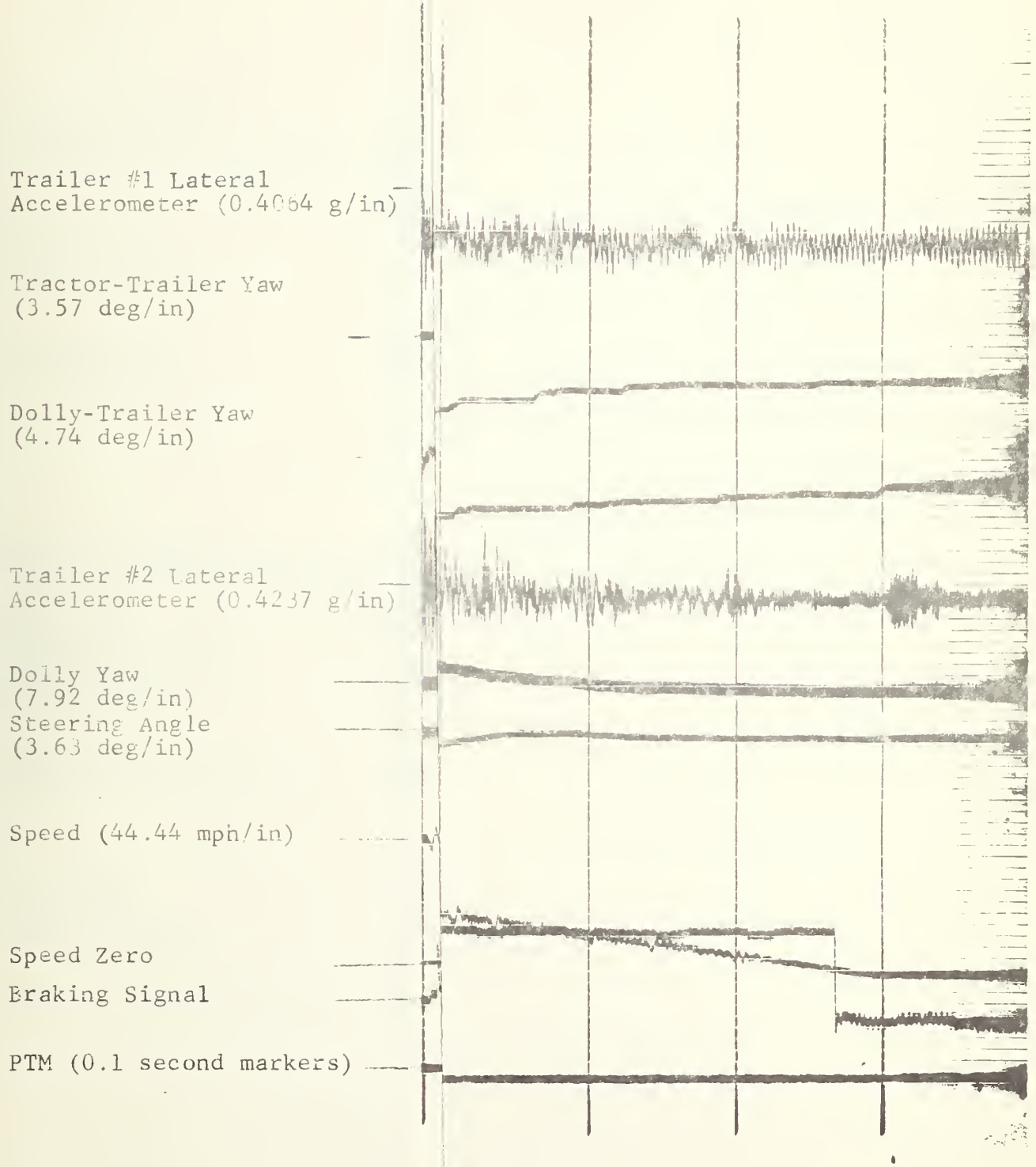


Figure 11. (concl)



record of a double articulated vehicle
n evasive maneuver.

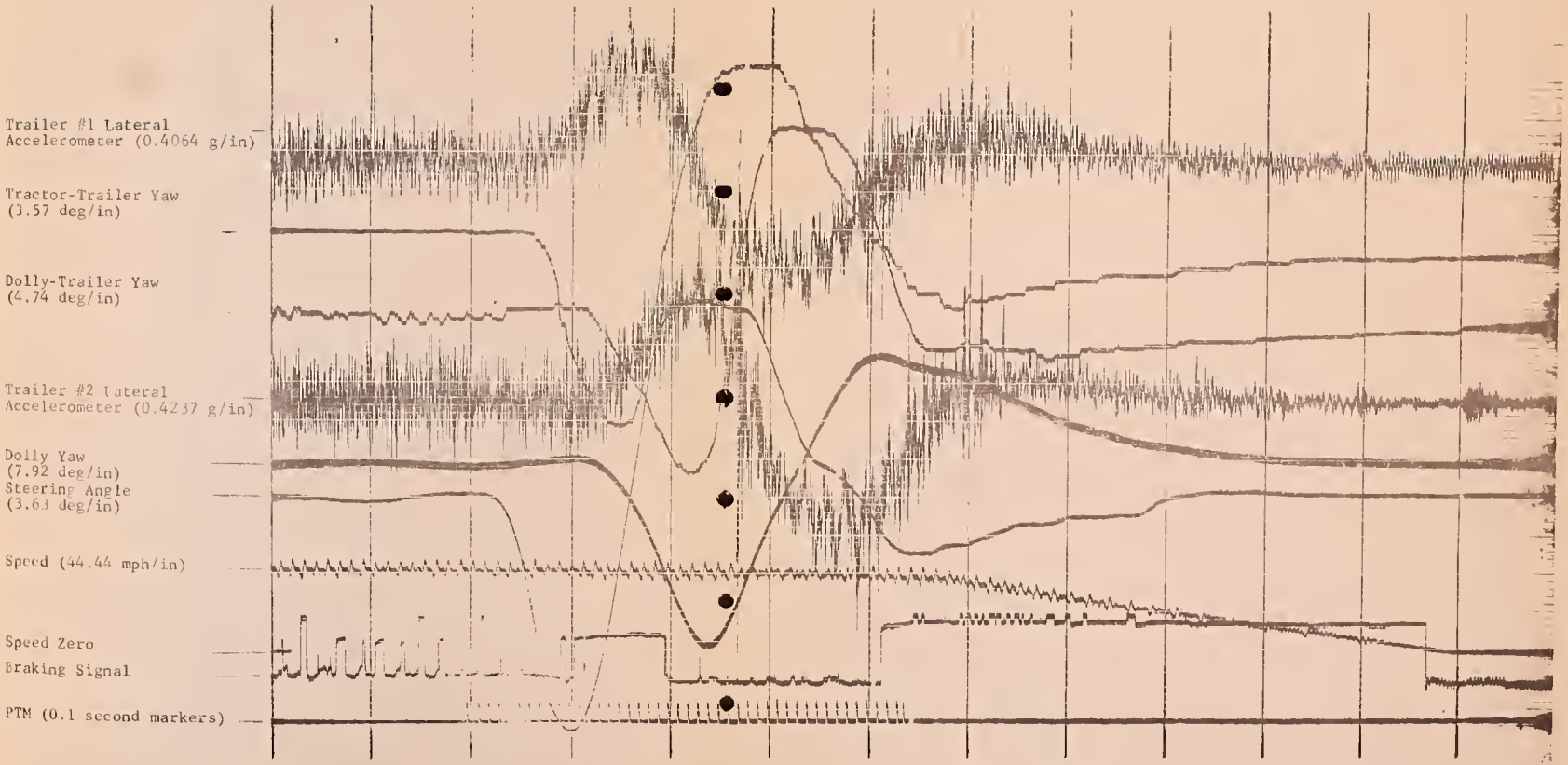


Figure 12. Experimental record of a double articulated vehicle performing an evasive maneuver.

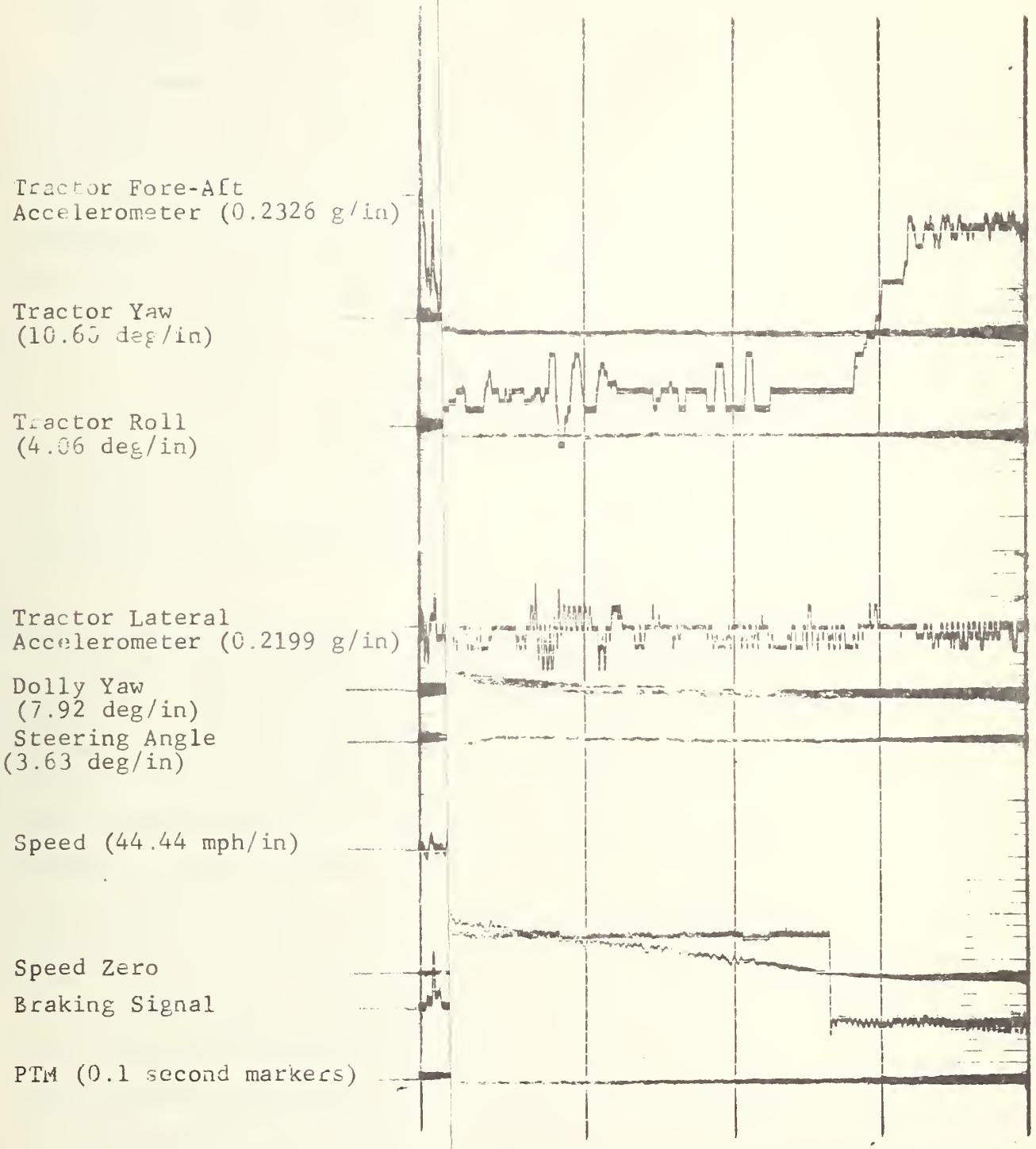


Figure 12. (concl)

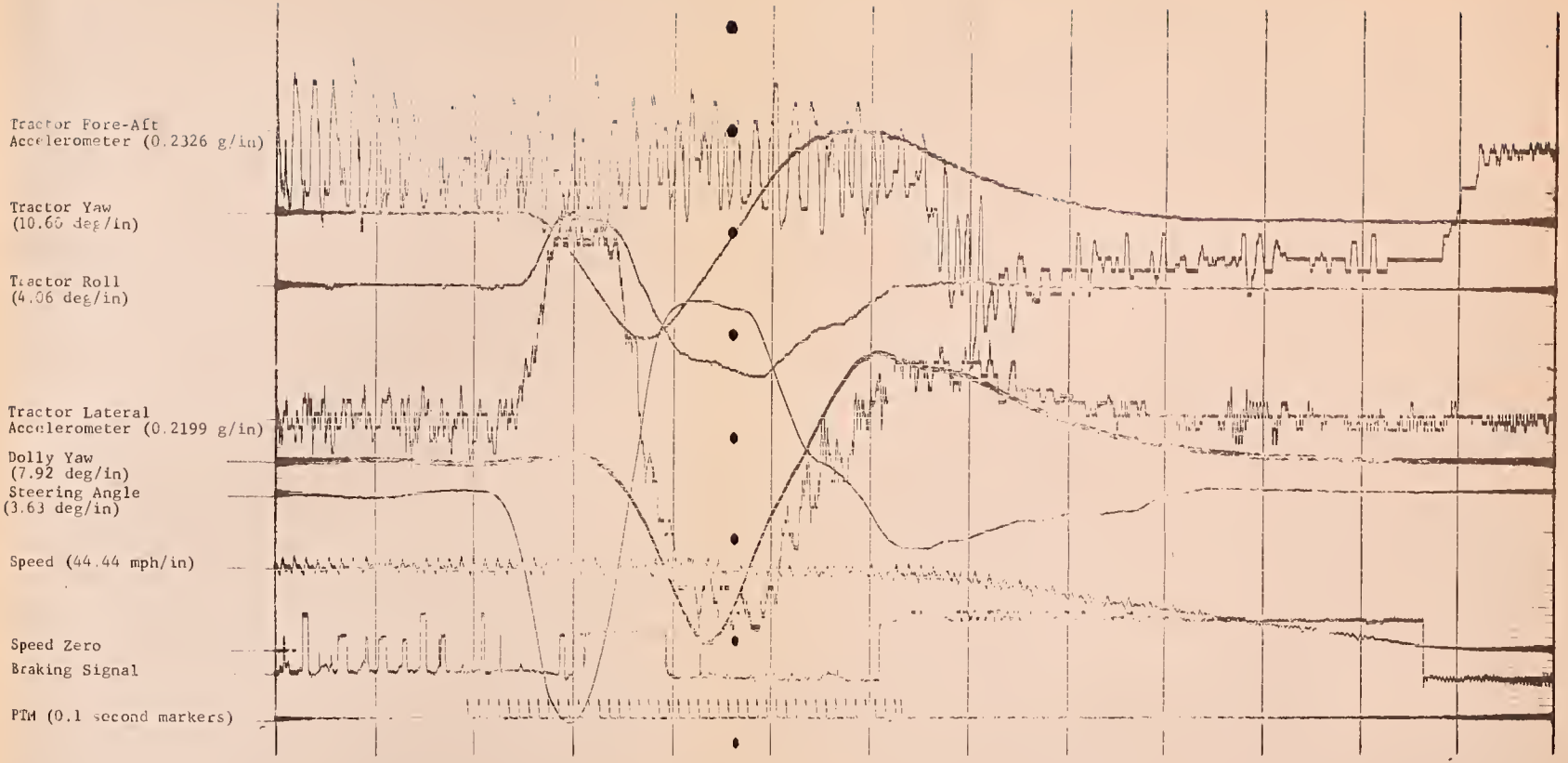


Figure 12. (concl)

6. STABILITY MODEL VALIDATION

The articulated vehicle experimental program reported in Section 5 was conducted to develop and verify stability and handling criteria. This section is concerned with the validation of the results of the AVDS3 computer simulation model and the stability criteria generated by it. Three basic vehicle configurations were tested with varied loads, speeds, tires, maneuvers and surface condition:

- Tractor - 45 ft semitrailer
- Tractor - 27 ft semitrailer
- Tractor - double 27 ft semitrailer unit

In this section it will be shown that the AVDS3 computer simulation model is capable of predicting the dynamic response including stability limits of tractor single- and double-semitrailer systems provided that tire data is available.

6.1 Validation Procedure

Twelve articulated vehicle tests were chosen to verify the AVDS3 simulated dynamic response of articulated vehicles. Table 7 shows the test configuration for each analytical validation; the validation procedure is shown in Figure 13. The complete test data for each limit maneuver was reduced and plotted in analog form. The tractor lineal response data including acceleration, velocity and position (track marker) were used as input to the AVDS3 model. In this series of simulations, tractor forward and lateral accelerations were used as input to AVDS3. Direct test lateral accelerometer data and its integrated velocity were used along with direct forward accelerometer data when braking was prominent in the maneuver. Forward acceleration was obtained from the directional velocity change in nonbraking maneuvers. The fifth wheel measured

Table 7

SUMMARY OF ANALYTICAL-EXPERIMENTAL RESPONSE FOR ARTICULATED VEHICLES PERFORMING LIMIT MANEUVERS

Simulation Number	Test Number	Vehicle Configuration	Surface	Maneuver	Tractor Accel. (ft/sec^2) Forward	Tractor Accel. (ft/sec^2) Lateral Exper.	Tractor Accel. (ft/sec^2) Lateral Anal.	Steering (deg) Exper.	Steering (deg) Anal.	Tractor Yaw (deg) Exper.	Tractor Yaw (deg) Anal.	Tractor-1st Trailer Yaw (deg) Exper.	Tractor-1st Trailer Yaw (deg) Anal.	Dolly Yaw (deg) Exper.	Dolly Yaw (deg) Anal.	Dolly-2nd Trailer Yaw (deg) Exper.	Dolly-2nd Trailer Yaw (deg) Anal.	Trailer Yaw (deg) Exper.	Trailer Yaw (deg) Anal.	Maximum Tire Slip (deg)
1	2- 6	Double 27' Trailer (empty)	D	Lane Change with braking	-12.10	+ 8.68 - 6.51	------	+ 5.23 - 5.08	+ 4.95 - 3.90	+10.21	+11.18	+ 4.22 - 2.61	+ 4.19 - 3.21	+ 8.47	+10.86	+ 2.88 - 1.69	+ 3.18 - 2.74	------	------	- 3.61
2	2-14	Double 27' Trailer (full)	D	Evasive	------	+11.08 -14.57 + 7.67	------	+ 6.33 - 6.44 + 3.33	+ 4.93 - 6.14 + 3.01	+10.28 - 9.34	+ 9.82 -12.01	+ 2.46 - 4.31 + 4.30	+ 4.01 - 5.67 + 2.65	+10.30 -10.77	+10.60 -13.67	+ 3.66 - 4.85 + 3.80	+ 4.39 - 6.70 + 2.46	------	------	+ 5.22
3	2-15	Double 27' Trailer (full)	D	Evasive with braking	- 9.45	+ 9.33 - 7.50 + 3.82	------	+ 4.75 - 5.09 + 3.73	+ 3.87 - 4.36 + 2.51	+10.21 -10.41	+ 9.73 -12.70	+ 3.66 - 4.27 + 3.87	+ 3.43 - 4.10 + 3.57	+ 9.42 - 8.55	+ 9.86 -13.59	+ 3.47 - 2.85 + 1.35	+ 3.53 - 4.77 + 5.60	------	------	- 5.10
4	2-39	Double 27' Trailer (empty)	D	Lane Change	------	+14.94 -15.99	------	+ 7.41 - 8.97	+ 8.43 - 8.13	+13.41 - 1.74	+15.38 - 1.37	+ 6.55 - 6.63	+ 6.85 - 8.08	+14.23 - 4.24	+16.53 - 3.70	+ 6.34 - 9.29 + 4.08	+ 7.21 - 9.13 + 4.65	------	------	+ 6.81
5	2-41	Double 27' Trailer (empty)	D	Evasive	------	+14.64 -13.56 + 4.08	------	+ 9.04 - 7.41 + 2.25	+ 7.94 - 5.80 + 1.66	+13.62 - 9.22	+13.61 - 6.93	+ 7.18 - 6.07 + 2.88	+ 6.66 - 5.74 + 1.59	+14.93 - 8.80	+14.76 - 7.33	+ 7.57 - 8.35 + 2.31	+ 6.43 - 6.38 + 1.31	------	------	- 6.35
6	2-51	Single 27' Trailer (empty)	D	Lane Change	------	+12.71 -11.24	------	+ 6.99 - 6.35	+ 6.34 - 5.17	+11.00	+11.81	+ 4.10 - 5.12	+ 5.42 - 5.08	------	------	------	------	+ 9.43	+11.20	- 4.74
7	2-53	Single 27' Trailer (empty)	D	Evasive	------	+13.94 -13.61 +12.29	------	+ 8.27 - 7.04 + 7.33	+ 7.49 - 5.75 + 5.35	+ 2.77 -11.21	+13.24 -11.25	+ 5.27 - 5.71 + 3.66	+ 6.23 - 6.00 + 4.91	------	------	------	------	+10.45 - 9.35	+12.47 -11.07	- 5.69
8	1- 9	Single 45' Trailer (empty)	D	Lane Change	------	+ 9.13 - 9.13	------	+ 3.27 - 3.90	+ 3.02 - 2.99	+10.80	+ 9.90	+ 4.51 - 5.18	+ 5.43 - 5.15	------	------	------	------	+ 8.79	+ 8.21	- 2.50
9	1-19	Single 45' Trailer (empty)	W	Lane Change	------	+ 7.59 - 7.70	------	+ 2.92 - 2.29	+ 3.84 - 3.37	+ 9.21	+ 7.94	+ 4.04 - 4.00	+ 4.39 - 4.45	------	------	------	------	+ 7.72	+ 6.52	- 3.06
10	2-26	Double 27' Trailer (1st full, 2nd empty)	W	Lane Change	------	+ 9.31 -10.43	------	+ 5.99 - 7.26 + 2.50	+ 5.69 - 7.88 + 7.13	+11.83 - 1.76	+11.83 - 3.60	+ 3.68 - 5.03 + 2.03	+ 4.46 - 5.90 + 3.69	+11.72 - 4.12 + 1.58	+15.25 - 8.22 + 3.18	+ 3.88 - 7.13 + 3.04	+ 6.29 -12.02 +14.14	------	------	+ 6.82
11	2-37	Double 27' Trailer (1st full, 2nd empty)	D	Evasive with Braking	-13.50	+11.28 -11.60	------	+ 7.22 - 6.61	+ 5.82 - 5.21	+10.83	+12.15	+ 4.37 - 5.23	+ 4.62 - 4.51	+12.28	+13.24	+ 6.48 - 8.43	+ 5.69 - 4.59	------	------	- 4.42
12	1-12	Single 45' Trailer (empty)	D	Lane Change with braking	-15.68	+ 9.92 - 6.39	------	+ 1.98 - 2.52	+ 3.09 - 3.85	+ 9.45	+10.93	+ 4.72 - 3.70	+ 6.39 - 5.55	------	------	------	------	------	------	-11.85
13	1-15	Single 45' Trailer (empty)	D	Cornering 72.5' radius	------	+10.95	+16.50	+11.00	+12.85	------	------	+24.00	+24.71	------	------	------	------	------	------	- 7.16
14	2- 9	Double 27' Trailer (empty)	D	Cornering 82.0' radius	------	+12.18	+14.92	+19.90	+12.48	------	------	+12.32	+13.63	------	------	+11.56	+14.03	------	------	- 6.14
15	2-59	Single 27' Trailer (empty)	W	Cornering 74.17' radius	------	+12.25	+12.15	+17.71	+14.53	------	------	+15.83	+15.46	------	------	------	------	------	------	- 7.65

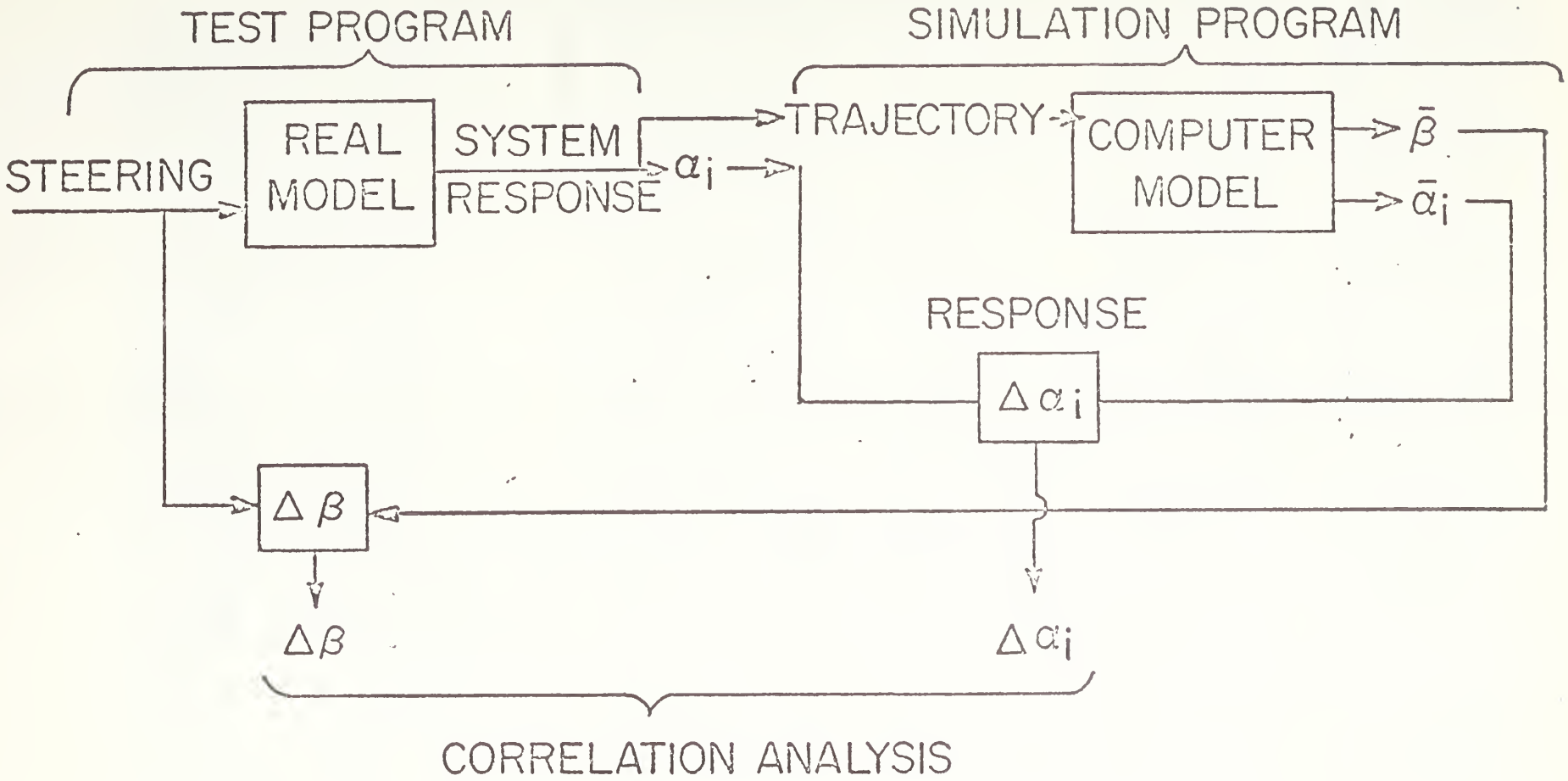


Figure 13. Analytical-experimental validation.

velocity, the tractor yaw and tractor yaw rate were used to obtain these values of lateral acceleration. The fact that engine vibration is predominant on acceleration records when rigid body accelerations due to handling are small, necessitates this alternate method of obtaining data for AVDS3.

In verification of stability limits, functional equations which simulate corners, lane changes and evasive maneuvers developed in Appendix A and in the previous NHTSA project[23] on articulated vehicle handling were used as input. It was found that in general, these mathematical functions simulate actual experimental trajectory data very well.

The tire model utilized in the simulations is described in Appendix D. It was developed from actual cornering data taken by the Goodyear Tire and Rubber Company[62,63] and from data published by Freudenstein[64]. This model is described in Section 3. Even though data were available only for Goodyear tires used on dry surfaces the same model was used[64,65] for all tires on dry and wet surfaces. The tire road friction coefficient was adjusted in the tire model to relate the model to test conditions. Skid trailer surface measurements guided this data. The results relate the applicability of the tire model to the limit maneuvers performed. The friction ellipse concept was used to relate cornering and tractive/braking forces during the maneuvers.

6.2 Error Evaluation

Before examination of the results, the sources of error should be placed in perspective because error occurs both from computation and modeling in the simulation and from measurements in the experimentation. Since full-scale vehicles were used in the experiments, no error occurs as a result of modeling in the experimental work. The total measurement system was calibrated for the range of parameters used in the tests to minimize error. Errors could occur with the

transducers (calibrated before each test run), the modifying instruments, and the data reduction. A total system error of no less than ± 5 percent and no more than ± 8 percent including data reduction was realized in obtaining the experimental data.

Estimation of error on the simulation must be made on the basis of individual experiments because the major source of error is the tire model. It will be shown that the simulation results are within 10 percent where tire information is known. In cases where the tire model is an uncertainty, higher values of error are obtained. Other modeling errors appear to be negligible in this work, as does numerical computational error. A small experiment was conducted where the computational time step size was varied for the same simulation and no descernable difference was obtained for the resulting vehicle responses.

In the analytical-experimental correlation procedure, input data to the simulation from the experimental records provide some error. However, the data reduction techniques practiced in this work combined with the proper selection of the trajectory have helped to minimize data input error.

6.3 Analysis of Results

The 15 tests listed in Table 7 represent a series of limit maneuvers conducted with the following varied parameters:

- Maneuver (lane change, corner, and evasive)
- Vehicle configuration (single 27, single 45 and double 27)
- Load configuration (empty, loaded, one trailer empty and the other loaded
- Surface (wet and dry)
- Tires (mixed, Goodyear, mixed Firestone and Goodyear)
- Braking

The simulations of limit maneuvers are compared on a time history basis throughout the maneuver to provide credibility of the AVDS3 simulation model to develop stability limits. Since the cornering maneuvers were performed at steady state, peak response values are compared for experiment and AVDS3 simulation. A summary of the peak values of analytical-experimental responses is shown in Table 7 .

In general it can be observed that the analytical-experimental results obtained in this project correlate well. No experimental data is available on tire slip angles; however, simulation is available. Even though the tire slip angle results appear to be low in magnitude, it must be recognized that most tires are utilized in the low load regime owing to multiple tires per axle where the slip angle at which the maximum side force occurs is low. Therefore, these values are near the tire force saturation slip angle.

In the ensuing experimental-analytical records it must be noted that the experimental response values lag the analytical calculations. This phenomenon was first observed in these limit maneuvers (at lower values of lateral acceleration this phase discrepancy was not noted). In addition because of the fact that the basic vehicle control is exerted at the tractor, errors in response (either phase or amplitude) are propagated through the vehicle. Therefore, the tractor double-semitrailer system accentuates both phase and amplitude errors. The phase lag observed in the experimental results is explained by the system lateral flexibility and damping not being modeled in AVDS3. In effect, the AVDS3, is a rigid body model forced directly by tire side and forward forces; whereas the experimental model (full-scale vehicle) is a rigid body mounted on flexible-damped springs and tires, with tire side and forward forces transmitted through the flexible-damped members. In a somewhat analogous simple, single degree-of-freedom system, the response at the mass lags the

forcing function due to system flexibility and damping. Since AVDS3 does not model lateral flexibility and damping, its results do not reflect the lag in response between the tire forces and the rigid body mass. It should also be noted that this phenomenon has little effect on the response magnitudes.

Figure 14 shows the empty double-articulated vehicle response in a lane change maneuver at 40 mph with braking on a dry surface. A combination of Firestone and Goodyear tires (see Figure 9) was used on this and the two suceeding tests discussed herein. This vehicle carried a full load during the test. The results show high peak forward and lateral accelerations (0.39 and 0.28 g respectively) with a tractor yaw of about 10 deg. Peak values throughout the simulation are within 10 percent and phase differences are minimal. In this experiment, the tire model works well in simulating system response even though the friction ellipse concept is utilized.

Test results of an evasive maneuver performed on a dry surface with a fully loaded double-articulated vehicle are shown in Figure 15. No braking was performed during this violent maneuver (0.47 g lateral acceleration). The simulation results show the phase lag increase as it is propagated back through the dolly to the second semitrailer; however, peak magnitudes of response variables coincide with experiment.

Another fully loaded double-articulated correlation record is shown in Figure 16. Again an evasive maneuver was performed; however, the application of braking limited the lateral acceleration in the maneuver requiring an extra 2 sec for completion of the maneuver. This record shows remarkably good correlation between analysis and experiment.

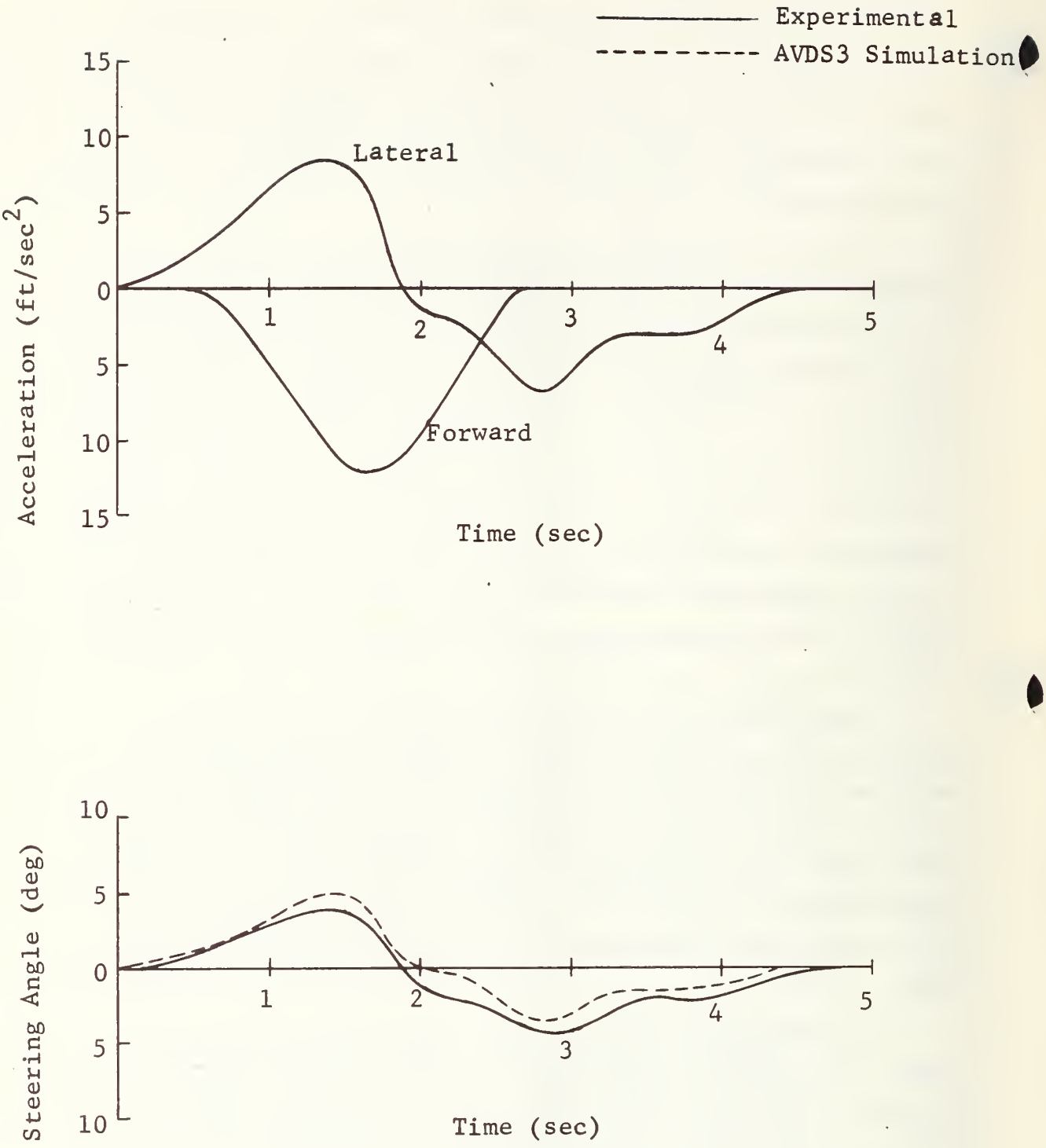
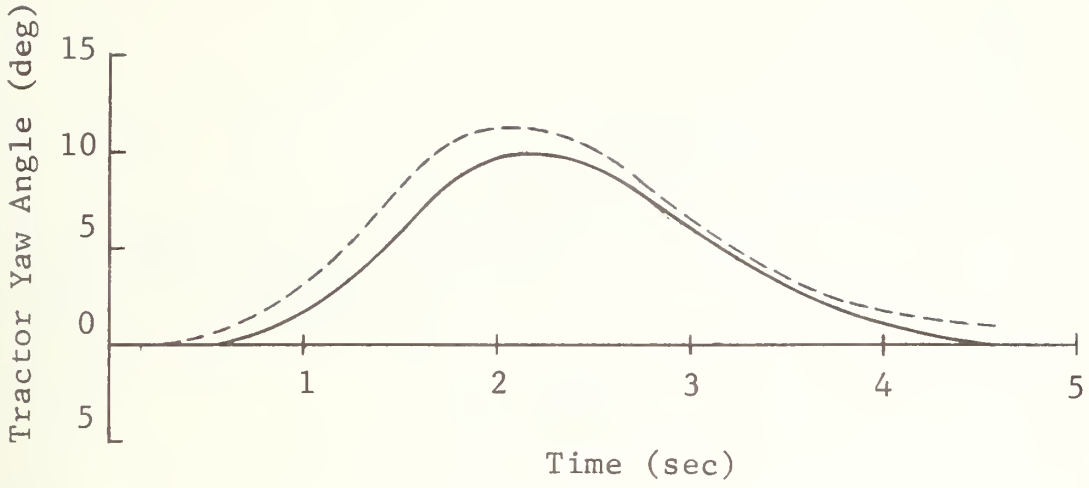


Figure 14. Analytical-experimental comparison of tractor double unloaded semitrailers (27 ft) response for experiment No. 2-6; lane change (13.9 ft x 218 ft) at 40 mph with braking.



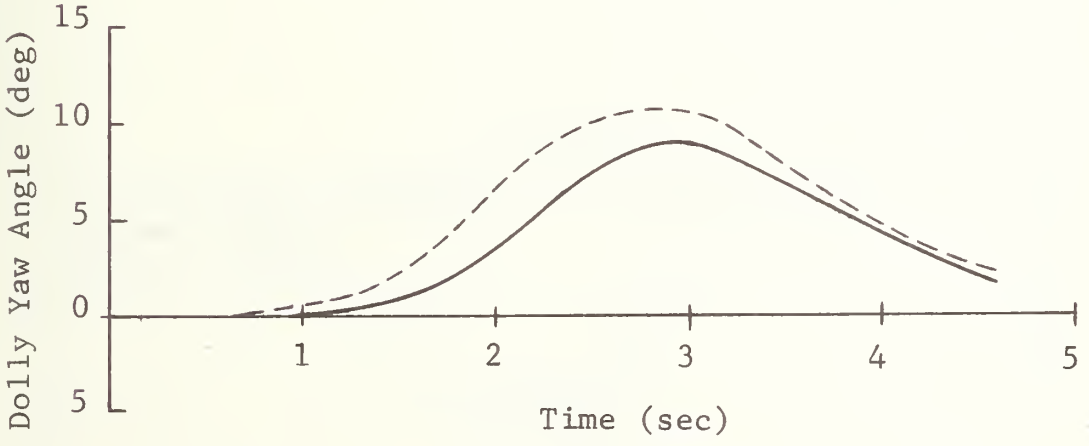
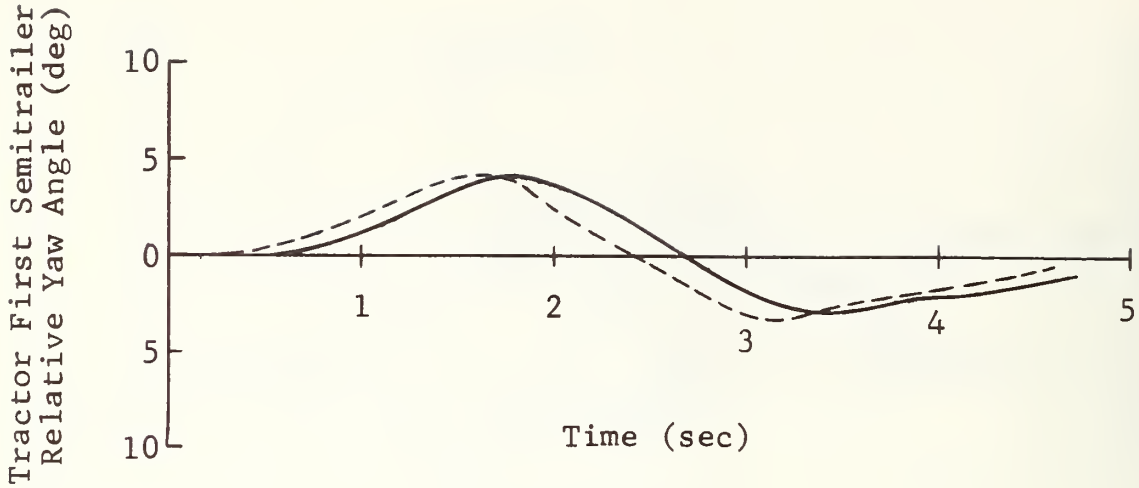


Figure 14. (contd)



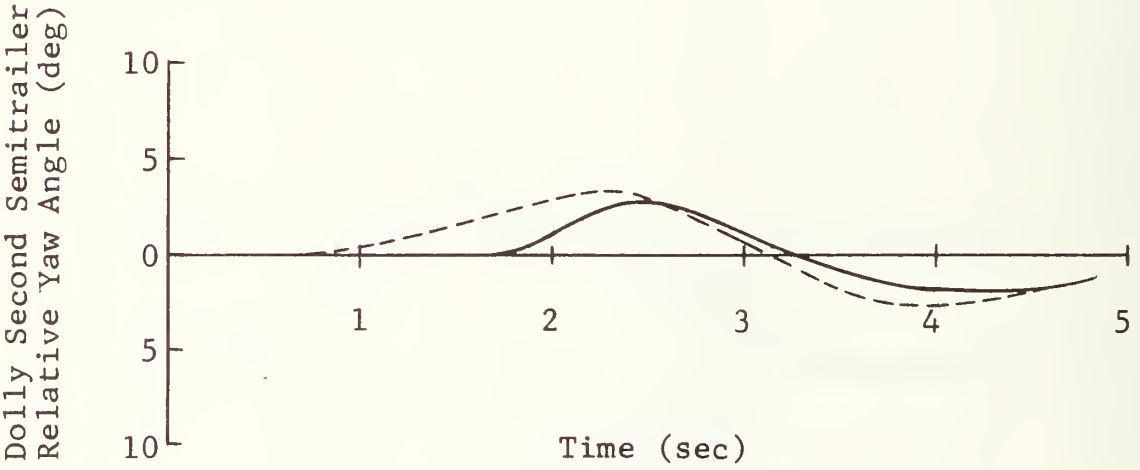


Figure 14. (concl)

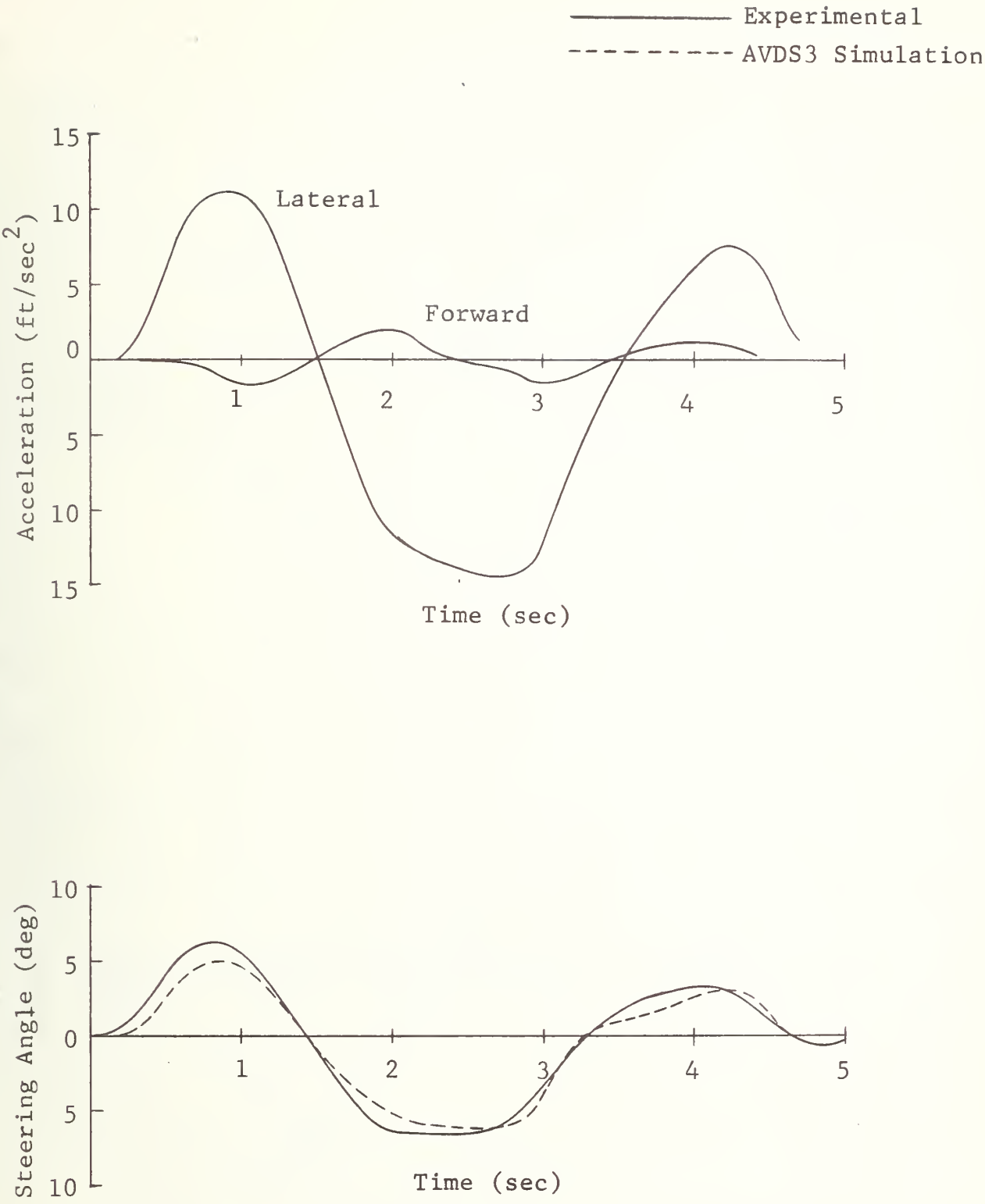
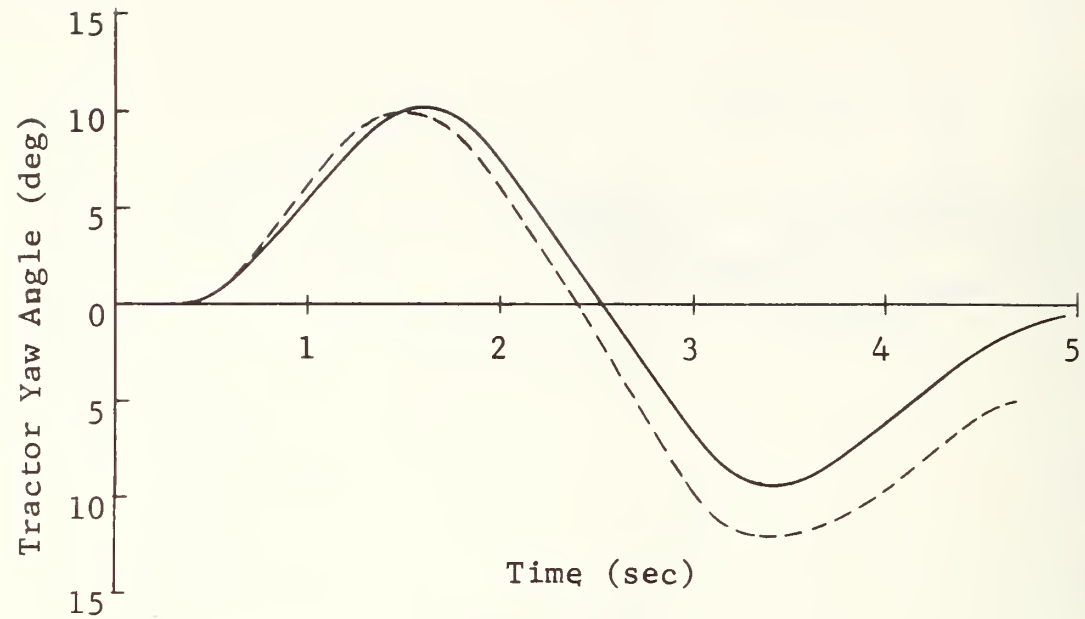
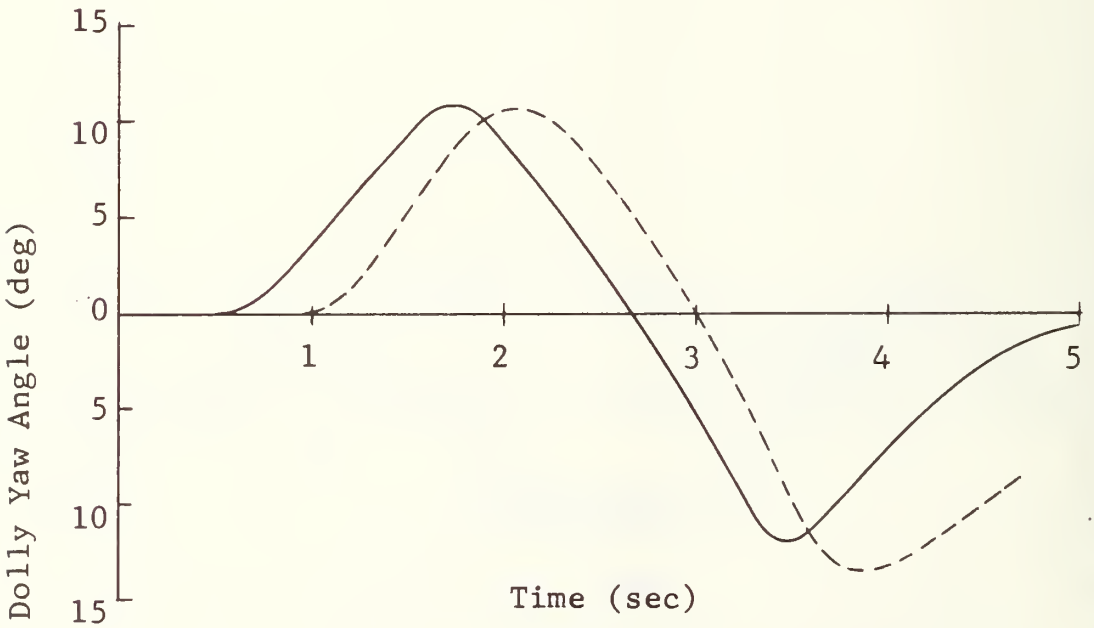


Figure 15. Analytical-experimental comparison of tractor double loaded semitrailers (27 ft) response for experiment No.2-14; evasive (11.5 ft x 315 ft) at 40 mph without braking.





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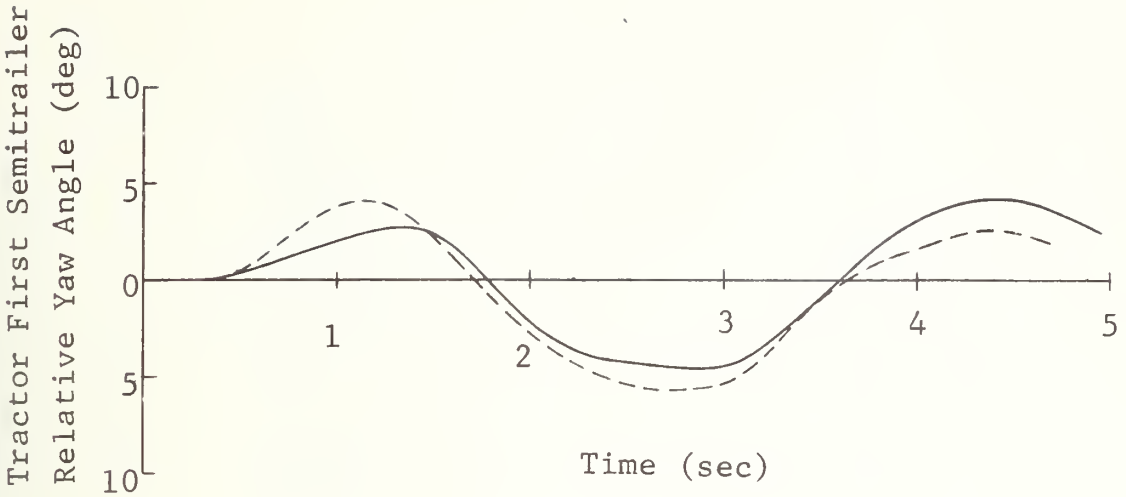


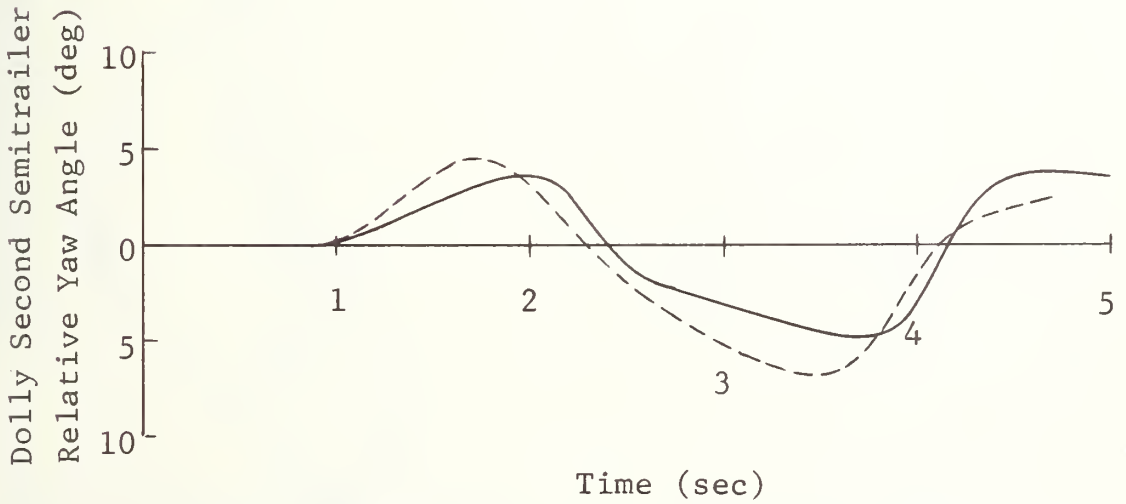


Figure 15. (concl)

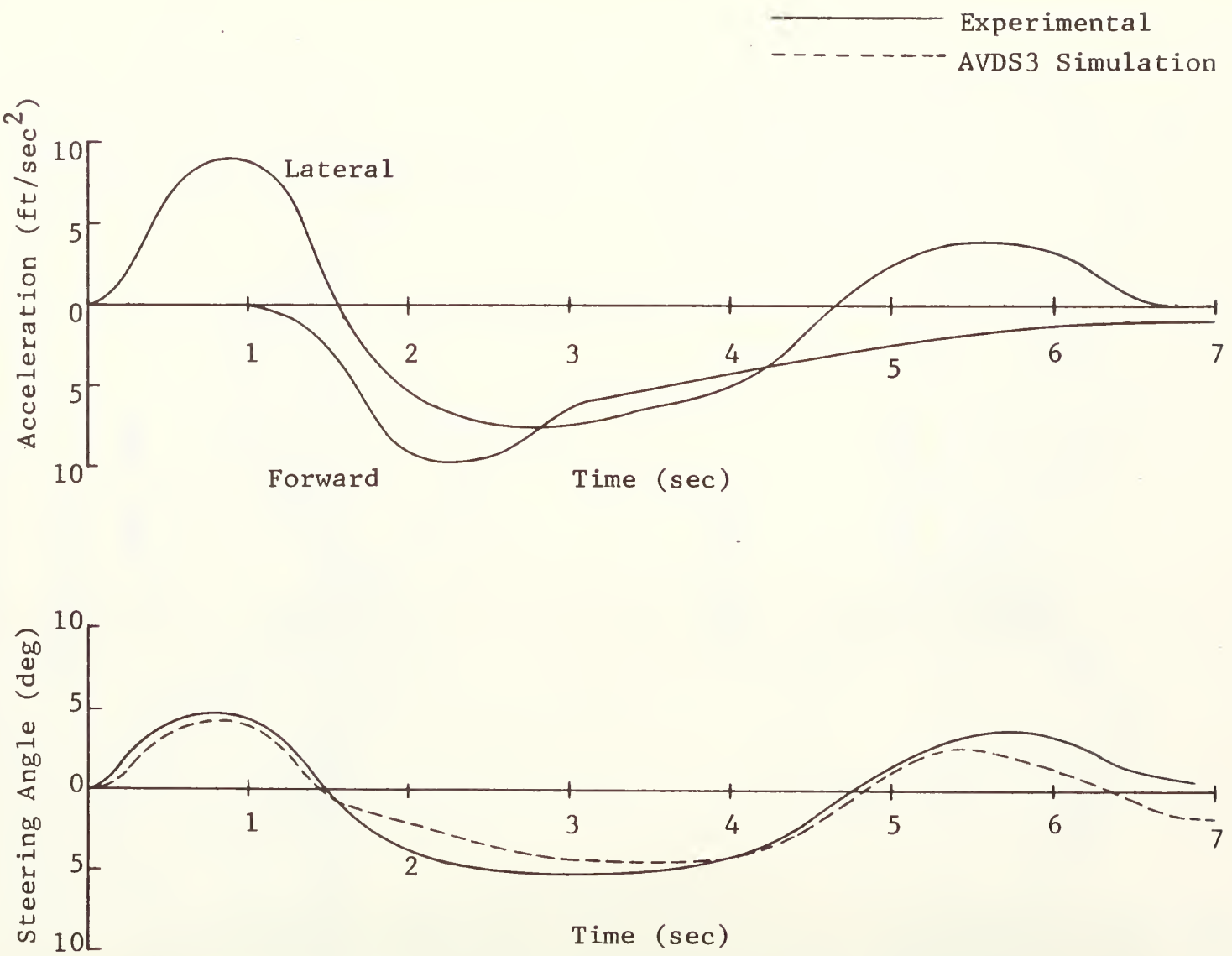
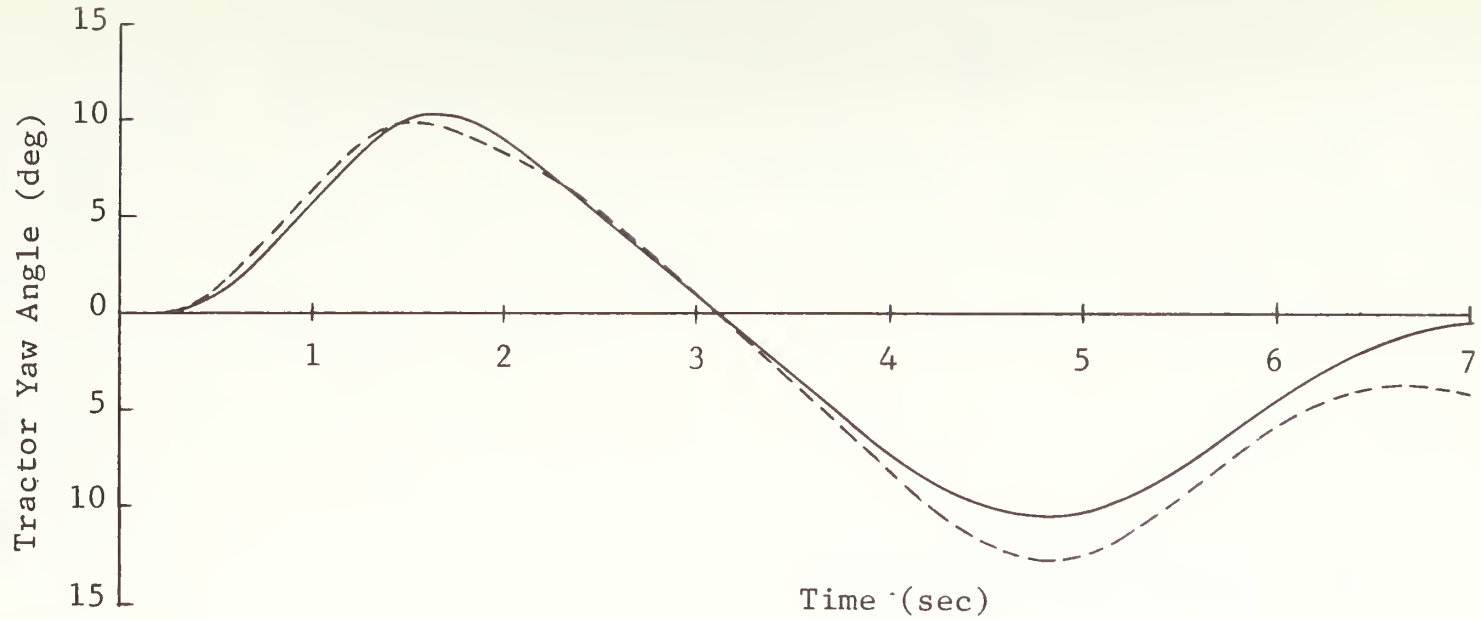


Figure 16. Analytical-experimental comparison of tractor double loaded semitrailers (27 ft) response for experiment No. 2-15; evasive (13.0 ft x 262 ft) at 40 mph with braking



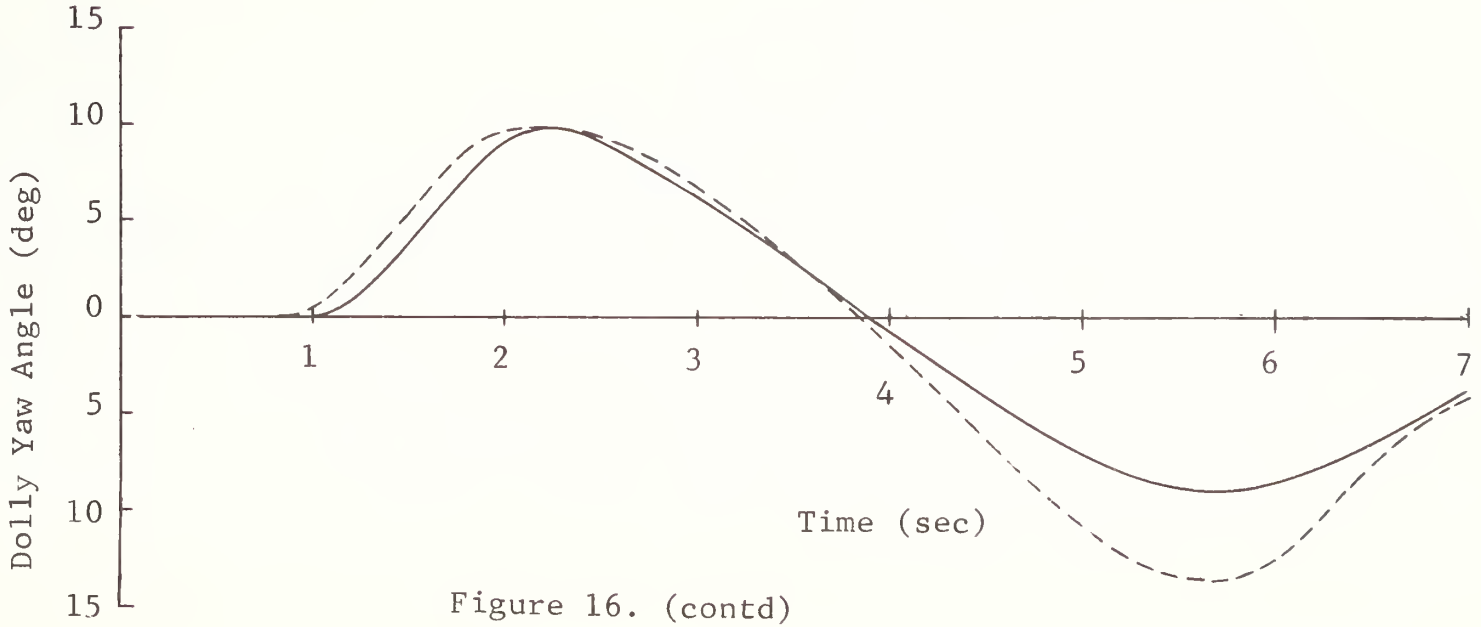


Figure 16. (contd)

Figure 16. (concl)

The response of a lane change maneuver without braking is shown in Figure 17. All Goodyear Super Hi-Miler tires were used in this configuration which was an empty double-27 articulated vehicle. The analytical results shown agree well with those of the experiment which is better than a 0.5 g maneuver. It is interesting to note that the driver steering response follows that of AVDS closely and simulates a decaying sine wave characteristic of a violent maneuver. The dolly second-semitrailer response is interesting because the shapes of the curves are almost exactly alike with the slight phase shift described earlier. Dolly yaw angles are higher in configuration than tractor yaw.

Results of a similar test configuration in an evasive maneuver are shown in Figure 18. The peak lateral acceleration is about 0.5 g. In this experiment, tractor-trailer yaw and dolly-trailer yaw angles are both about 7 deg. Taken with large dolly and tractor yaw angles (~15 deg) a large trailer swing was observed.

Figures 19 and 20 show analytical-experimental results of an articulated vehicle with a 27 ft trailer performing lane changing and evasive maneuvers on a dry surface. It is noted that the lateral accelerations (0.43 g peak) are less for the single articulated vehicle than those obtained for the double trailer unit. The simulation follows the experimental results closely. The steering record in the evasive maneuvers indicates that the driver was extended on this maneuver. These records show about 3 sec to complete a lane change and about 5 sec to complete an evasive maneuver.

Results of tests with an articulated vehicle with a single 45 ft trailer on dry and wet surfaces are shown in Figures 21 and 22 respectively. Mixed tires were mounted on this vehicle. Peak lateral accelerations are lower for this trailer indicating greater demand on tires by the larger vehicle. On the dry surface a peak lateral acceleration of 0.28 g is obtained; this drops to 0.23 g on the wet surface.

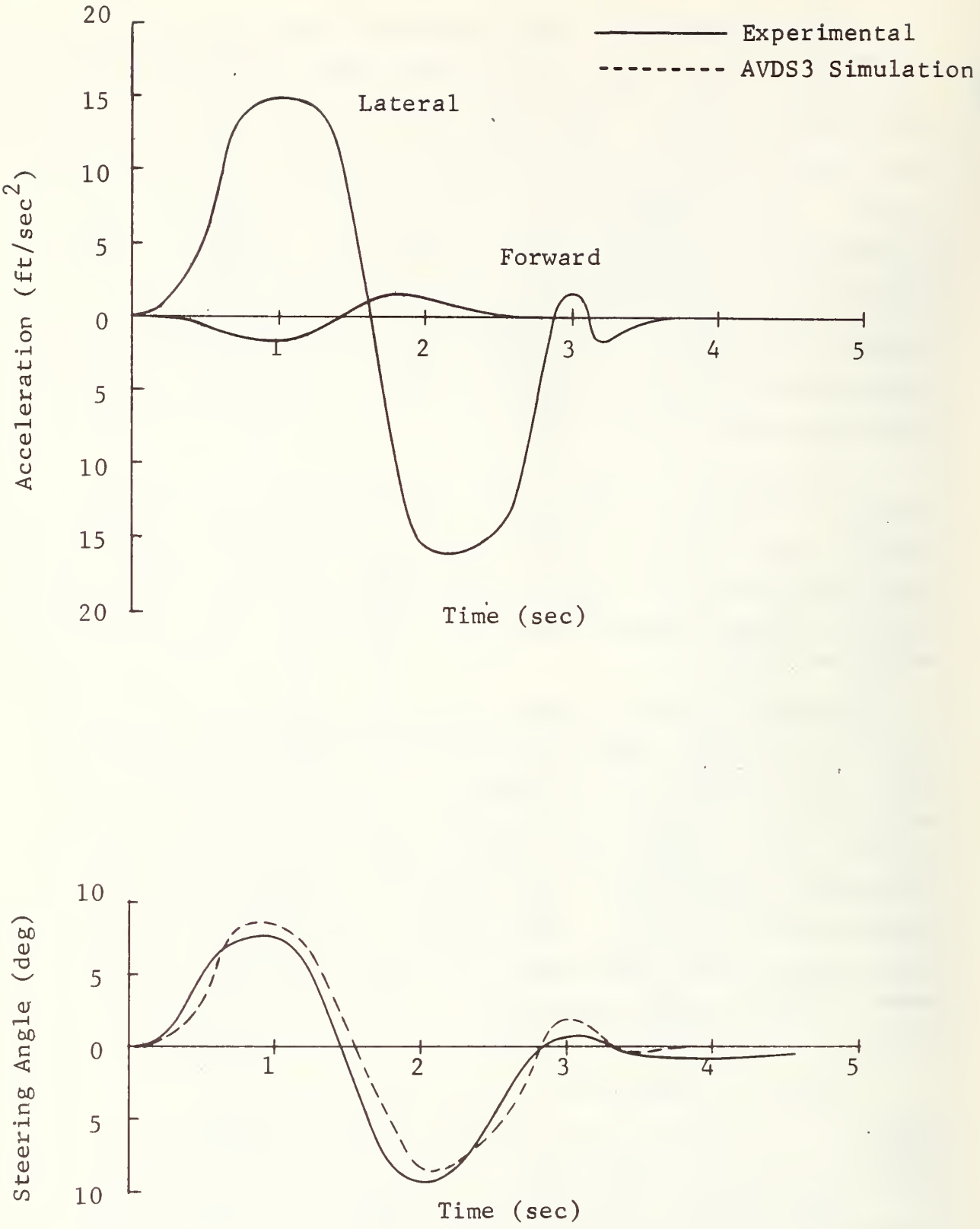
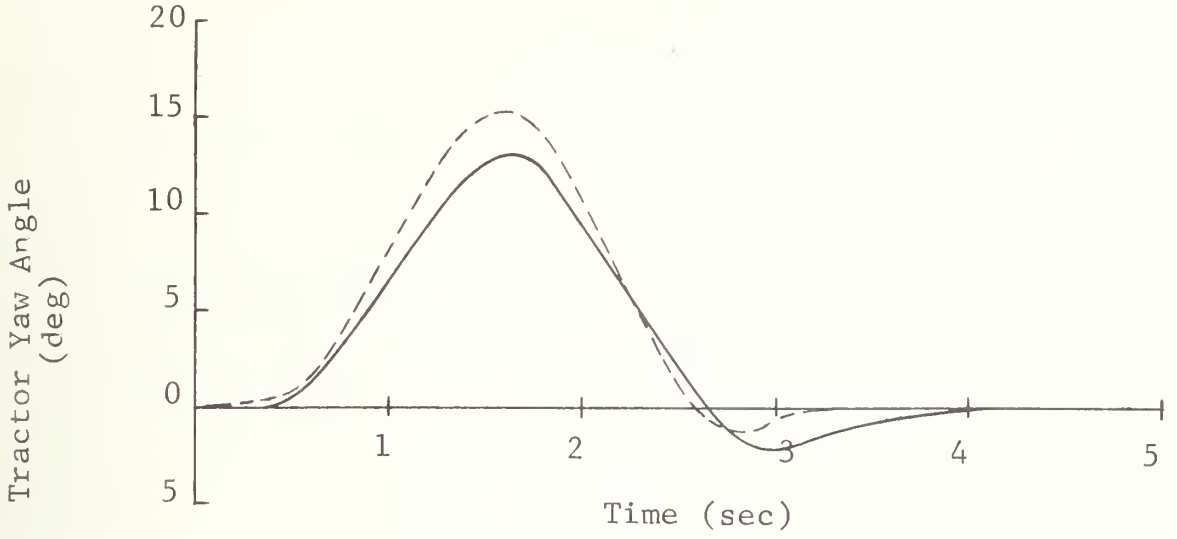


Figure 17 Analytical-experimental comparison of tractor double unloaded semitrailers (27 ft) response for experiment No. 2-39; lane change (14.4 ft x 191 ft) at 40 mph without braking.



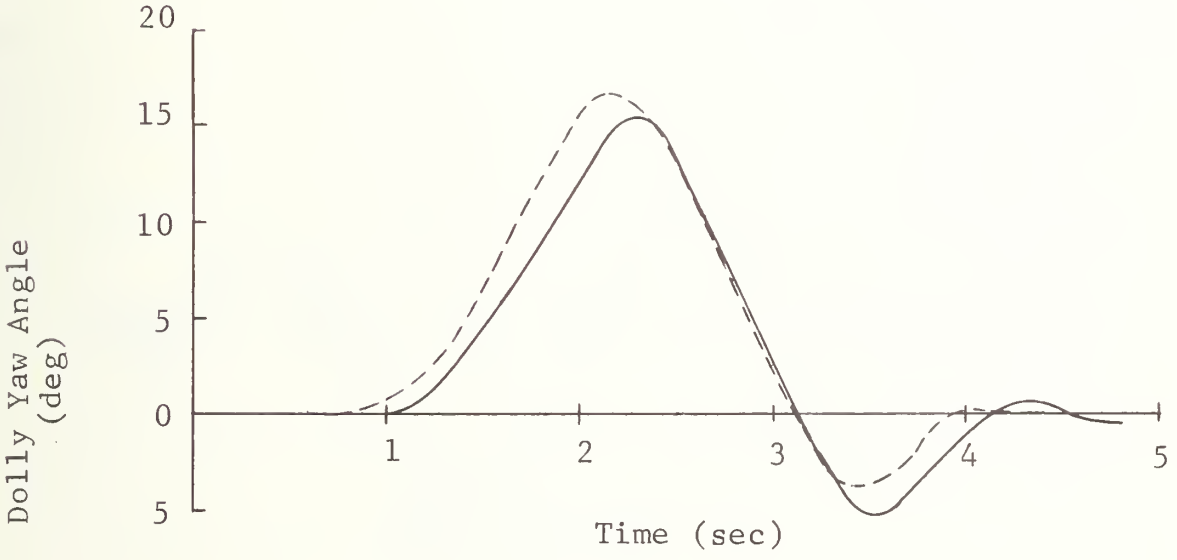
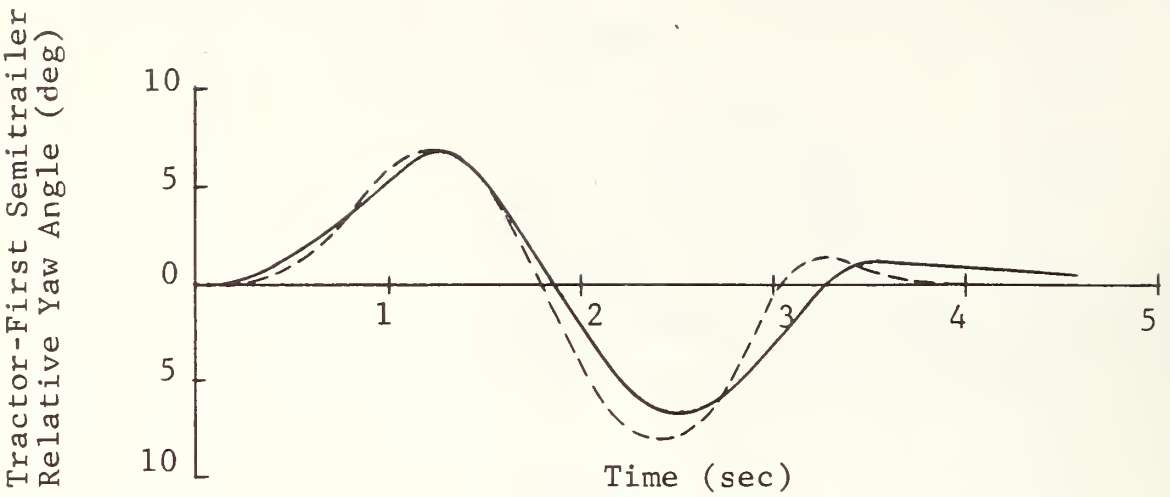


Figure 17 (Contd)



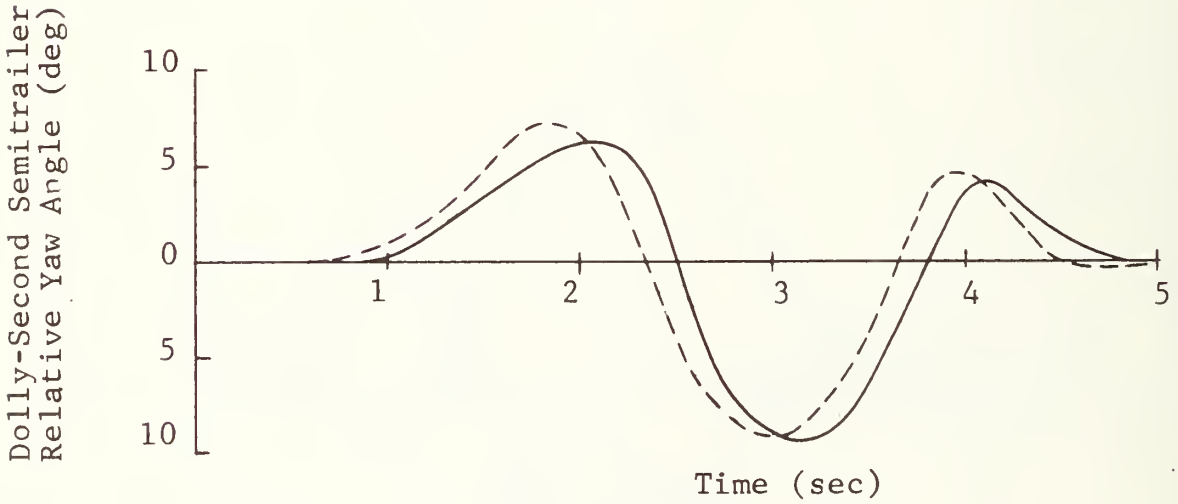


Figure 17 (Concl)

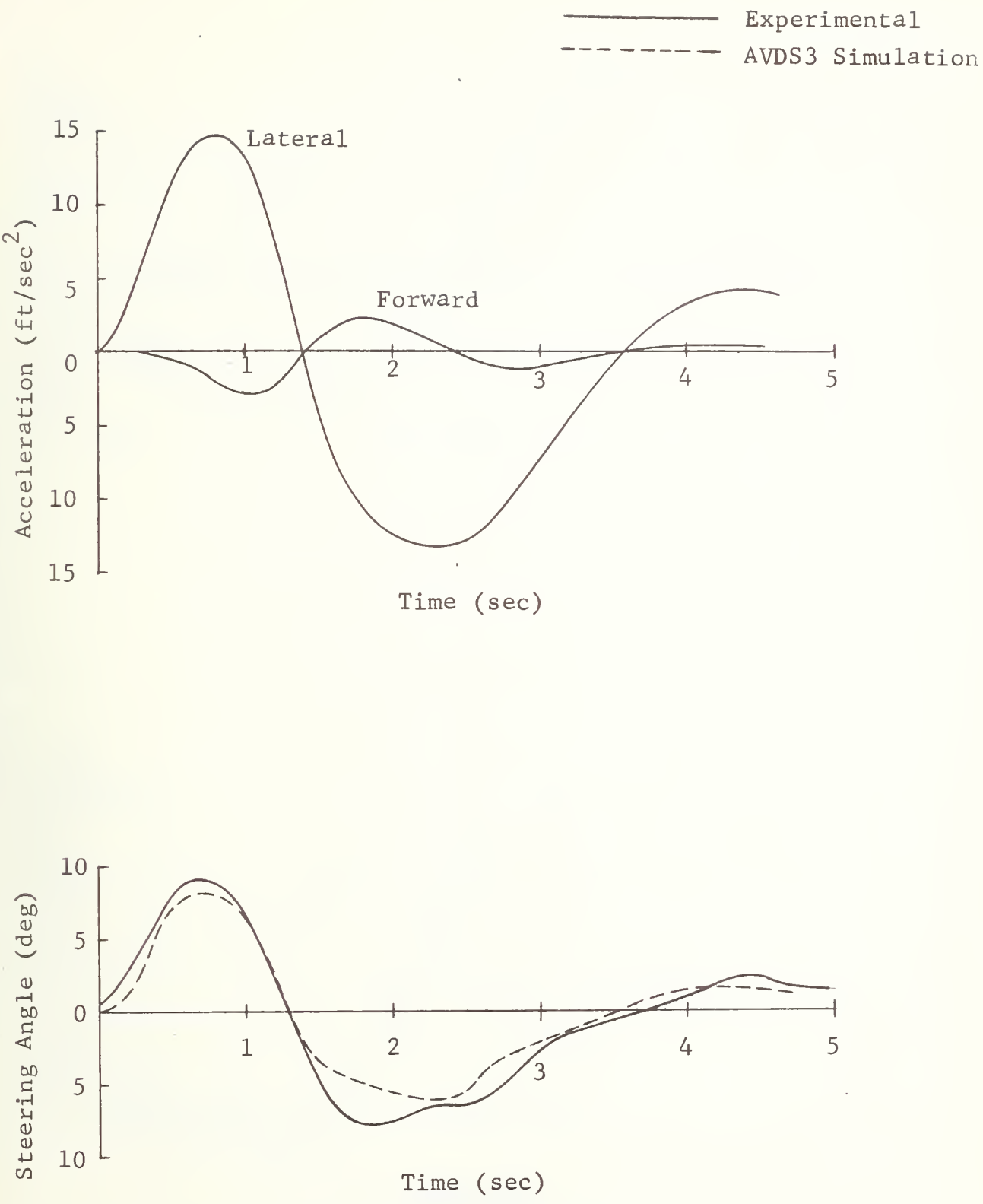
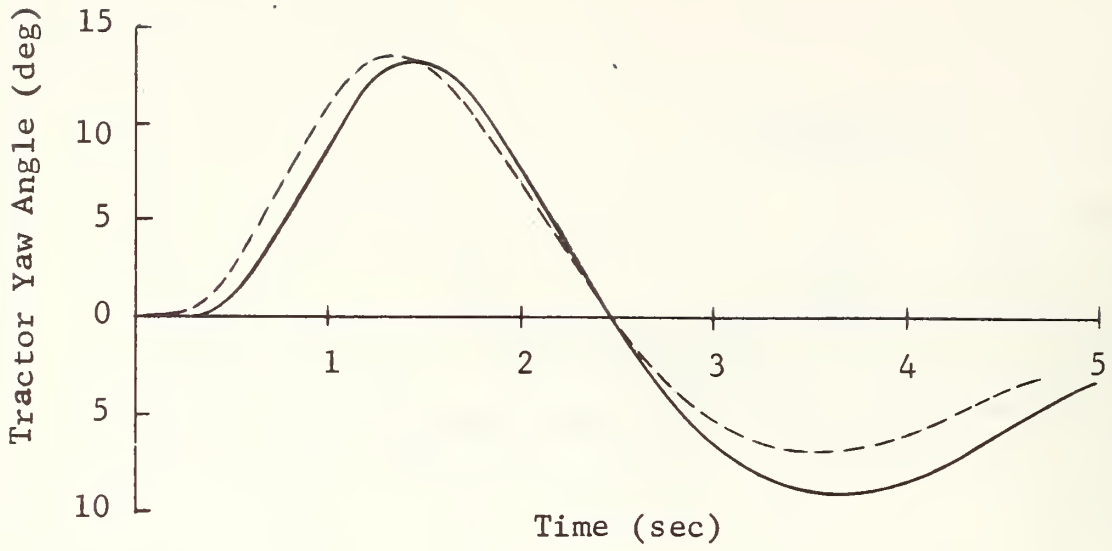


Figure 18. Analytical-experimental comparison of tractor double unloaded semitrailers (27 ft) response for experiment No.2-41; evasive (14.3 ft x 233 ft) at 40 mph without braking.



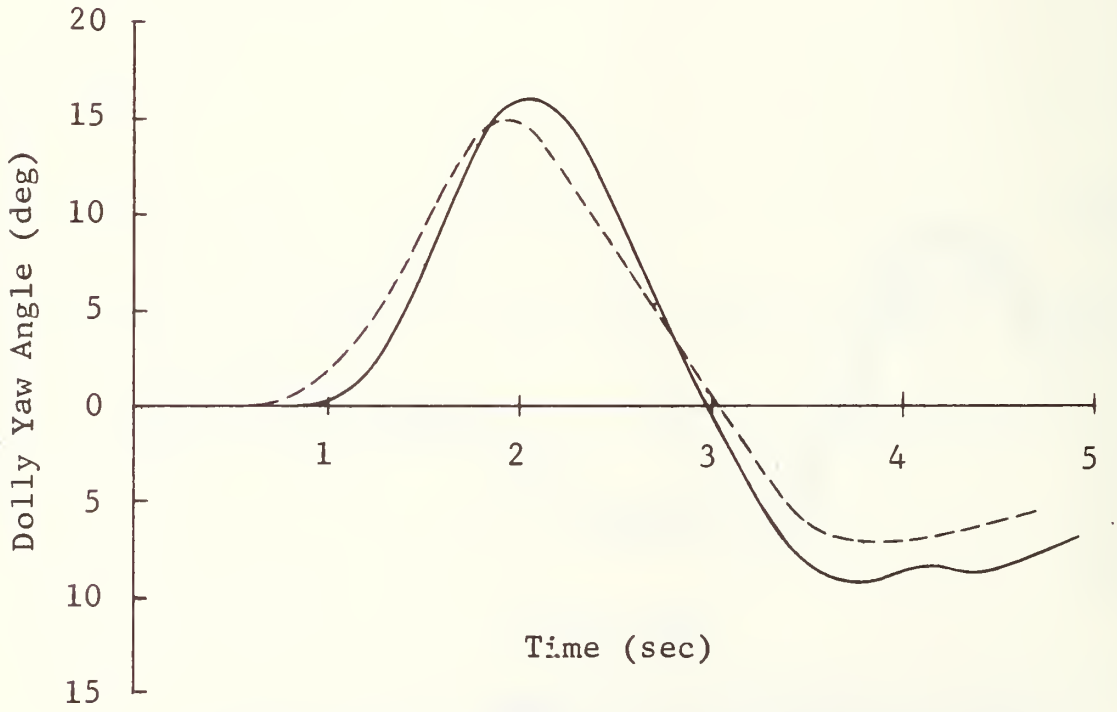
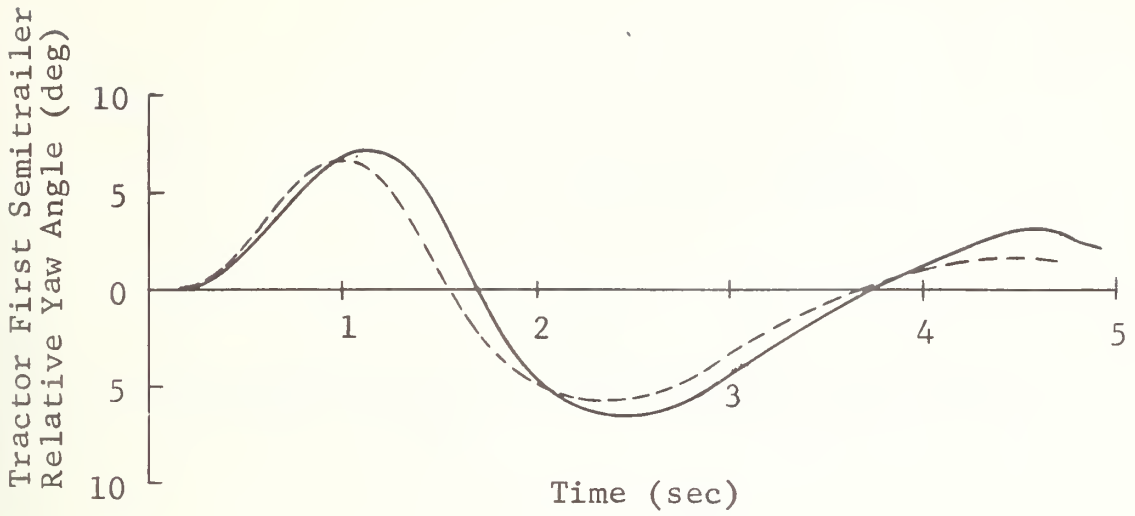
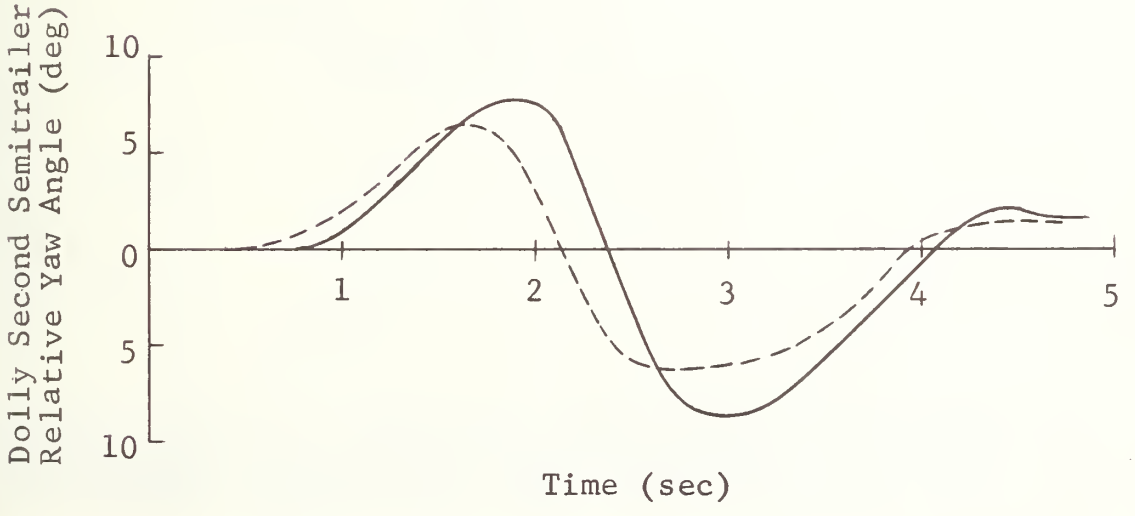


Figure 18. (contd)





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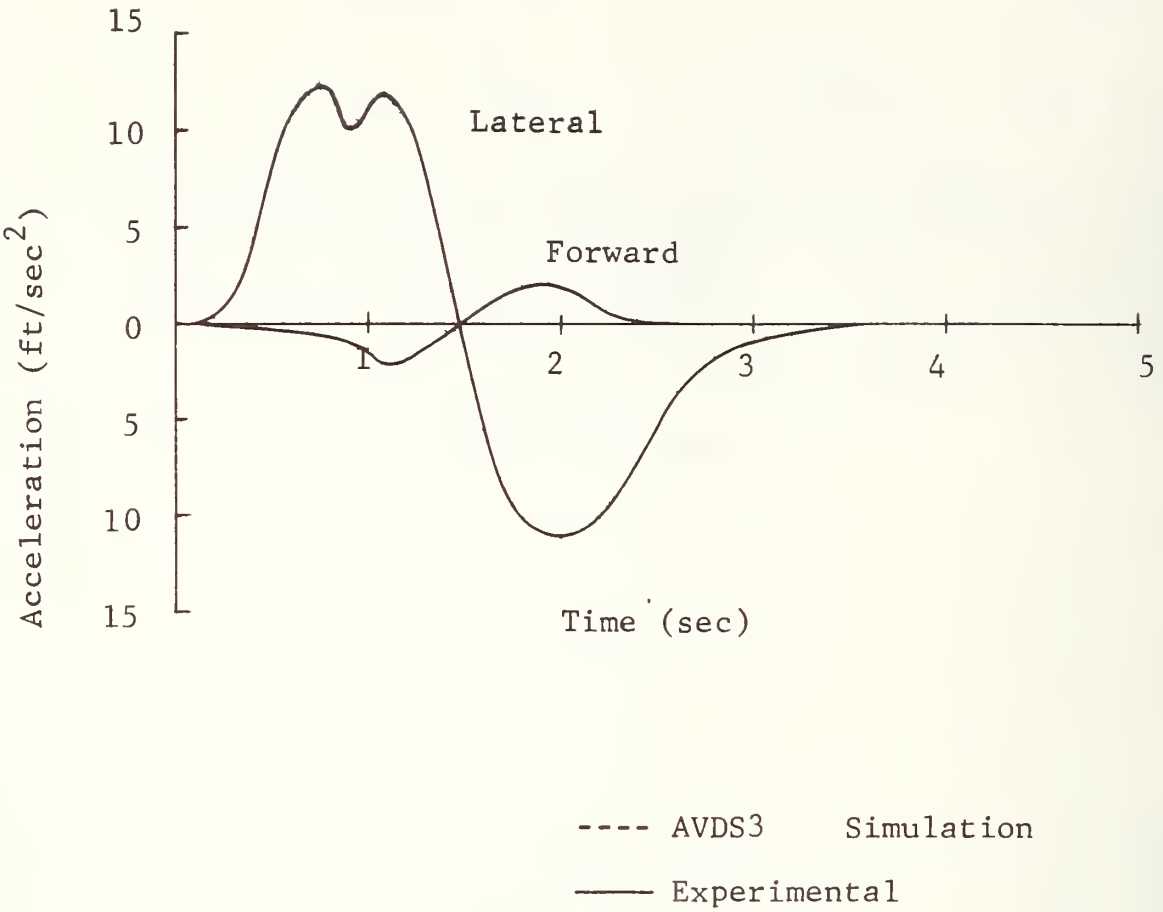
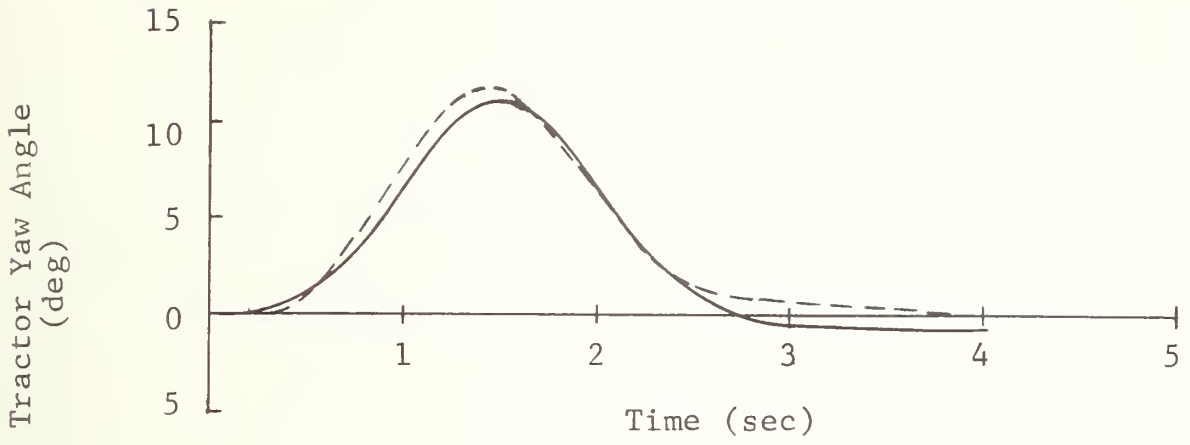
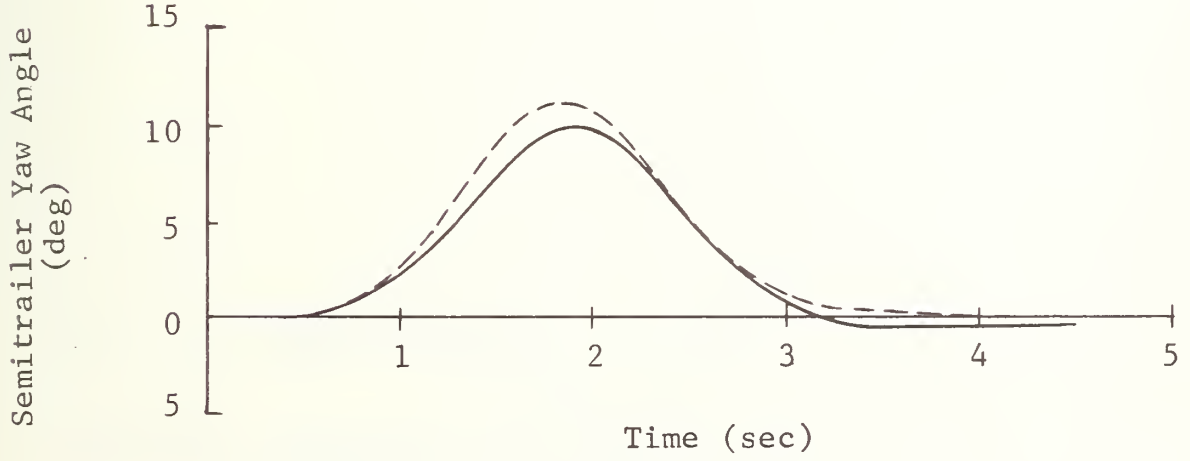


Figure 19. Analytical-experimental comparison of tractor single unloaded semitrailer (27 ft) response for experiment No.2-51; lane change (13.7 ft x 188 ft) at 40 mph without braking.





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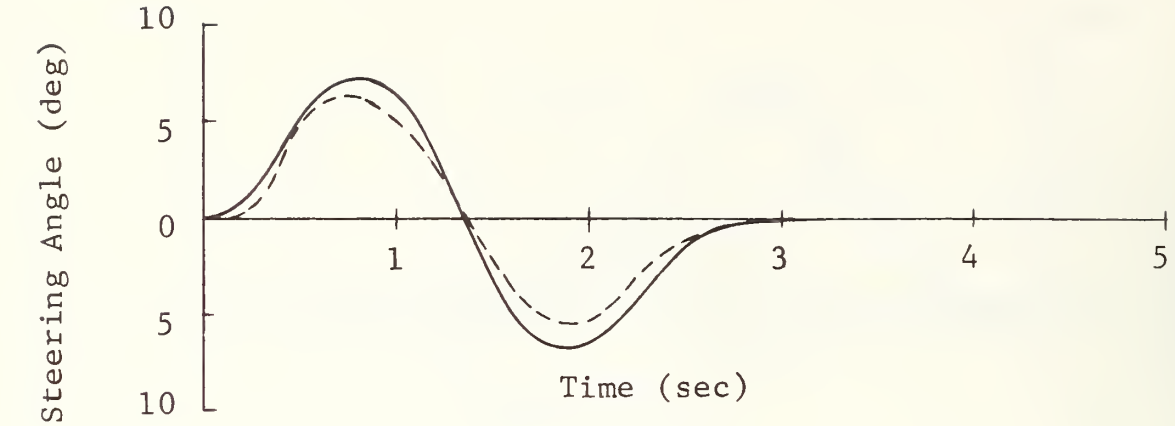


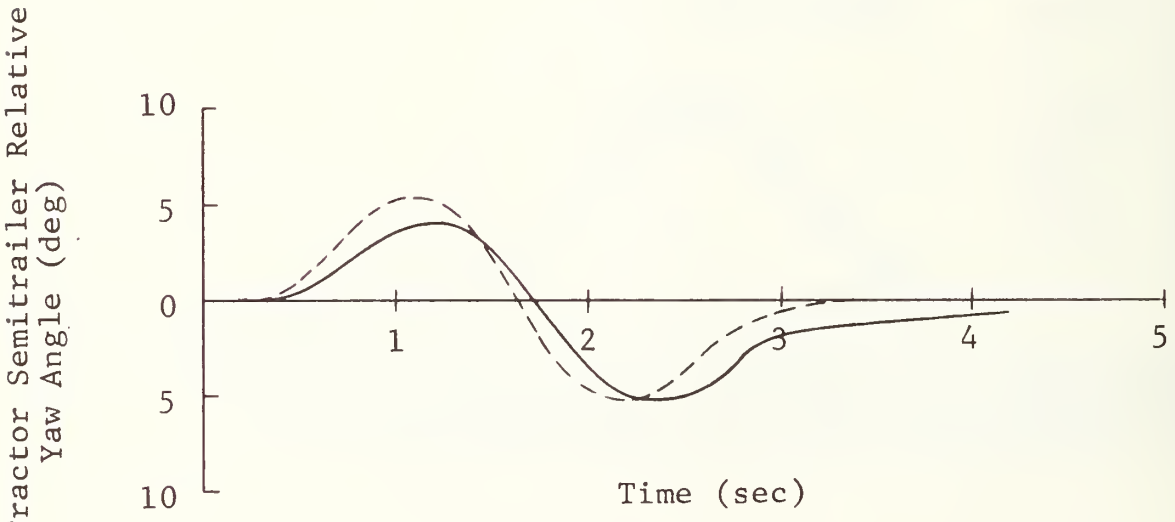
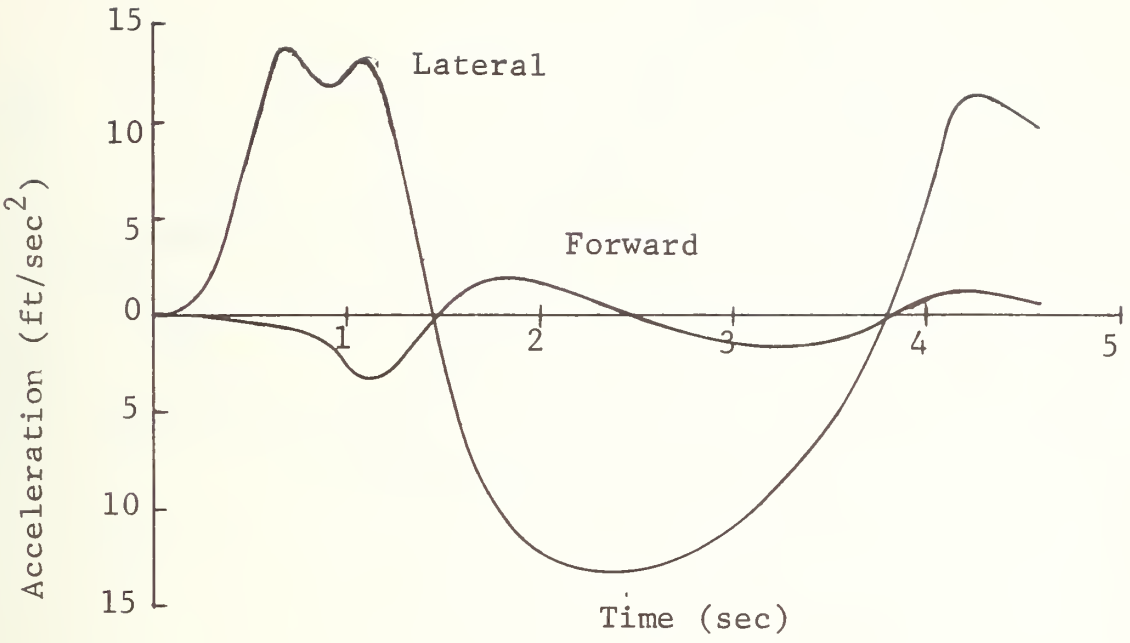


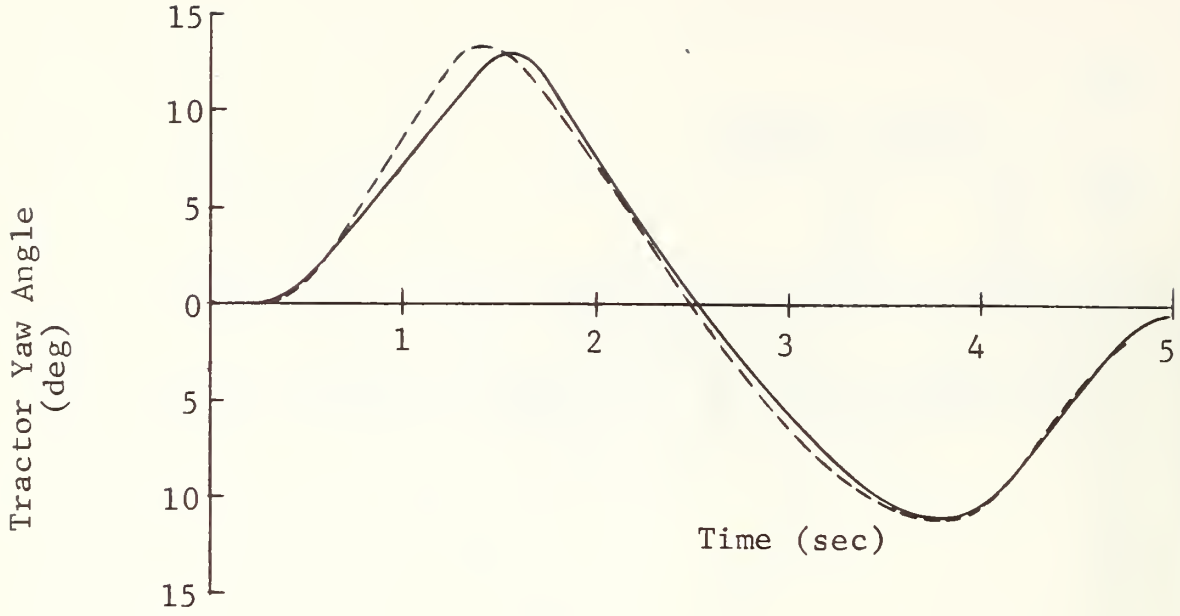
Figure 19. (concl)

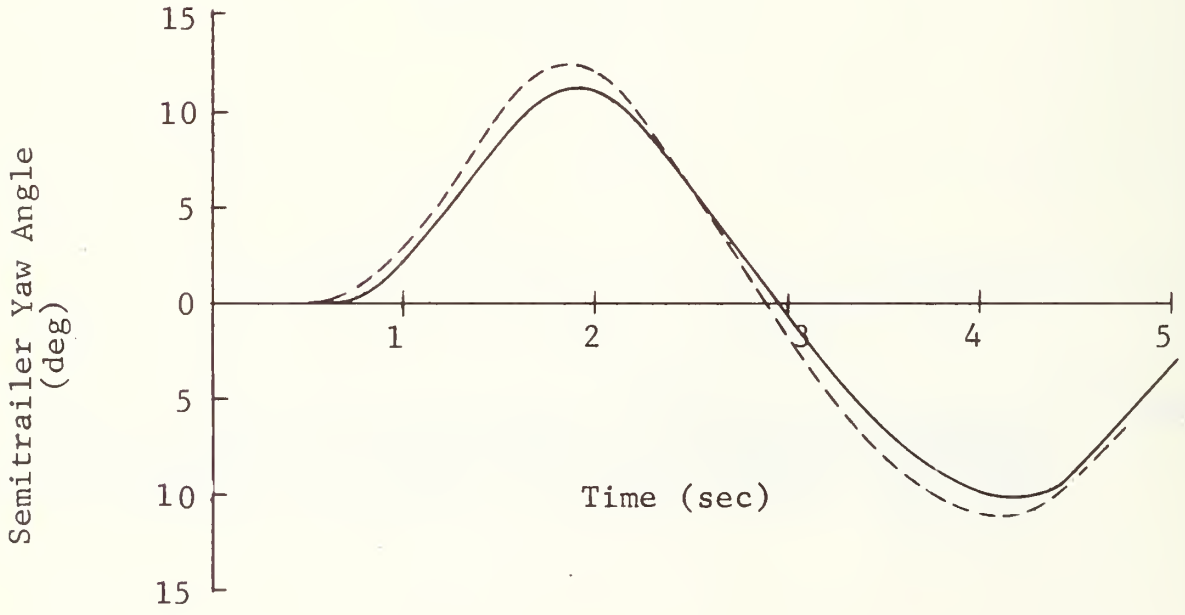


---- AVDS 3 Simulation

—— Experimental

Figure 20. Analytical-experimental comparison of tractor single unloaded semitrailer (27 ft) response for experiment No. 2-53; evasive (12.8 ft x 256 ft) at 40 mph without braking.





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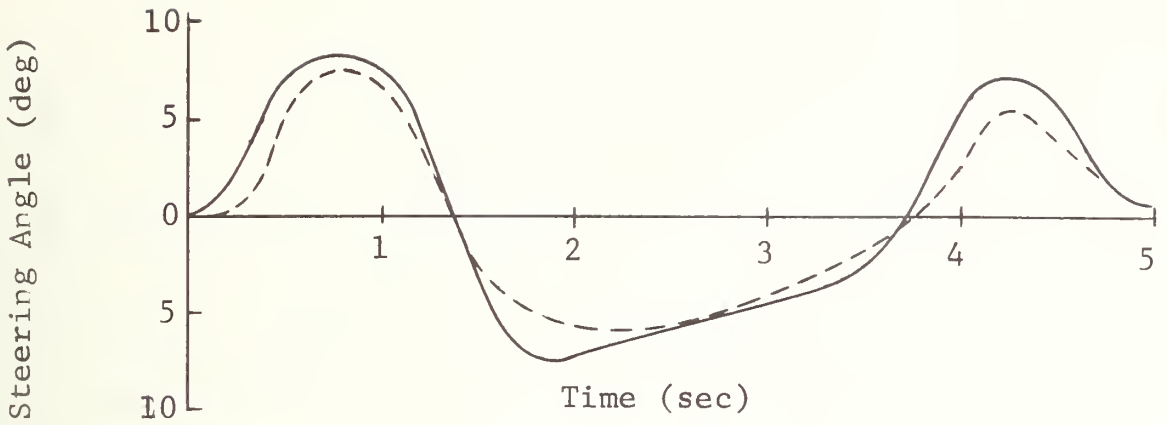

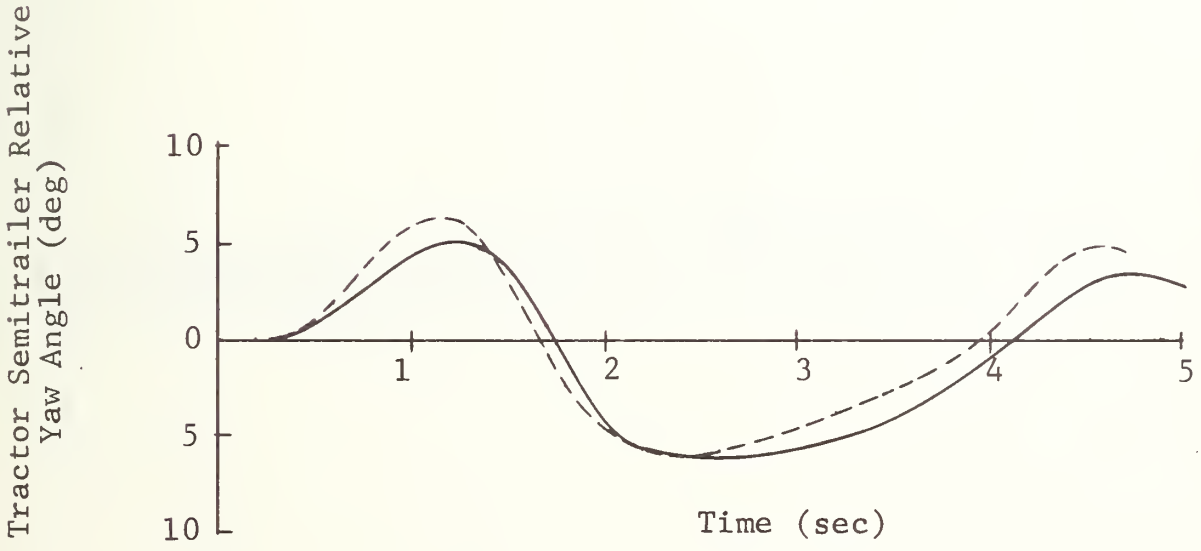

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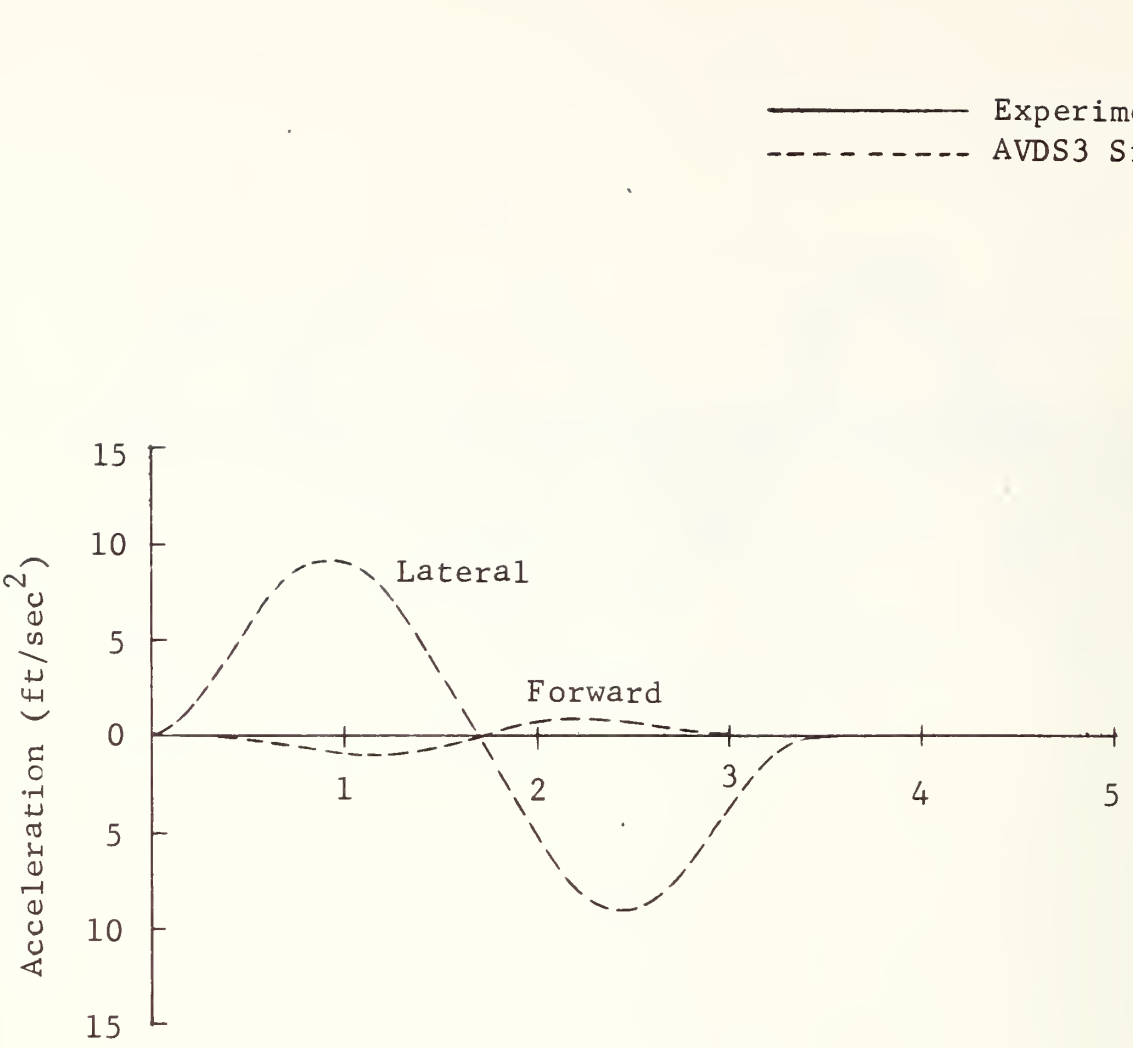
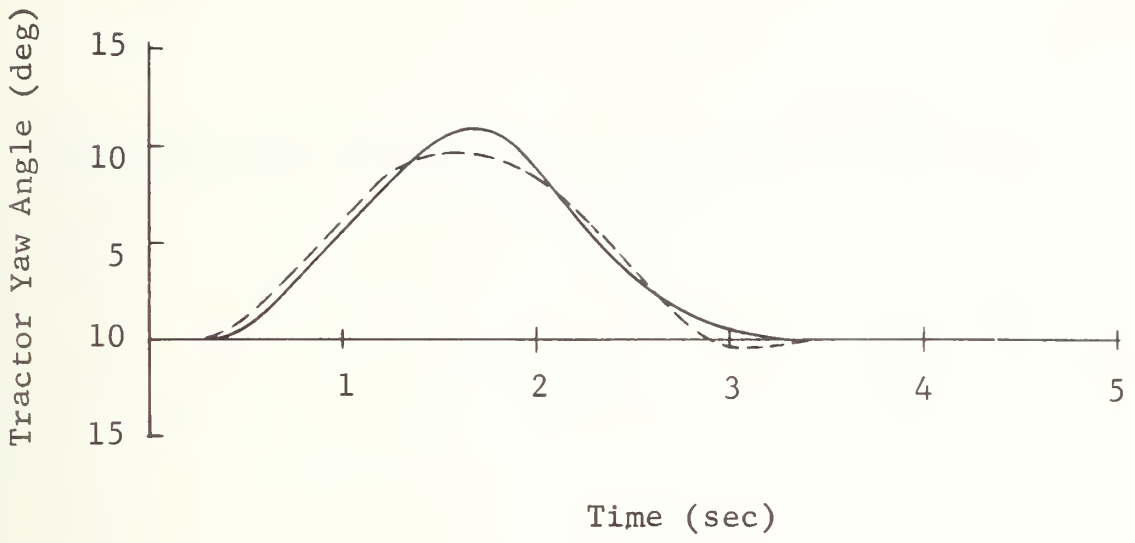


Figure 21. Analytical-experimental comparison of tractor single unloaded semitrailer (45 ft) response for experiment No.1-9; lane change (12.0 ft x 146 ft) at 40 mph without braking.



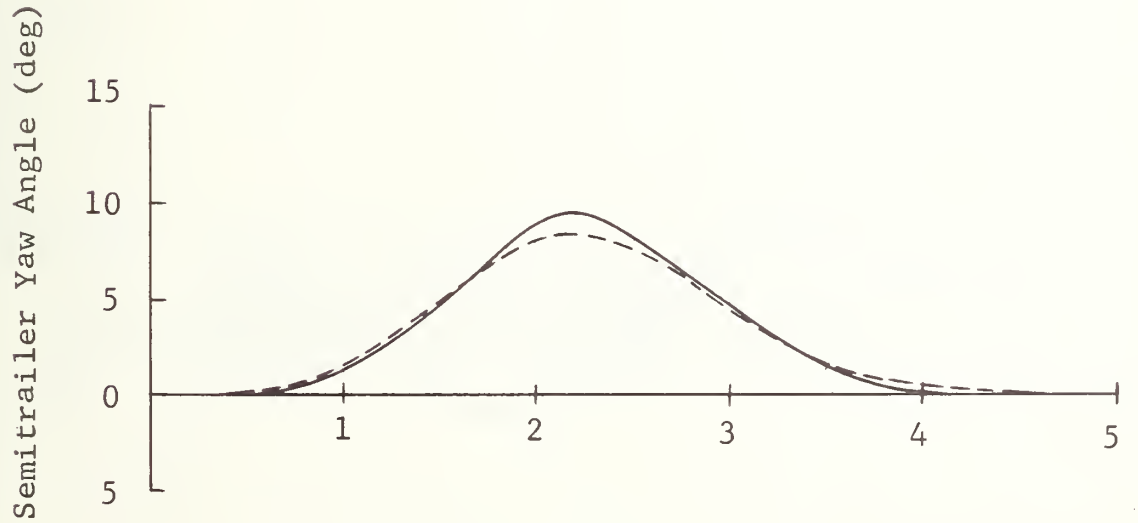
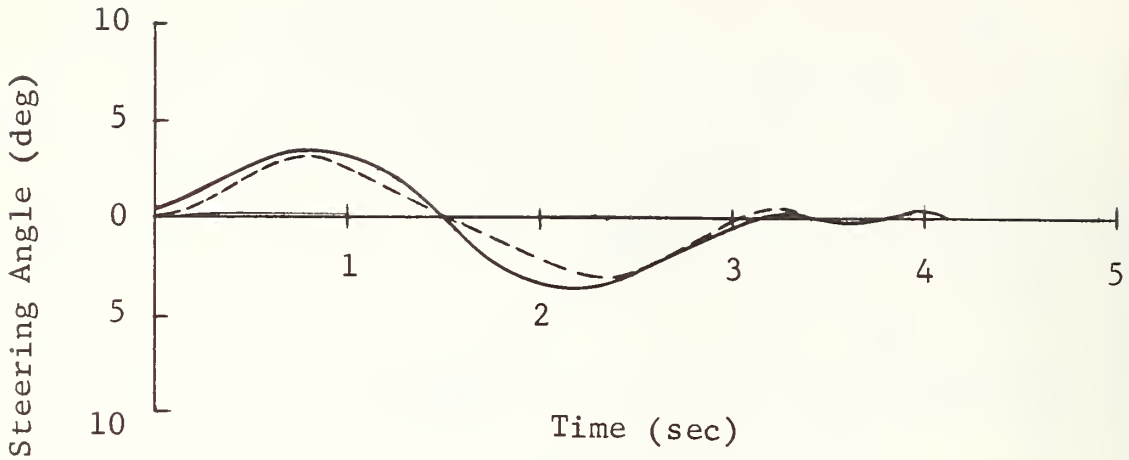


Figure 21. (contd)



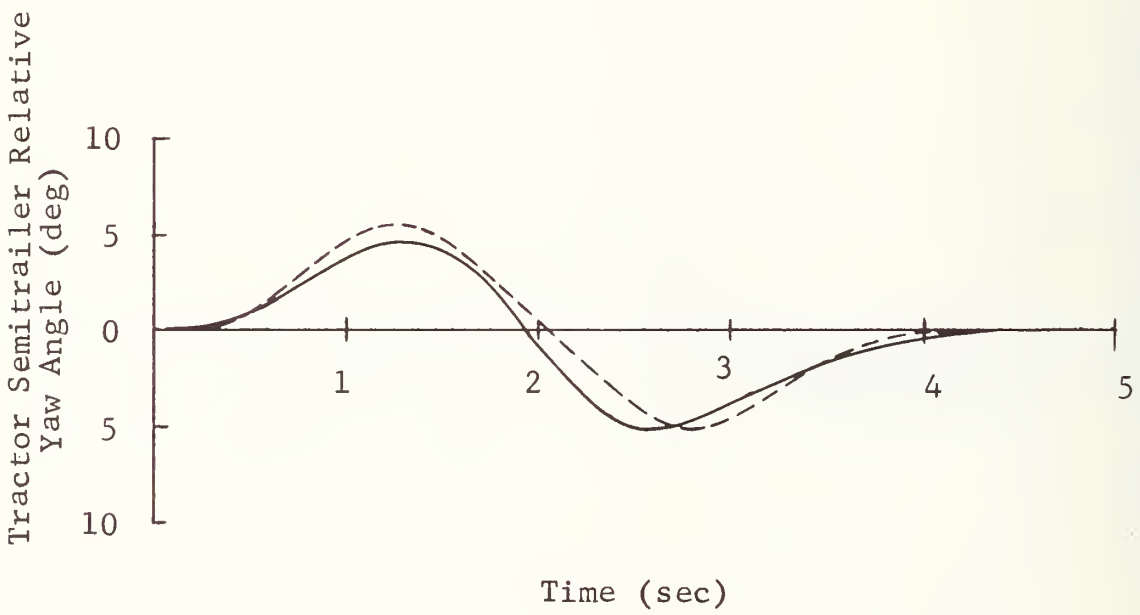


Figure 21.(concl)

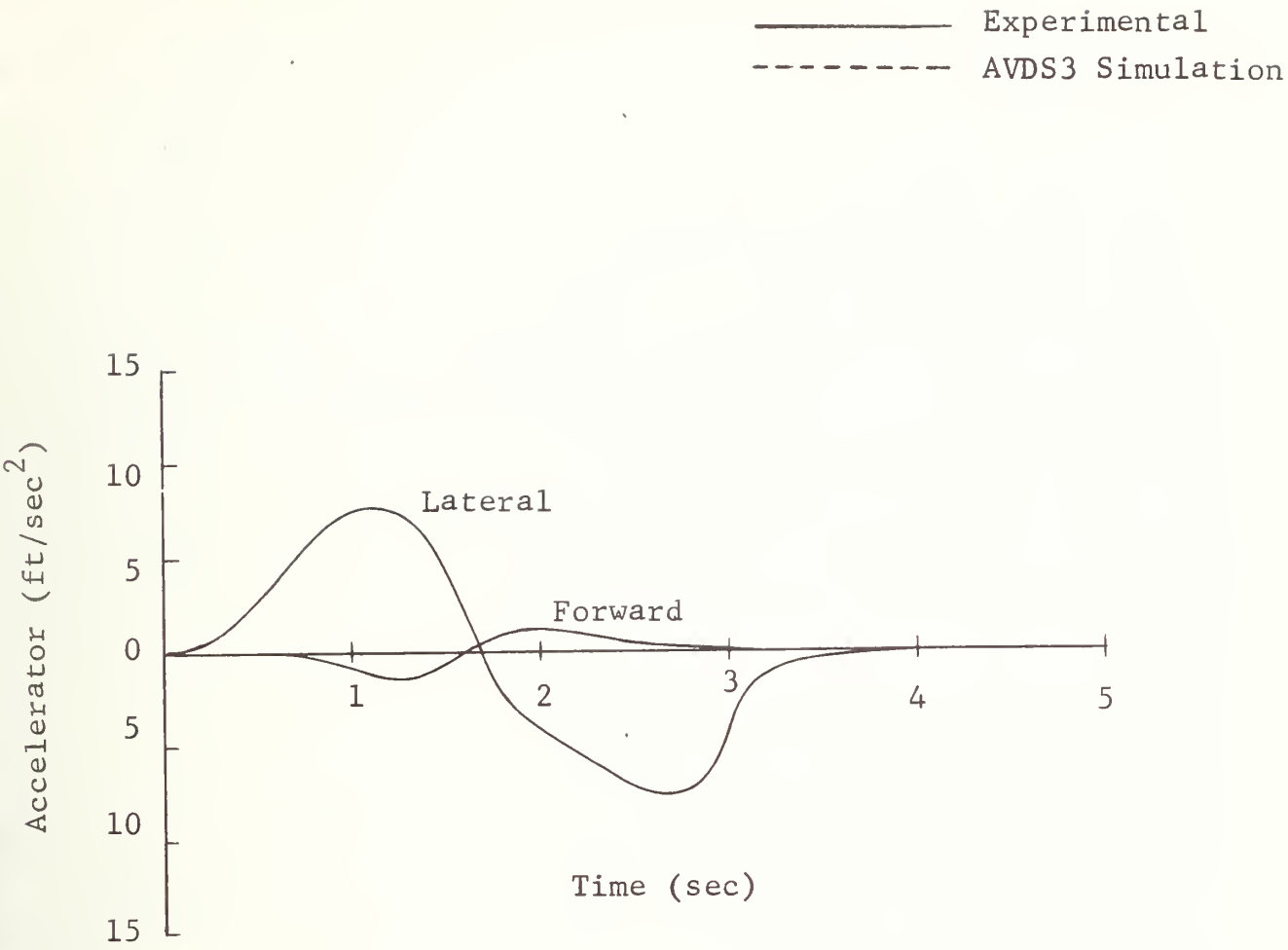
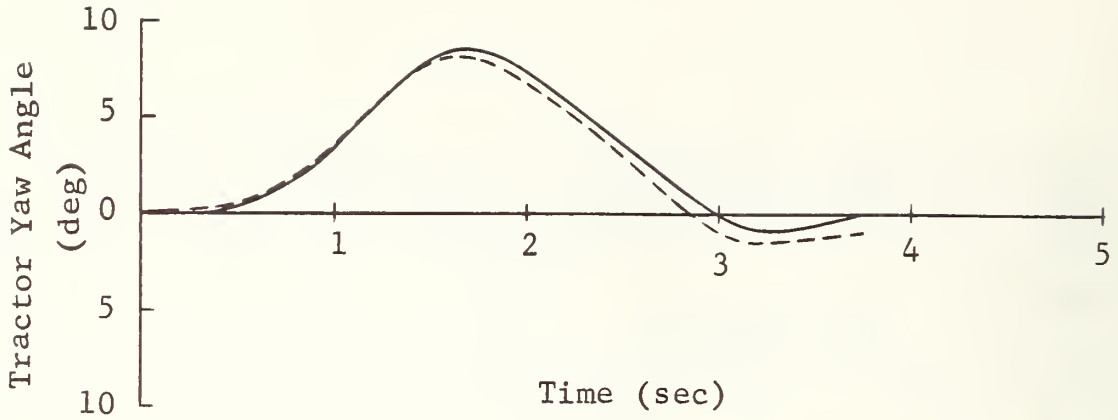
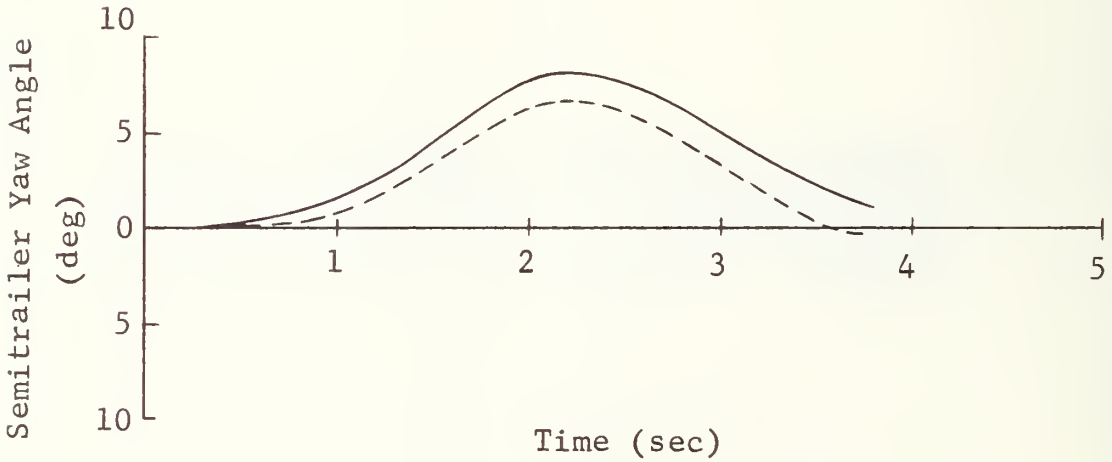


Figure 22. Analytical-experimental comparison of tractor single unloaded semitrailer (45 ft) response for experiment No.1-19; lane change (12.3 ft x 117 ft) at 40 mph without braking on a wet surface.





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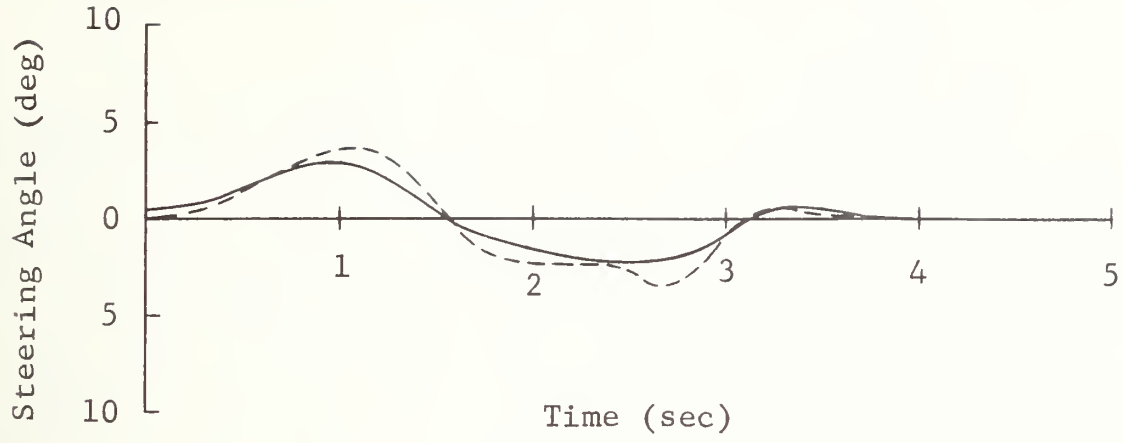


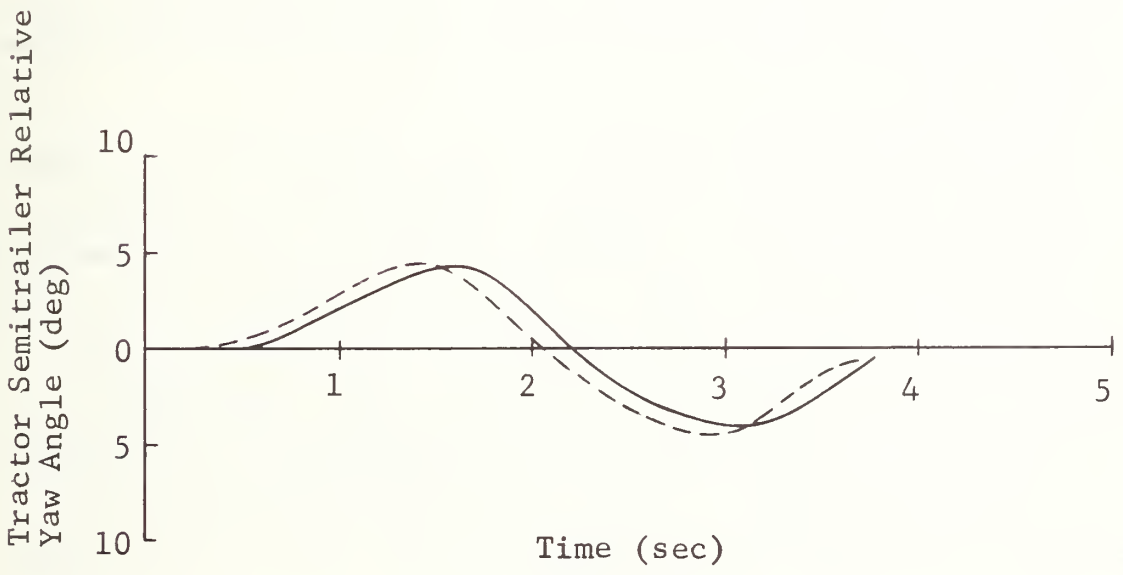


Figure 22. (concl)

Despite mixed tires, good correlation is obtained. This indicates that the functional form of the tire cornering force curve obtained from experimental data simulates well the varied tires on this vehicle.

Comparisons of experimental and analytical results are shown in Figures 23 and 24 for a tractor double-semitrailer system performing lane change and evasive maneuvers on wet and dry surfaces with the first trailer loaded and the second semitrailer unloaded. The first lane change maneuver was performed on a wet surface and some anomalies are shown in the steering simulation. It appears that the simulation may have been sensitive to the high rate of change of acceleration with respect to time approaching a numerically unstable situation. The simulation peaks follow the experimental results well except for the dolly-trailer yaw. It appears that the extension of the tire model to a wet surface did not work well in this instance. The simulation of the similar evasive maneuver with braking is extremely good; however, this maneuver was performed on a dry surface where the tire characteristics are known.

In the lane change maneuver with violent braking (Figure 25), the simulation was barely acceptable. It appears that the friction ellipse concept as presently formulated did not properly apportion tire cornering and braking forces. In the simulation, tractor and trailer yaw angles were about 25 percent higher than those obtained in the experiments. This indicates a lack of sufficient cornering force available in the simulation model. The friction ellipse concept constrained the available cornering force more than the presence of braking would have, in reality.

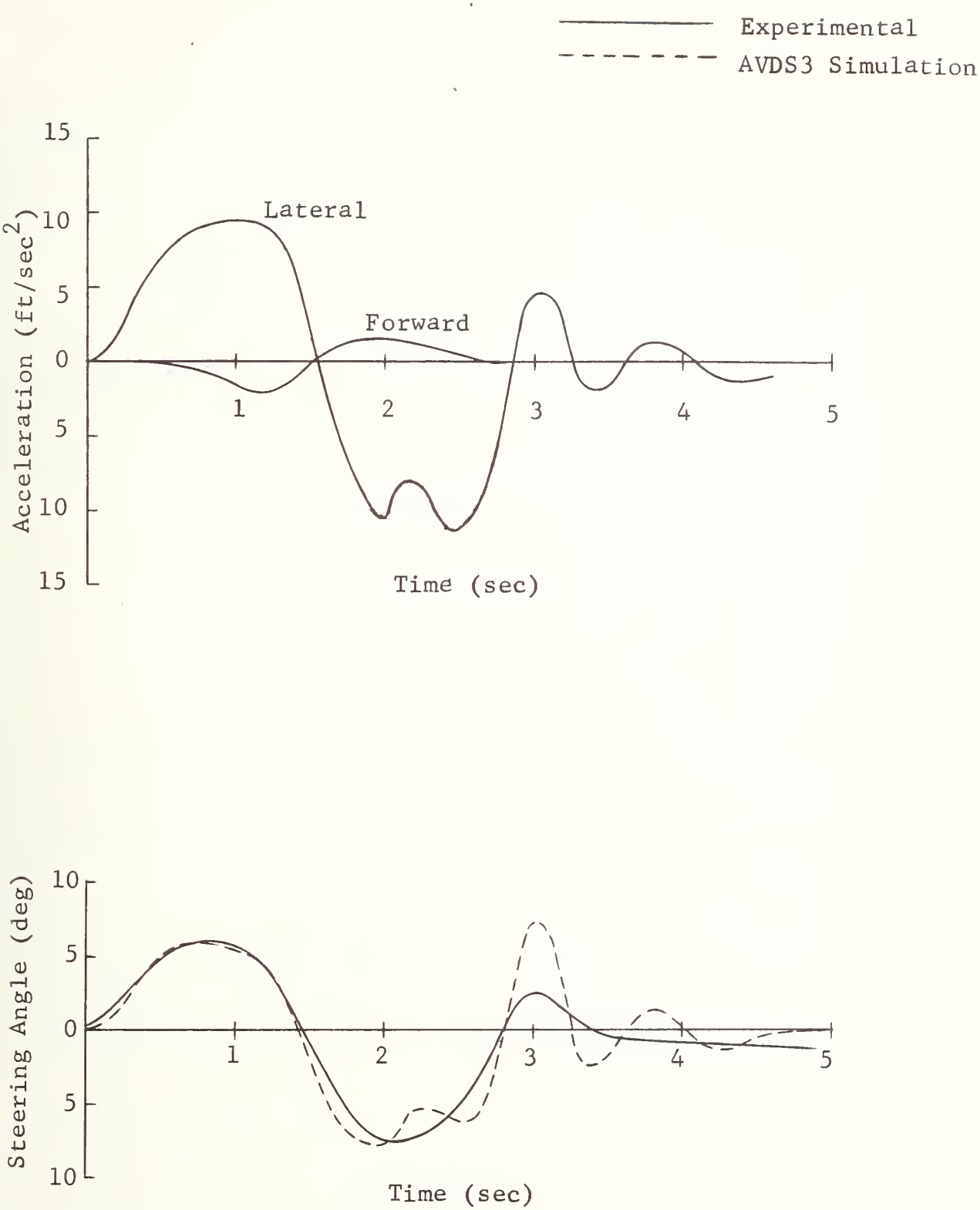
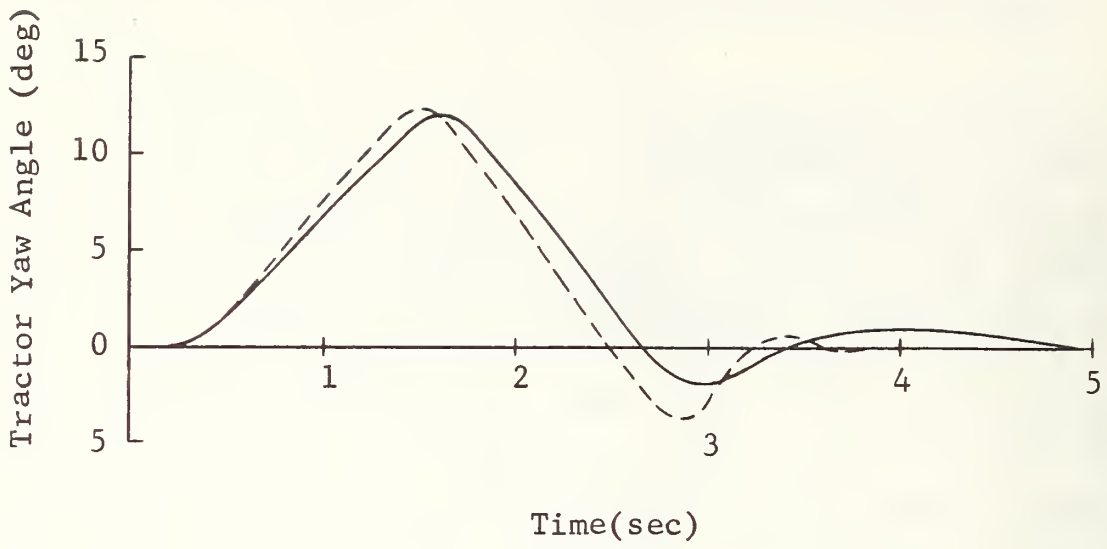


Figure 23. Analytical-experimental comparison of tractor double (1st loaded, 2nd empty) semitrailers(27 ft) response for experiment No.2-26; lane change at 40 mph without braking on a wet surface.



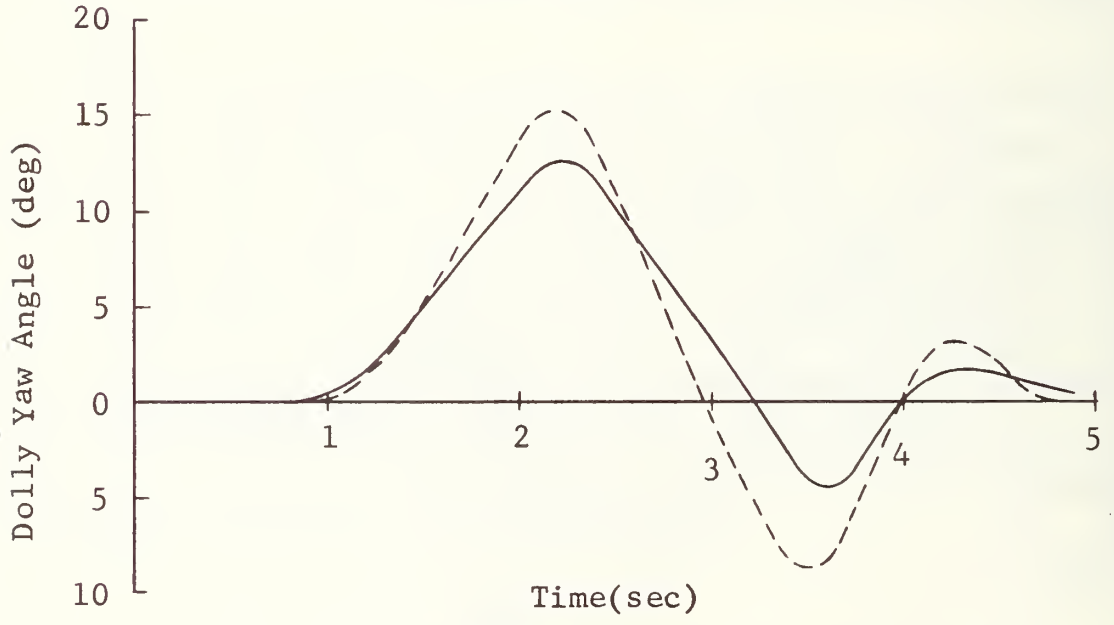
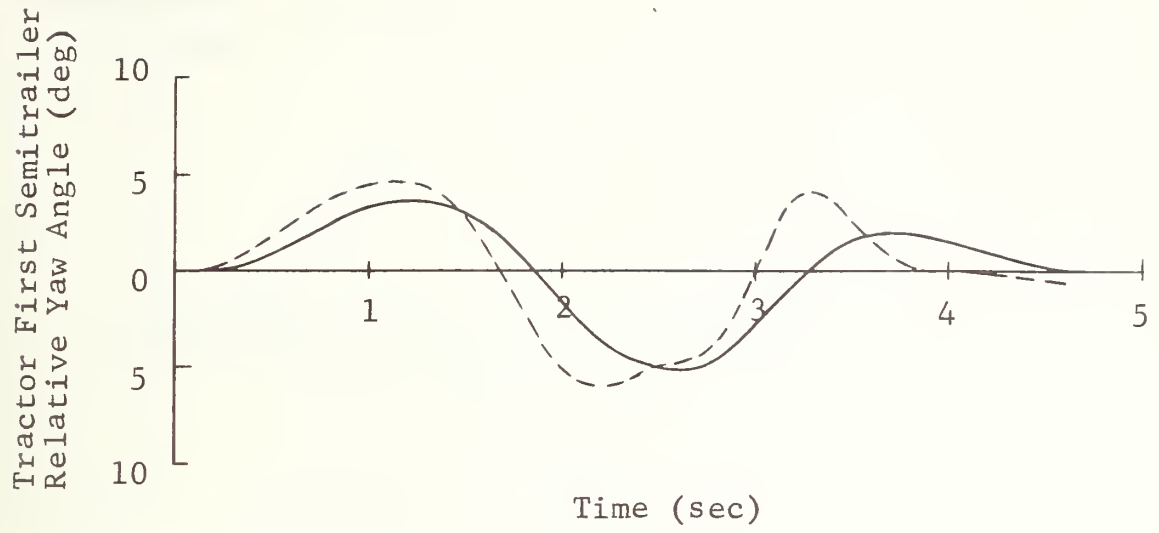


Figure 23.(contd)



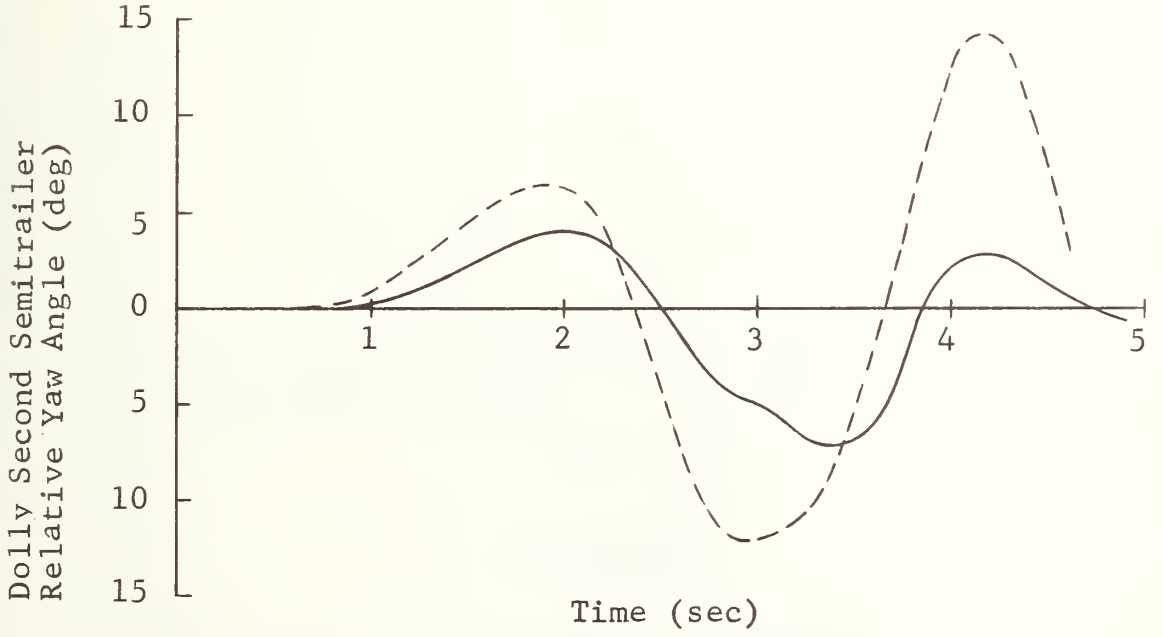


Figure 23. (concl)

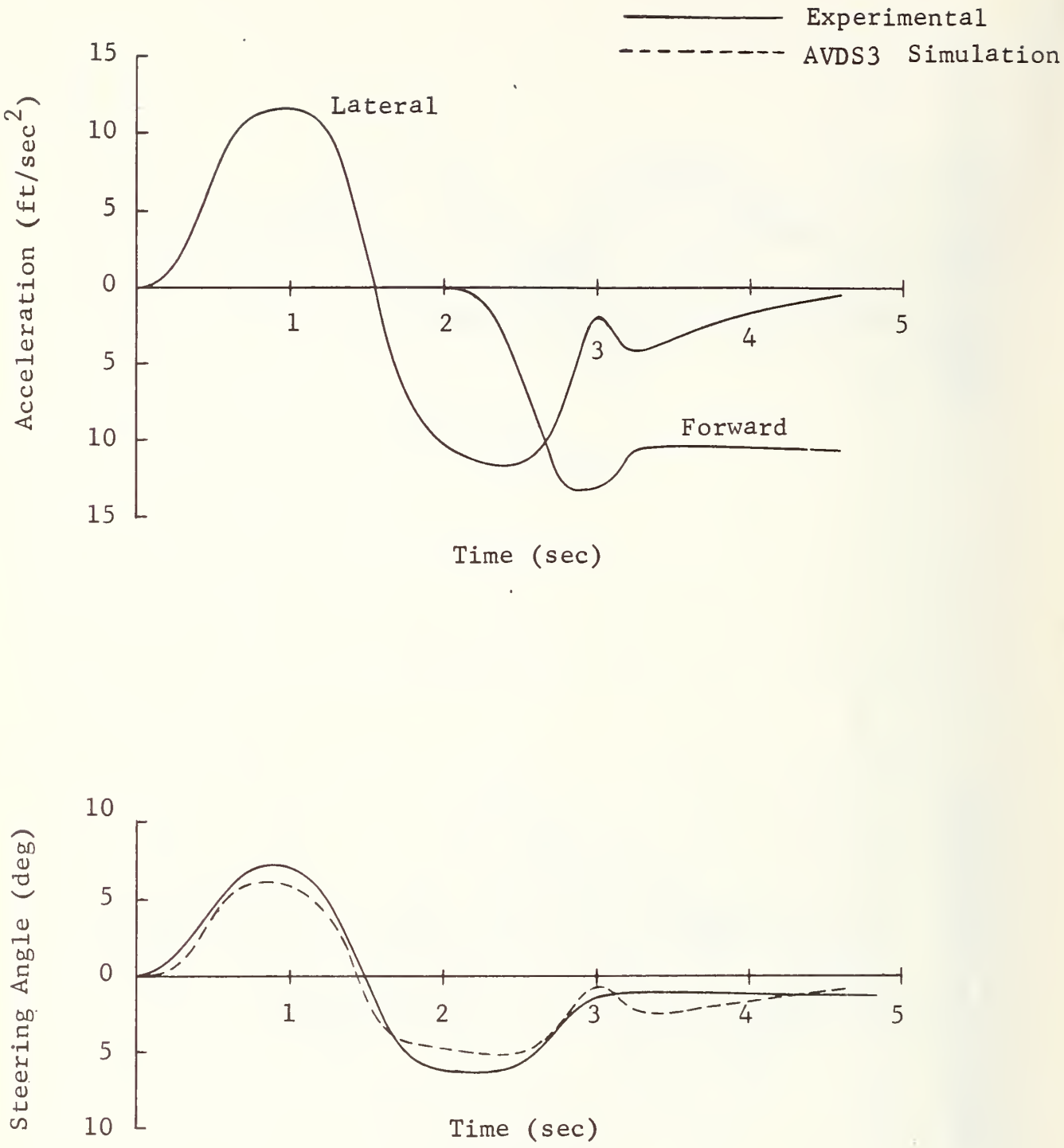
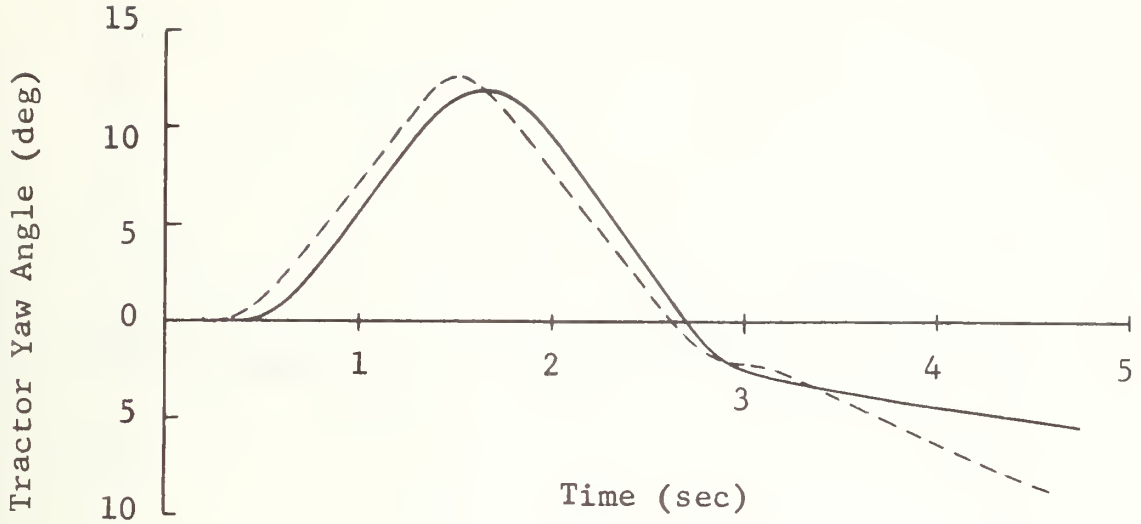


Figure 24. Analytical-experimental comparison of tractor double (1st loaded, 2nd empty) semitrailers (27 ft) response for experiment No.2-37; evasive (12.1 ft x 251) ft at 40 mph with braking.



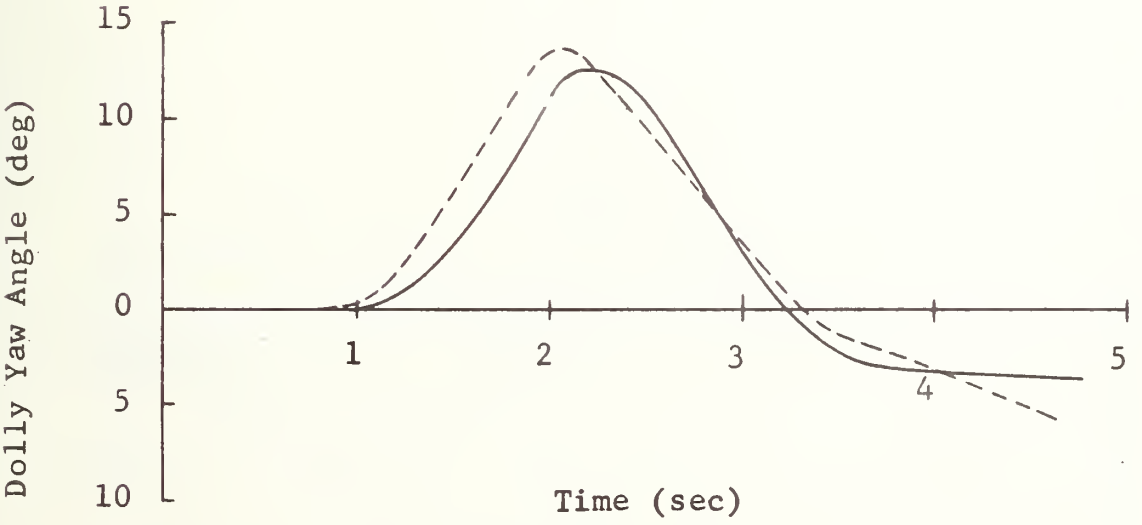
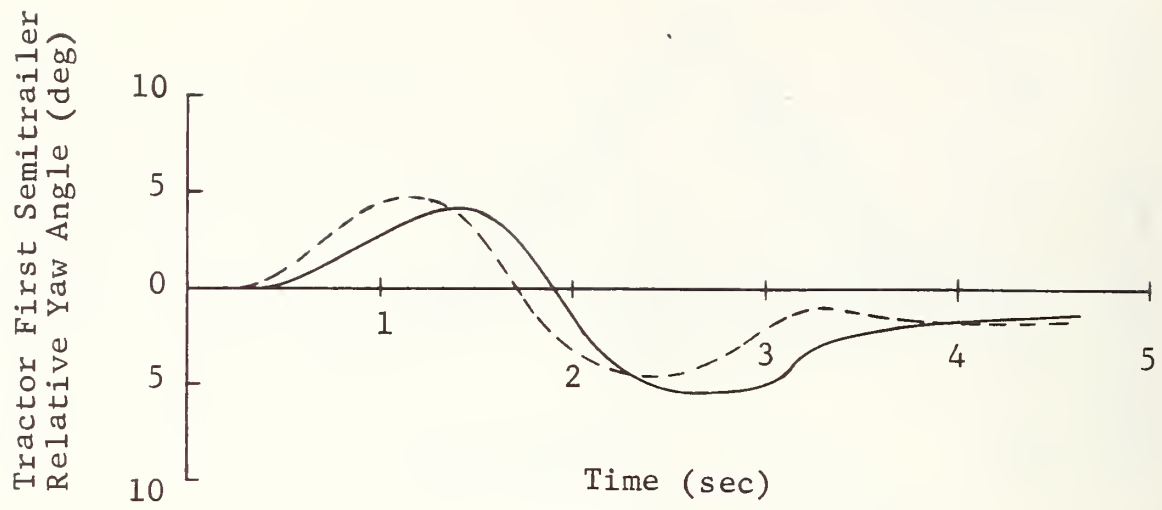


Figure 24. (contd)



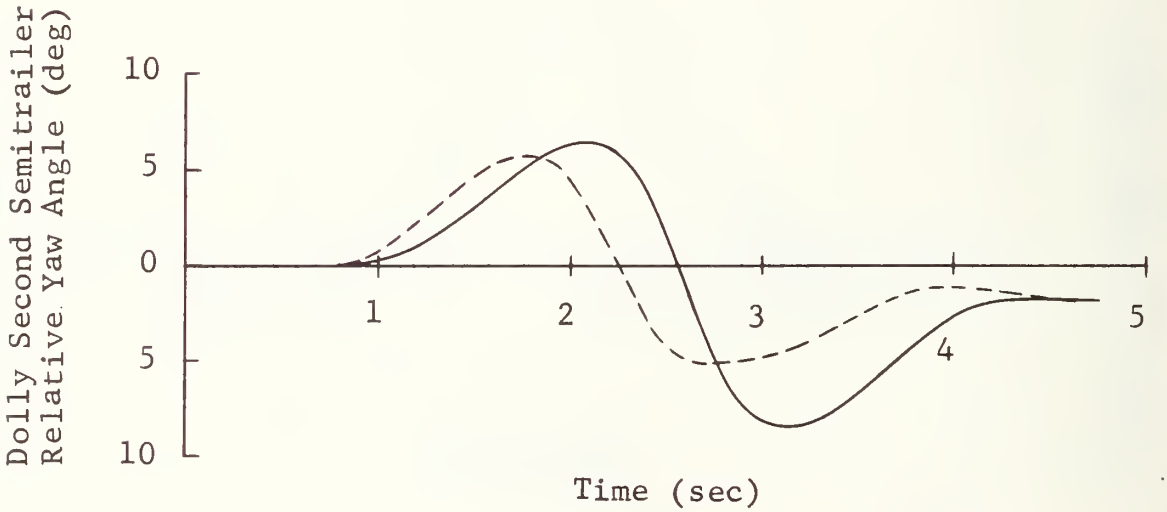


Figure 24. (concl)

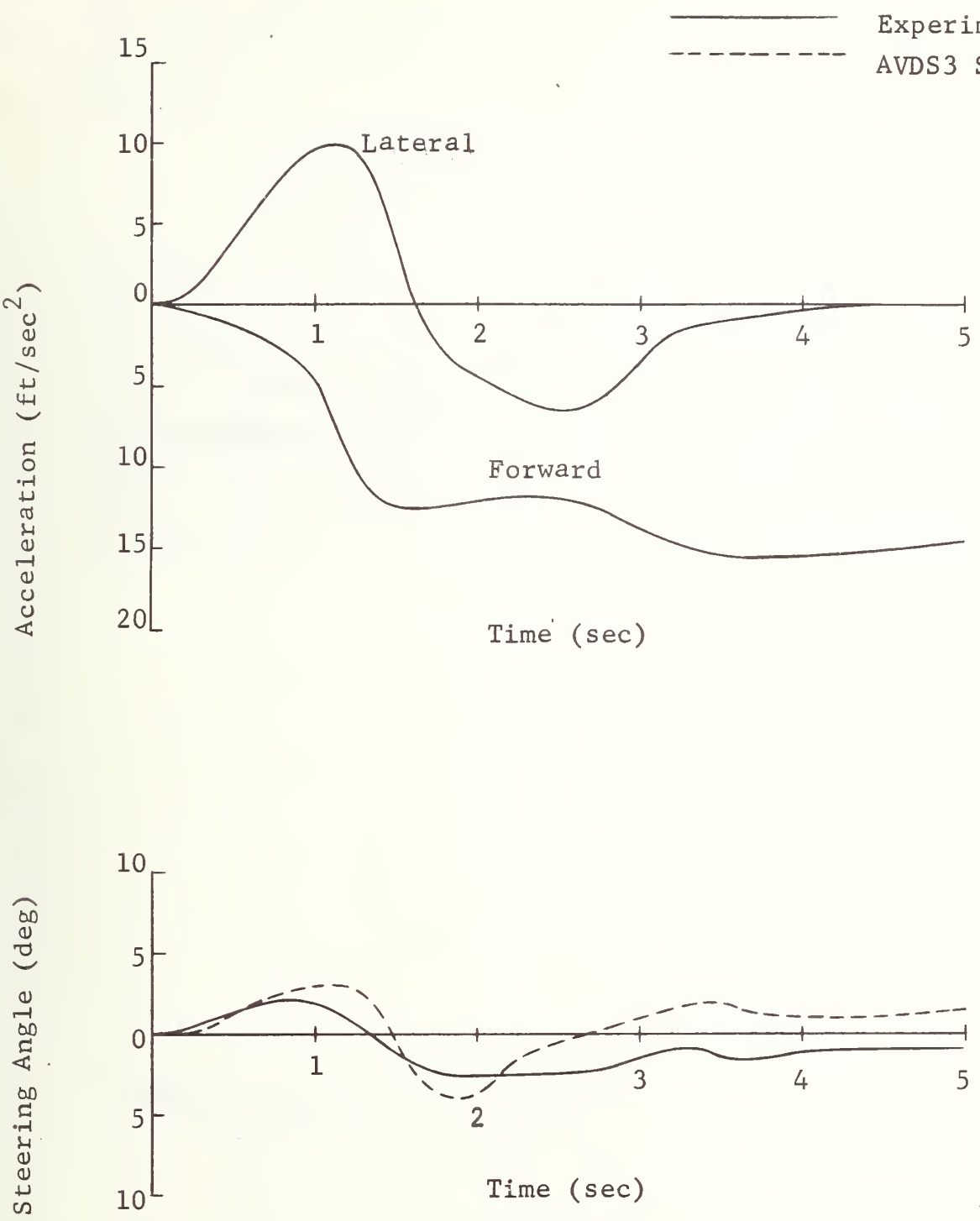
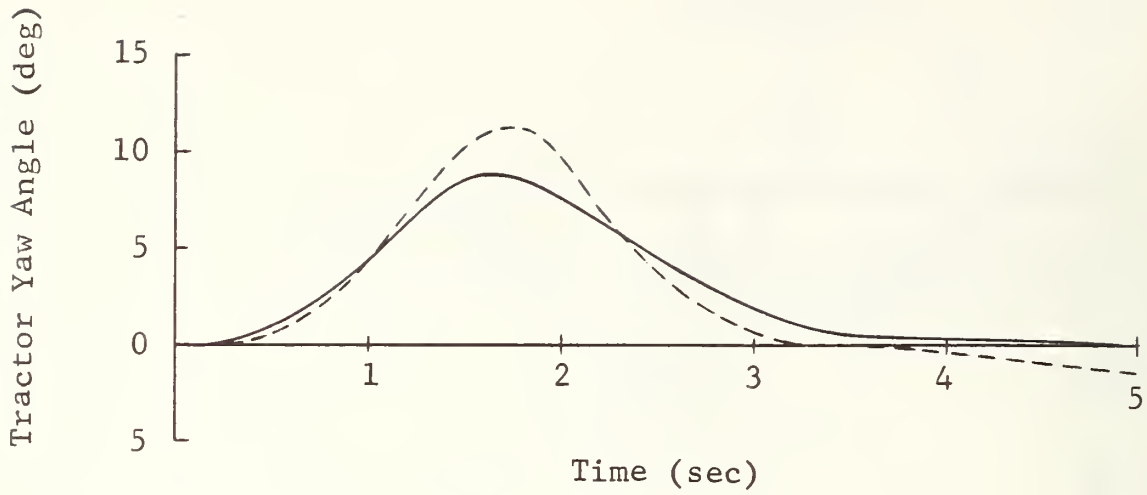


Figure 25. Analytical-experimental comparison of tractor single unloaded semitrailer (45 ft) response for experiment No. 1-12; lane change (14.3 ft x 222 ft) at 40 mph with braking.



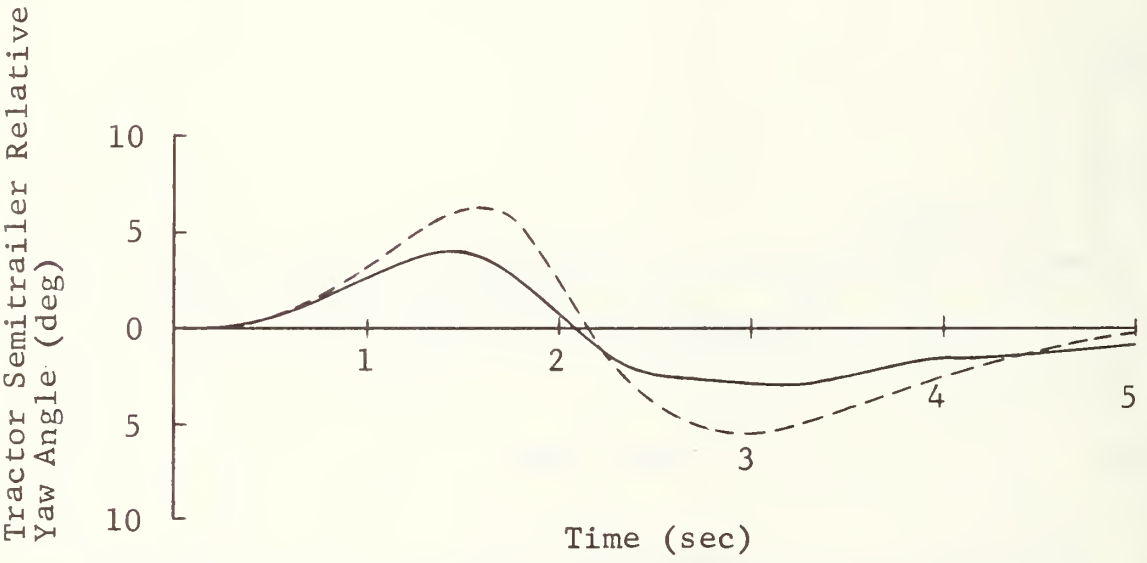


Figure 25. (concl)

The last three tests used to validate the simulation model were cornering maneuvers performed by the following vehicle configurations.

- Tractor 45 ft semitrailer (Figure 26)
- Tractor double 27 ft semitrailers (Figure 27)
- Tractor single 27 ft semitrailer (Figure 28)

Since these maneuvers are steady state in nature only, the peak measured parameters are compared to simulation results. Simulation results follow the experiments closely with the largest discrepancy occuring on the steering demand of the double unit. Results show similar yaw angles between the tractor first semitrailer and the dolly second semitrailer. Lateral acceleration and steering are similar for the single and double unit.

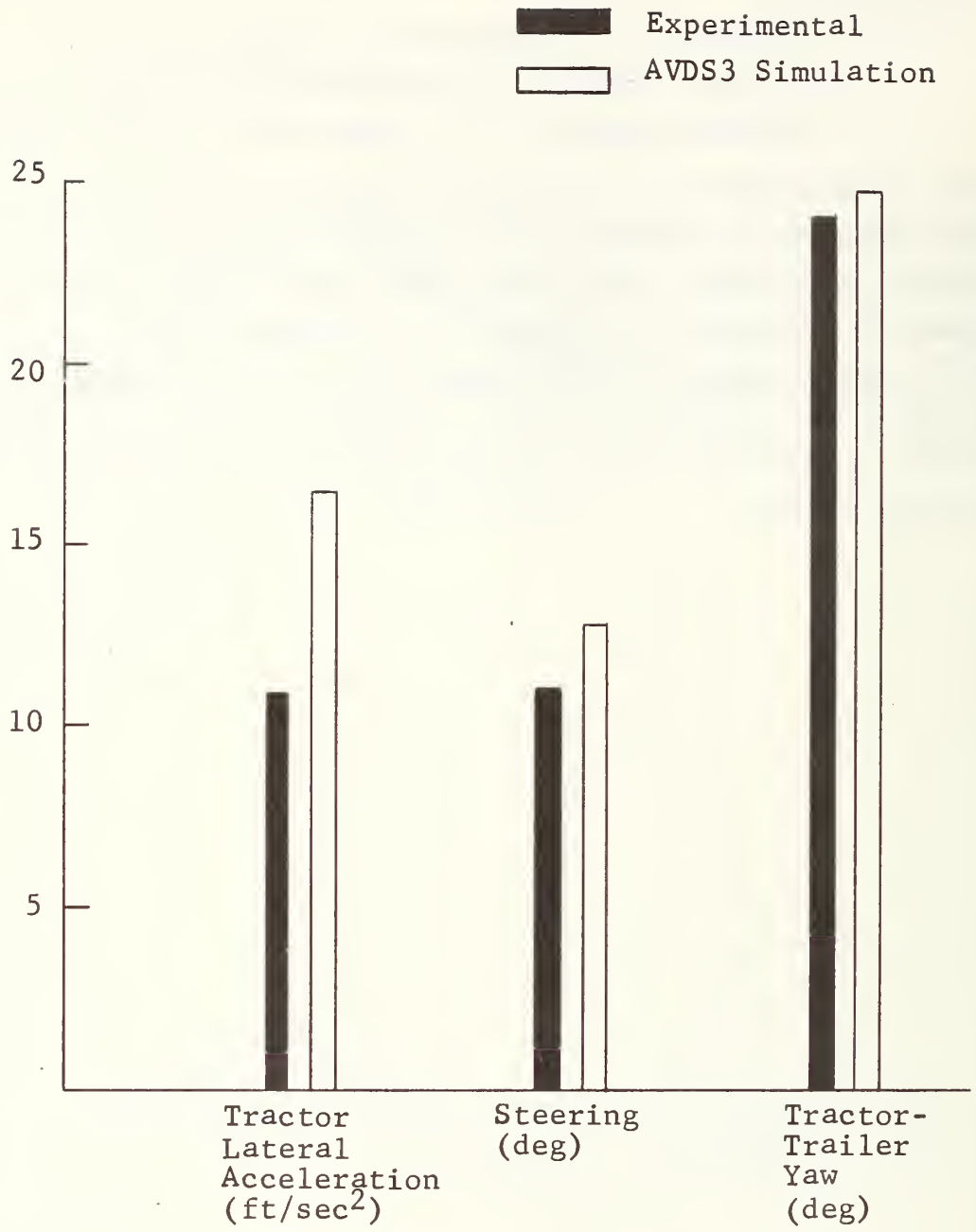


Figure 26. Peak response of a tractor single unloaded trailer (45 ft) in a steady state cornering maneuver on a dry surface; experiment No. 1-15.

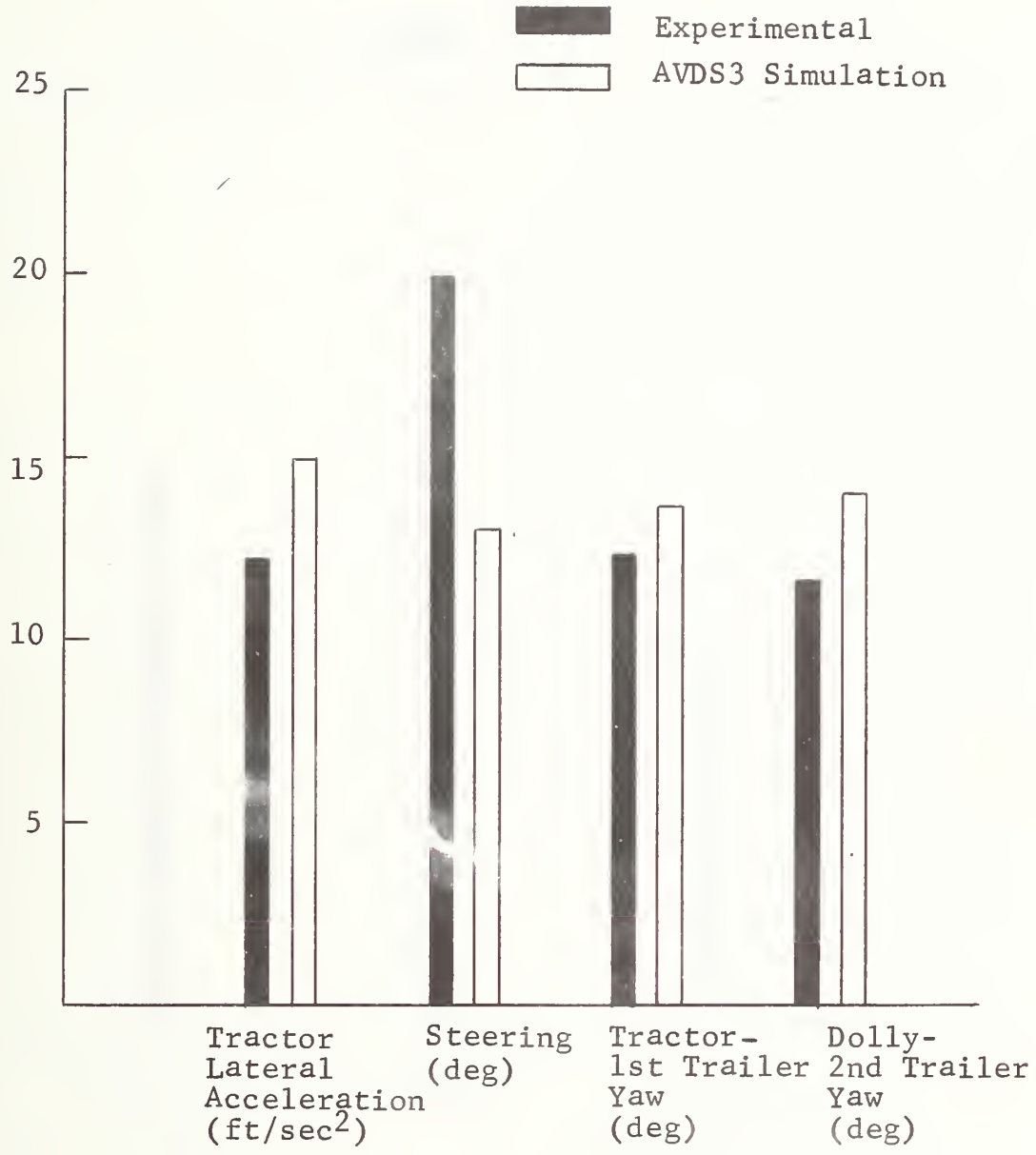


Figure 27. Peak response of a tractor double unloaded semi-trailers (27 ft) in a steady state cornering maneuver on a dry surface; experiment No. 2-9.

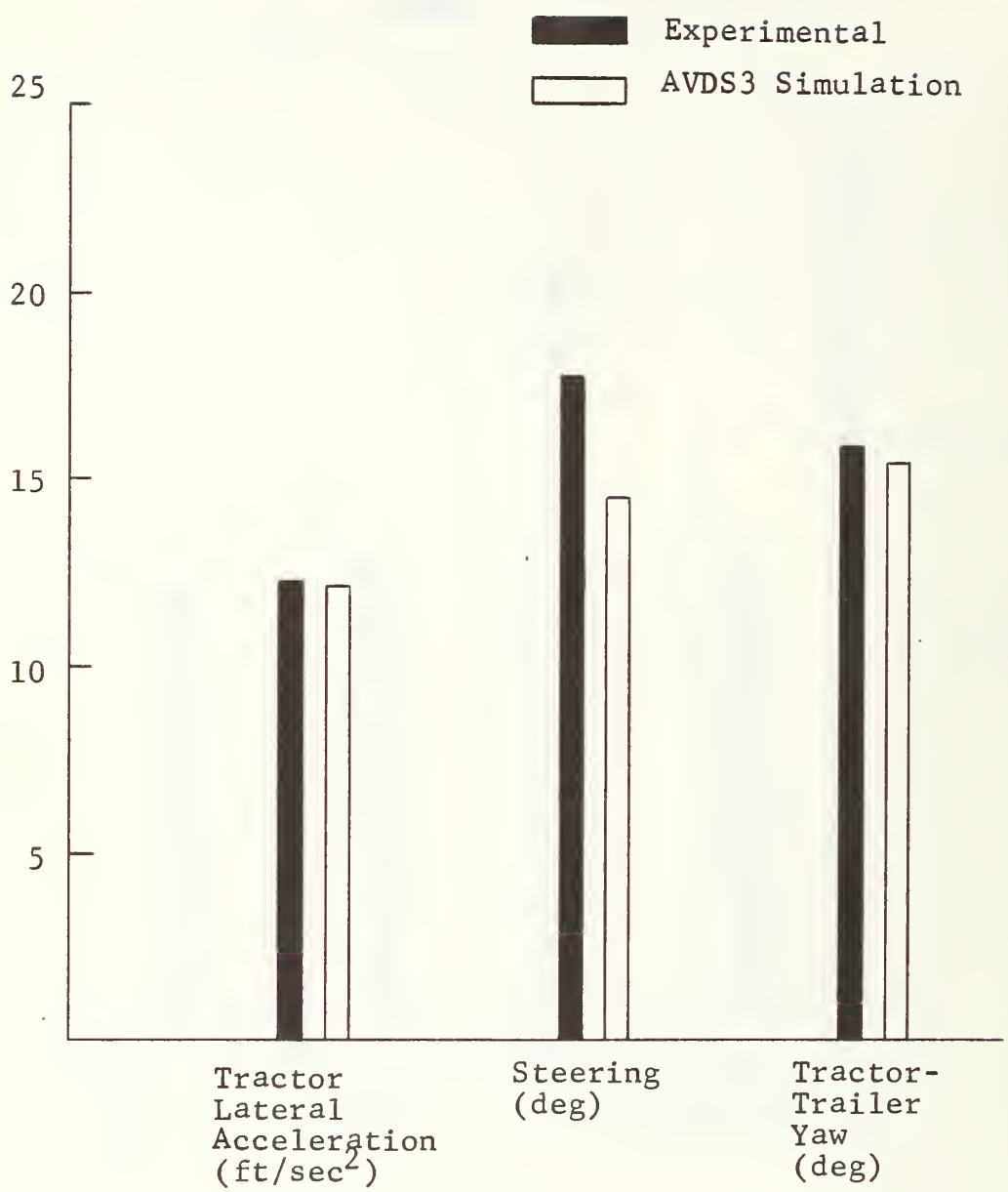


Figure 28. Peak response of a tractor single unloaded trailer (27 ft) in a steady state cornering maneuver on a wet surface; experiment No. 2-59.

7. PARAMETER STUDY

A parameter study was conducted on the vehicles described in Table 2 (the test vehicles utilized in this program) to establish stability limits for varied maneuvers, braking, road-tire interface, loading and speed. The stability criteria utilized to discern these stability limits are described in detail in Section 4. Basically, loss of side force on the tire at the road interface constitutes instability. Most of the results of this section have been verified by selected test points. Stability limits are given herein in terms of tire-road friction. For this reason, at best two verifications (wet and dry road surface) are given per limit diagram.

An experimental-analytical comparison of specific stability results generated for single and double articulated vehicles in this program is shown in Table 8. In general the stability limits generated by Lyapunov and AVDS3 are higher than the experimental results.

The first series of analytical and experimental results are for cornering of tractor semitrailer systems cornering without braking. Limit velocities obtained through simulation are about 10 percent higher than the experimentally obtained limits. This indicates that the stability criterion of requiring the total front axle to skid at the inception of instability yields limits which are unattainable. This is realistic since the AVDS3 simulation yields physical stability limits for the perfect driver, vehicle and road condition which should be unattainable in practice. The same conclusion can be drawn for lane changing and evasive maneuvers on a dry surface. For lane changing on a wet surface, test results (tests 28-2, 57-2 and 26-2) indicate that the driver exceeded the AVDS3 generated limits. This result can be attributed to either an incorrect assessment of the road surface/tire friction interface or the fact that the driver can tolerate some tire skid during the maneuver. Specific results of the parameter study are discussed individually in the following sections.

Table 8

EXPERIMENTAL-ANALYTICAL COMPARISON OF ARTICULATED VEHICLE STABILITY LIMITS

Vehicle Config.	Maneuver	Surface	Load Config.	Test No. Series	Limit Velocity Exp. (ft/sec)	Anal. (ft/sec)
27 ft single	C	Wet	U	59-2	30.18	31.75
27 ft double	C	Dry	U	9-2	31.50	36.50
		Wet	U	43-2	25.32	31.60
		Dry	H	31-2	32.40	36.50
		Wet	H	30-2	25.50	31.60
		Dry	F	16-2	32.80	36.50
		Wet	F	17-2	26.25	31.60
45 ft single	C	Dry	U	14-1	33.80	37.50
		Wet	U	21-1	25.90	31.75
		Wet	L	23-1	29.25	31.75
					Limit Lateral Acceleration (g, ft/sec^2)	
27 ft single	E	Dry	U	53-2	0.415	0.580
		Wet	U	57-2	0.409	0.400
	L.C.	Wet	U	55-2	0.396	0.400
27 ft double	E	Dry	U	41-2	0.441	0.600
		Wet	U	47-2	0.396	0.400
		Dry	H	36-2	0.422	0.640
		Wet	H	28-2	0.402	0.335
		Dry	F	14-2	0.400	0.625
27 ft double	L.C.	Dry	U	39-2	0.484	0.600
		Wet	U	45-2	0.368	0.400
		Wet	H	26-2	0.359	0.335
45 ft single	L.C.	Dry	U	9-1	0.340	0.580
		Wet	U	18-1	0.258	0.400

*U = unloaded
H = second trailer unloaded
F = fully loaded
L = loaded

C = circle
E = evasive
L.C = lane change

Stability limits in terms of tractor speed as a function of road-tire friction coefficient are shown in Figure 29 for a tractor-semitrailer (45 ft) in a 14 x 145 ft lane change maneuver without braking. Test 9 of series No. 1 conducted on a dry surface shows good correlation with the results of the AVDS3 simulation. In Figure 30, stability limits generated by AVDS3 and Lyapunov are shown for the same vehicle in a 75 ft cornering maneuver. Tests 15 (dry) and 21 (wet) confirm these results.

In Figure 31, the effect of load on the stability of a tractor double 27 ft semitrailer system performing an 82 ft radius cornering maneuver without braking is shown. The results obtained from the simulation model AVDS3 showed indistinguishable limit velocites for the three cases. The test results shown on this figure lead to a similar conclusion; however, there is a fine difference in limit velocity consistently showing highest limit for a loaded vehicle and lowest for an unloaded vehicle both on wet and dry surfaces. The difference in limit velocity is only 1 ft/sec.

Figure 32 shows stability limits for single tractor semitrailers performing 74 ft cornering maneuvers without braking. The AVDS3 generated stability limits show little discernable difference between the two test configurations (see Table 2) which were slightly different. A 45 ft trailer with tandem axle was used in the first series of experiments (tests 14 and 23) and a 27 ft trailer with single axle was used in test 59. Experimental results show slightly lower limits for the 45 ft trailer. Good experimental-analytical correlation is obtained. Figure 33 is a plot of vehicle kinematics showing the relationship between tractor c.g. lateral acceleration and maneuver velocity and radius for cornering. Figure 34 shows the stability of tractor single, double, and triple semitrailer systems performing cornering maneuvers without braking.

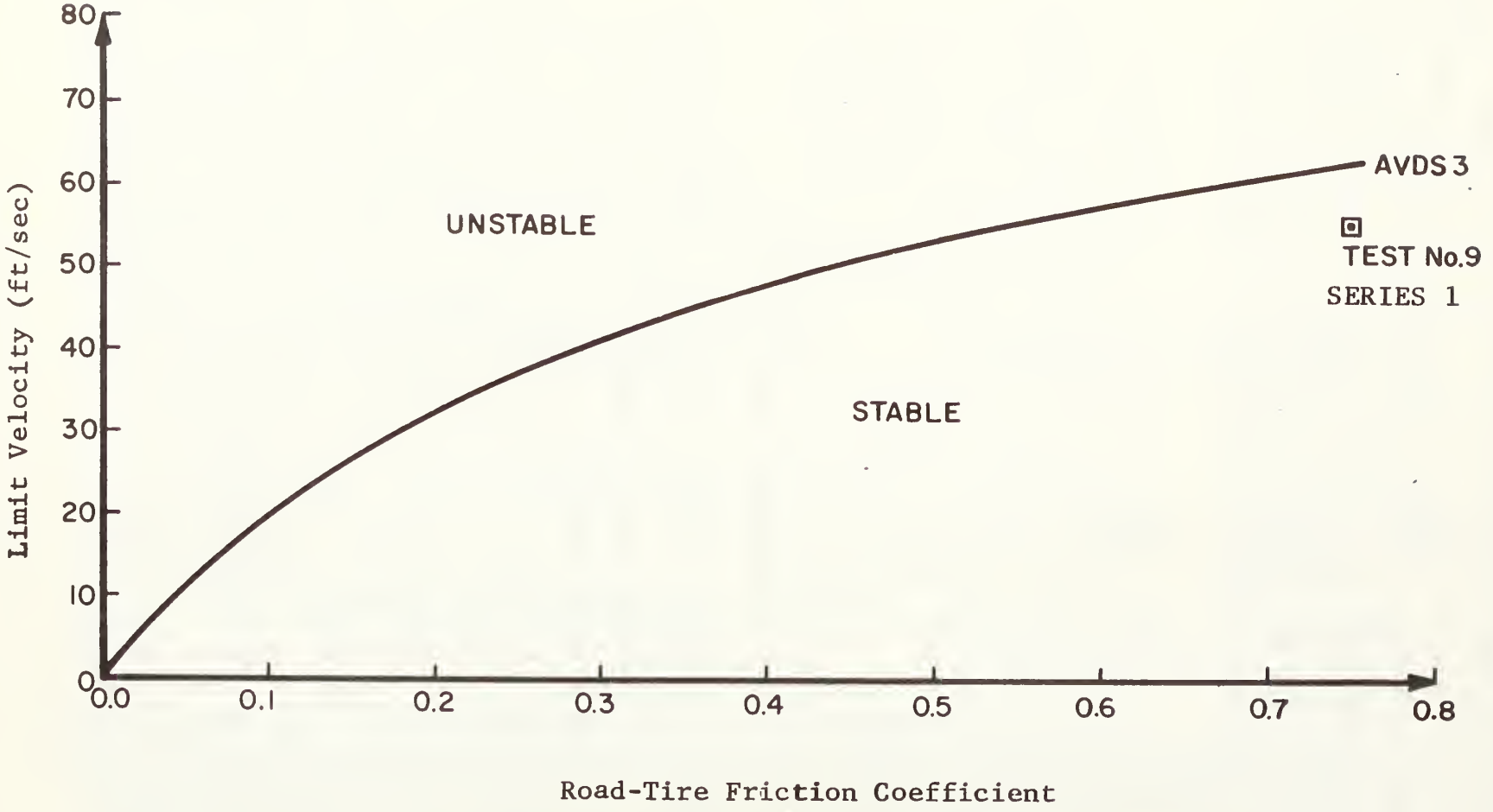


Figure 29. Tractor single-trailer (45 ft) vehicle stability limit for a 14.0 ft x 145.0 ft lane change maneuver without braking.

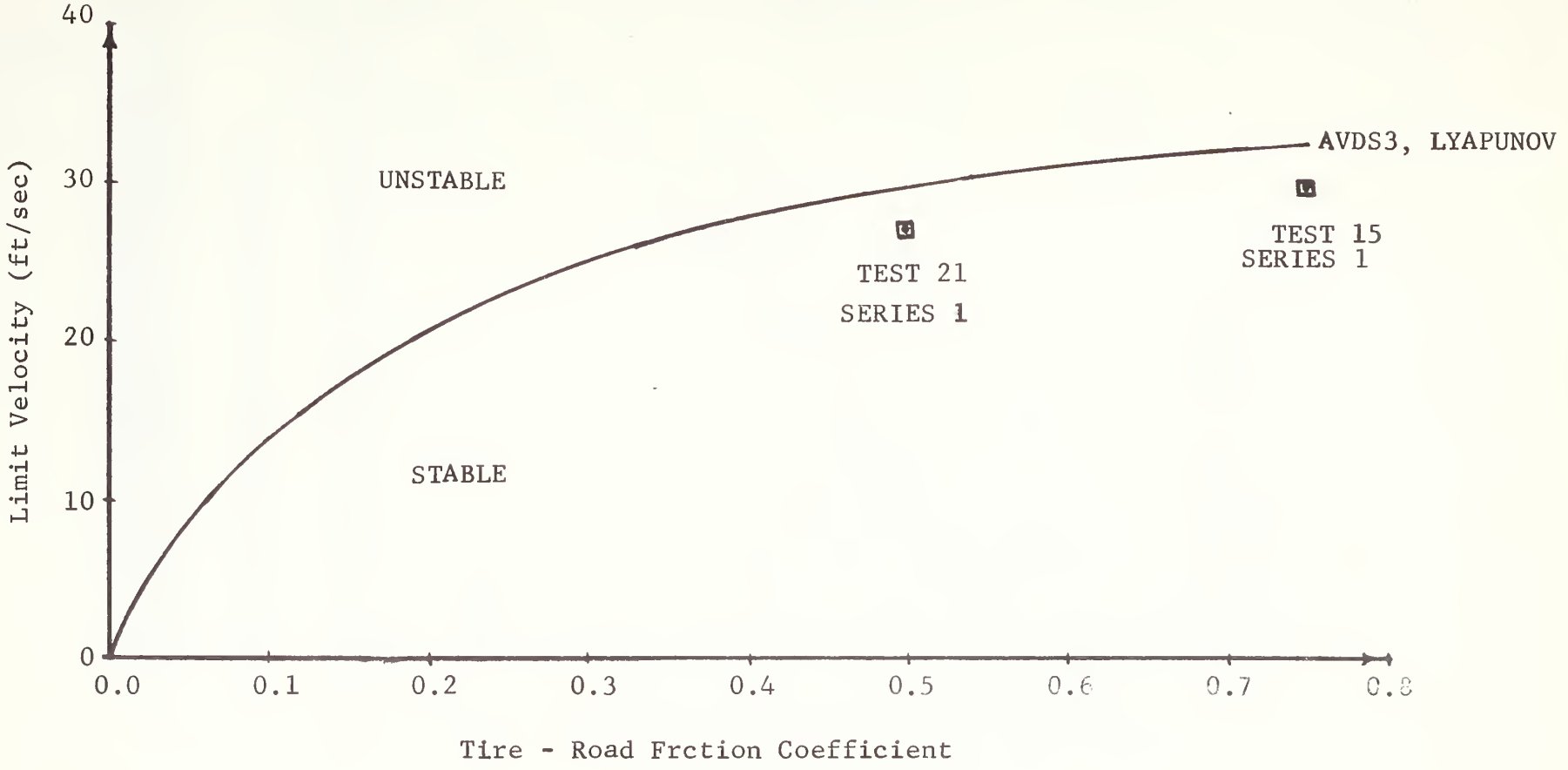


Figure 30. Tractor single-trailer (45 ft) vehicle stability limit for a 75 ft radius circular maneuver without braking.

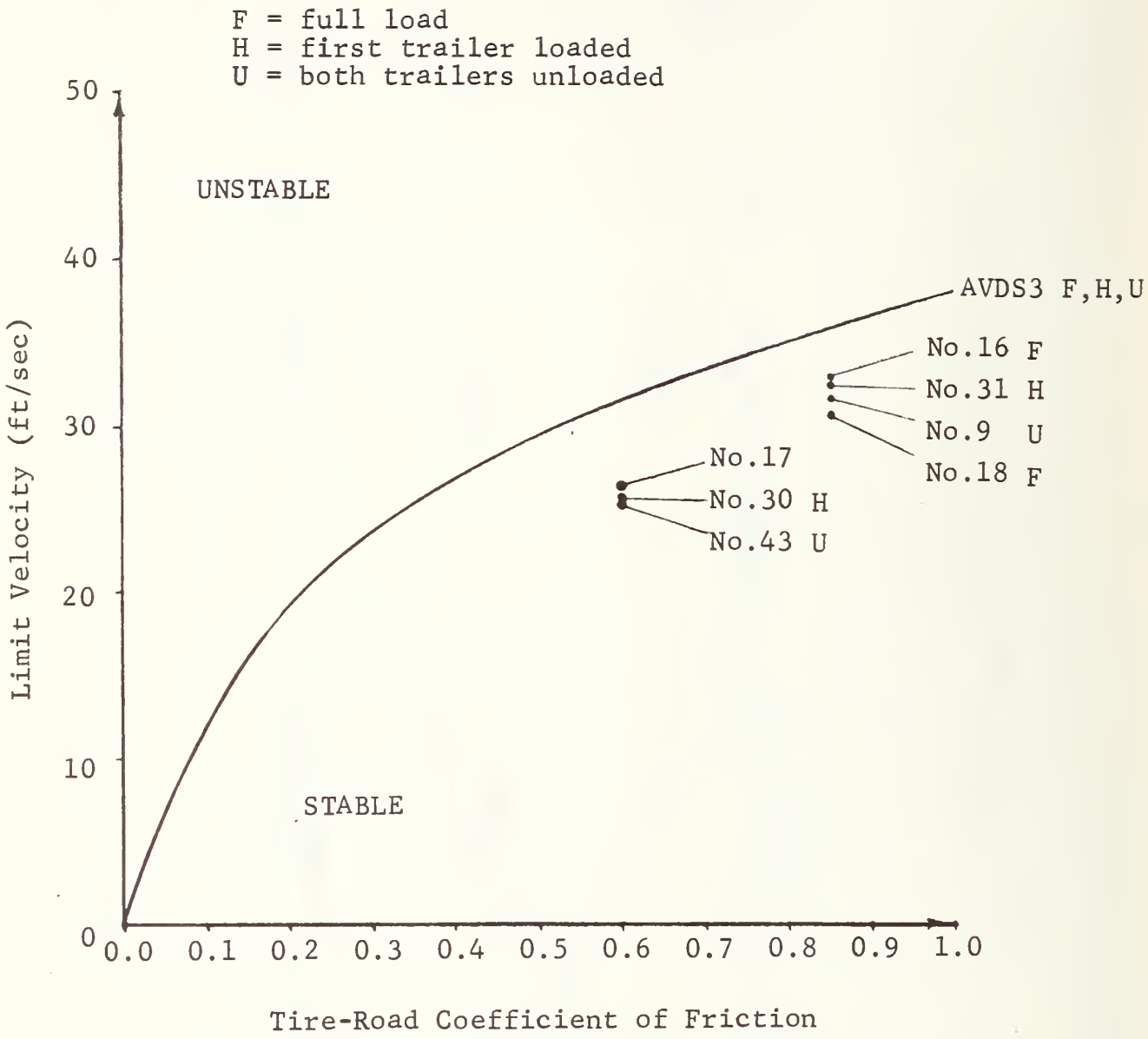


Figure 31. The effect of load on the stability of a tractor double 27 ft semitrailer system performing an 82 ft radius cornering maneuver without braking (all tests from test series No. 2).

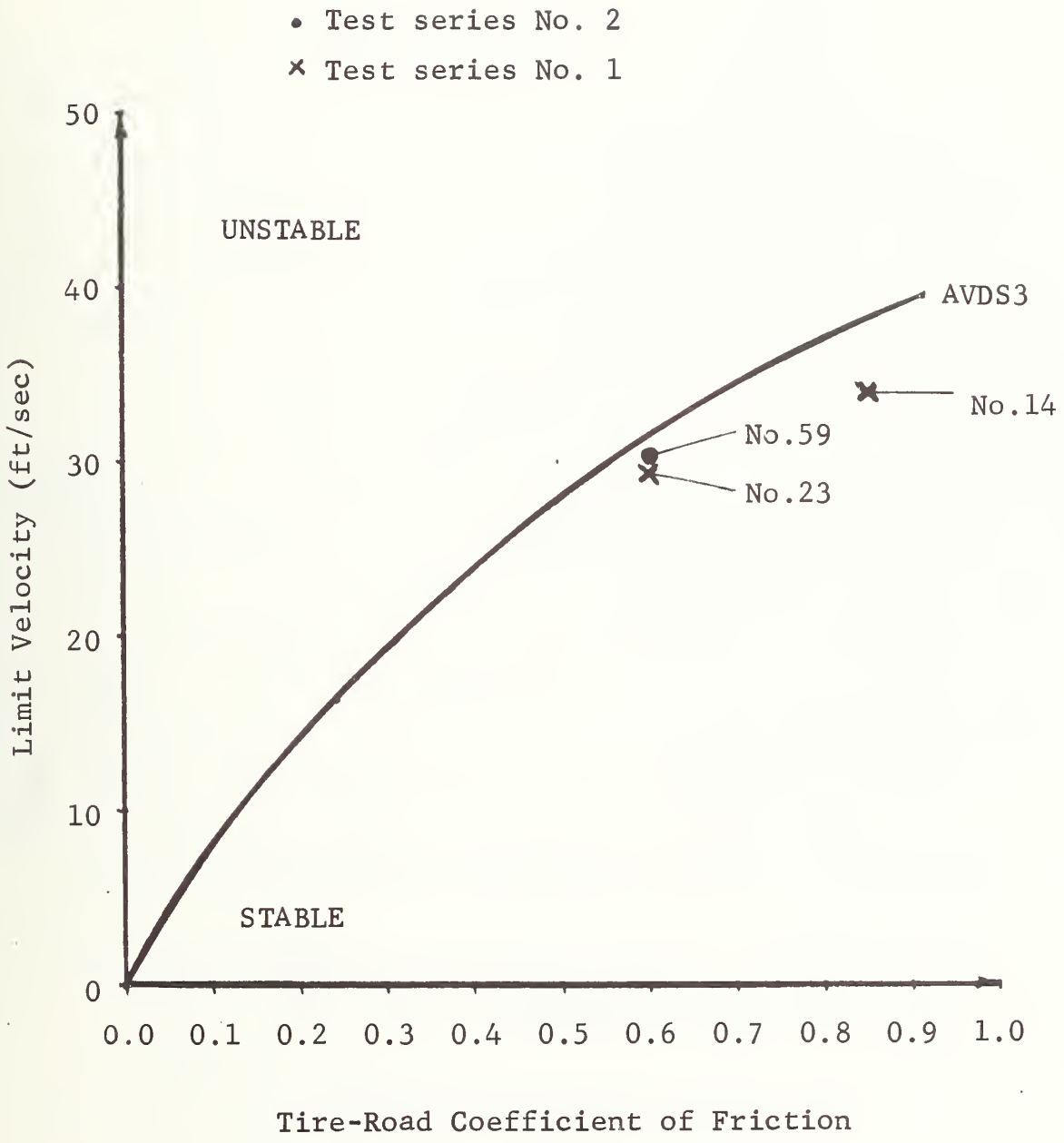


Figure 32. Stability limit for unloaded tractor semitrailers (27 ft and 45 ft) performing a 74 ft cornering maneuver without braking.

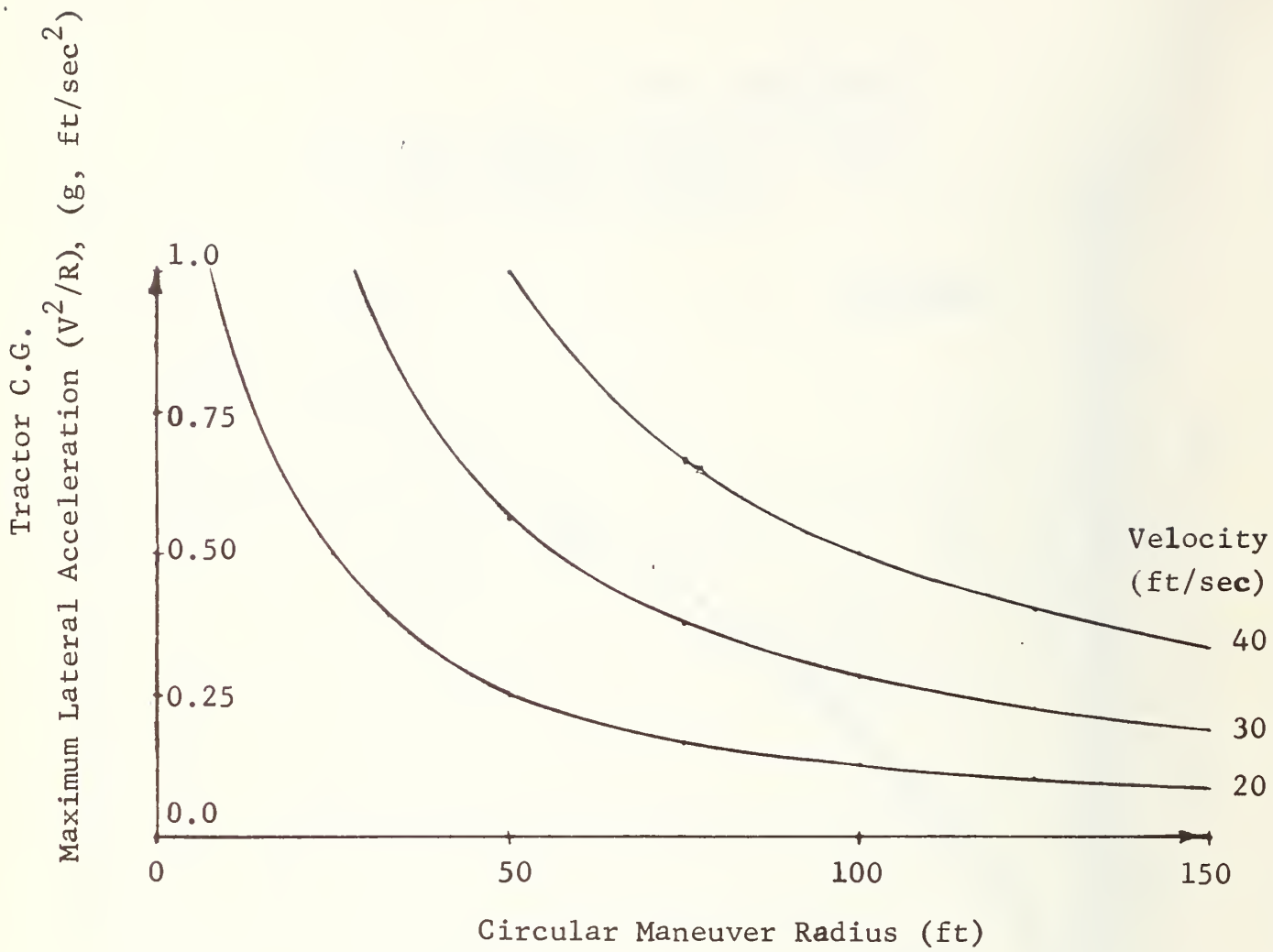


Figure 33. Relationship between tractor c.g. maximum lateral acceleration and circular maneuver radius during maneuvers without braking,

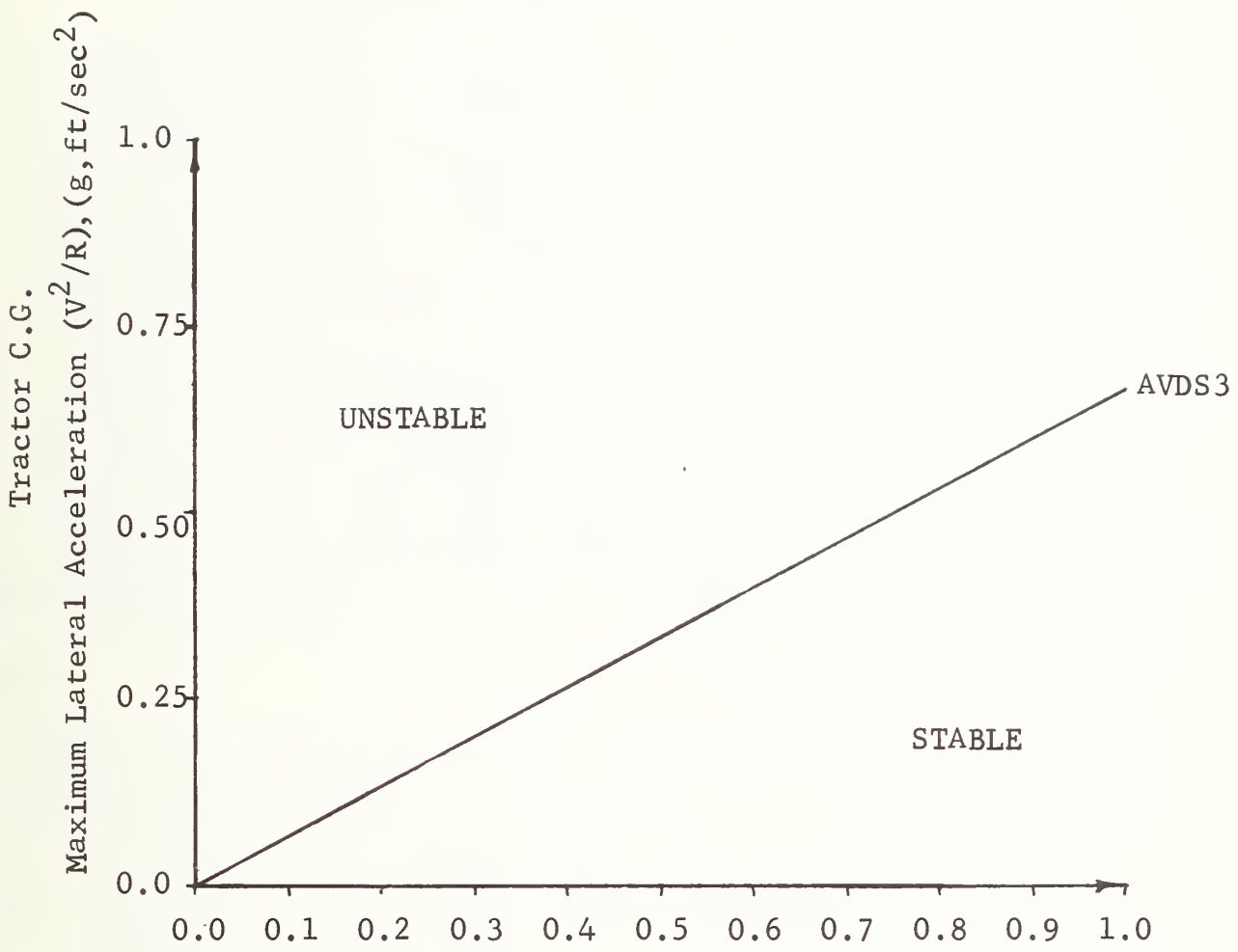


Figure 34. Stability of tractor single, double and triple 27 ft semitrailer systems performing cornering maneuvers without braking.

In Figure 35 peak lateral accelerations for 12 ft lane change maneuvers (the standard function described in Appendix A) of varied length and speed are shown. It has been found that the tractor peak lateral acceleration is a common descriptor of vehicle stability limits. This curve can be used to relate lateral acceleration to specific lane change maneuvers.

The relationship between load and tire-road friction coefficient for a tractor double-semitrailer vehicle performing lane change and evasive maneuvers is shown in Figure 36. It was found in the generation of the stability limits that similar peak lateral accelerations were obtained for evasive and lane changing maneuvers. In addition, the limiting curves show an insensitivity to load conditions. These results are not surprising since the available tire side force depends on the normal load. Therefore, additionally generated side forces due to lateral acceleration are compensated by greater tire side force availability resulting from the same additional vehicle weight. The results for dry surface tests show good correlation; however, for wet surface tests results show that the driver may have allowed tire skid.

40

Figure 37 shows stability limits for single tractor semitrailers performing lane changing and evasive maneuvers without braking. The results of tests conducted on a wet surface correlate with the AVDS3 stability limit whereas the results of dry surface tests are lower than the limit curve. The preceding two curves are essentially nondimensional and through the use of Figure 35, a limit velocity can be obtained for a range of lane changing maneuvers.

Figure 38 shows a comparison of stability results from tractor single-, double- and triple-semitrailer (27 ft) vehicles performing 12 ft lane change maneuvers. The results of the AVDS3 simulation indicate that the single articulated vehicle

is slightly more stable than the double and triple articulated vehicle while lane changing. The test results indicate this conclusion is valid for wet surfaces but not dry surfaces where test 39 showed a much higher value of lateral acceleration than test No. 53 with a single articulated vehicle.

Limit maneuvers performed with braking both experimentally and analytically showed a tolerance for hard braking without creating an unstable situation even when the vehicle was operating near its stability limit. The experimental and analytical stability limits shown in Table 9 for single- and double-articulated vehicles performing lane change maneuvers with braking on wet and dry surfaces correlate well. As expected, braking decreased the allowable lateral acceleration and brake unbalance (from front to rear) decreased the amount of allowable braking. The analytically derived limits were in general higher in magnitude than those obtained from testing. Analytically peak lateral and forward accelerations of 0.52 and -0.6 were obtained from balanced braking of a double articulated vehicle.

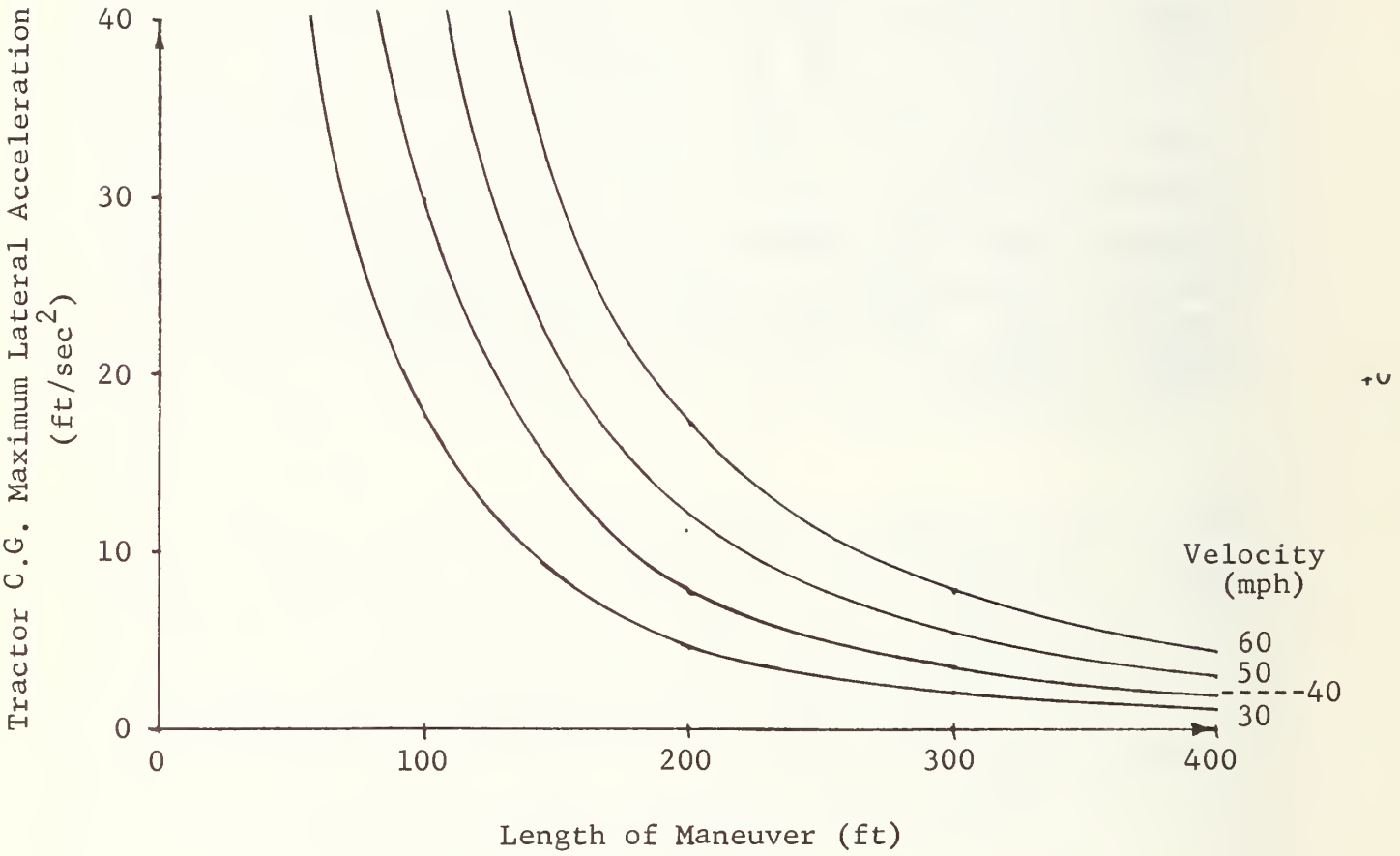


Figure 35. Relationship between tractor c.g. maximum lateral acceleration and length of maneuver during 12 ft wide lane change maneuvers without braking.

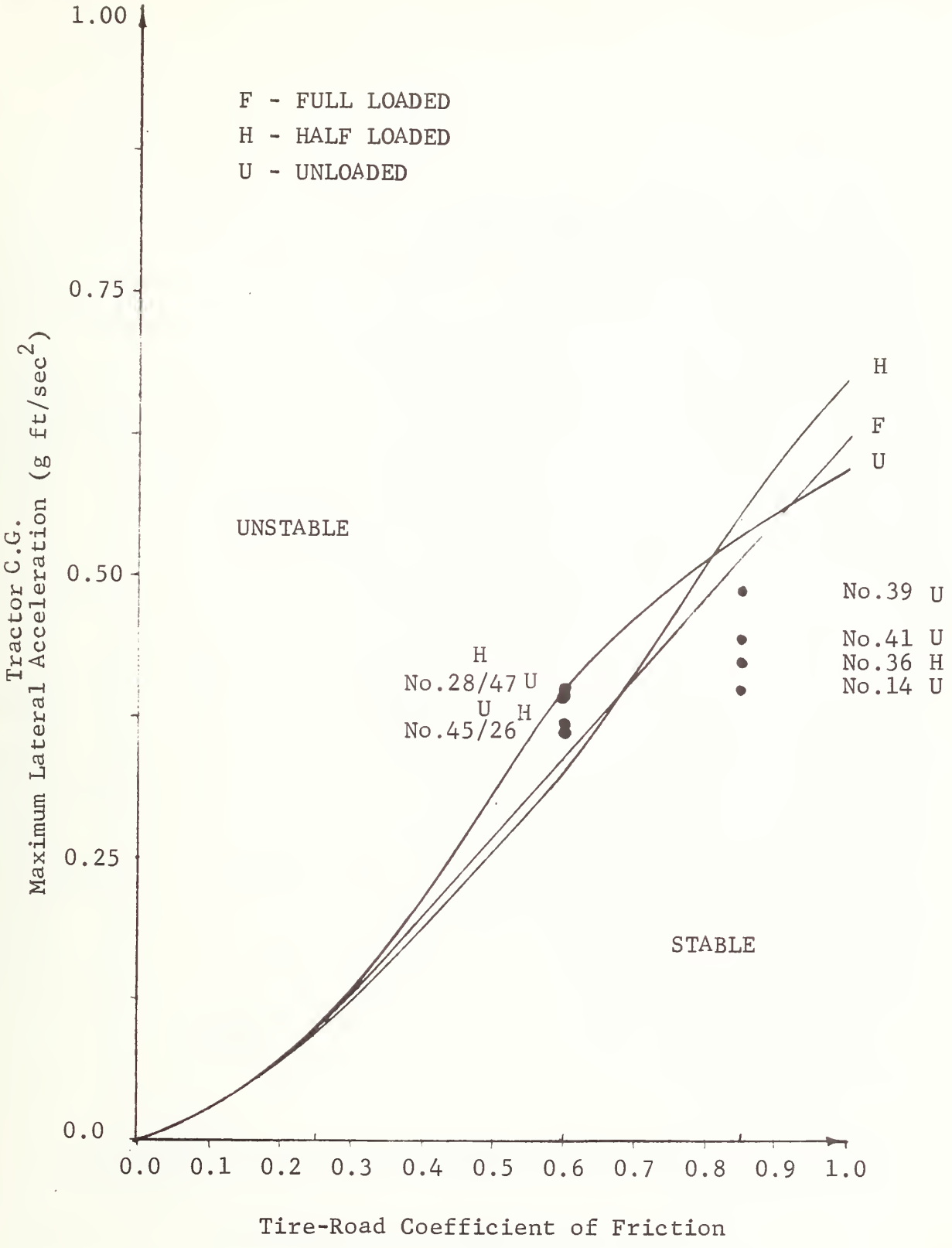


Figure 36. The effect of load on the stability limits of articulated vehicles performing a 12 ft lane change and evasive maneuvers without braking (all test are from series No. 2).

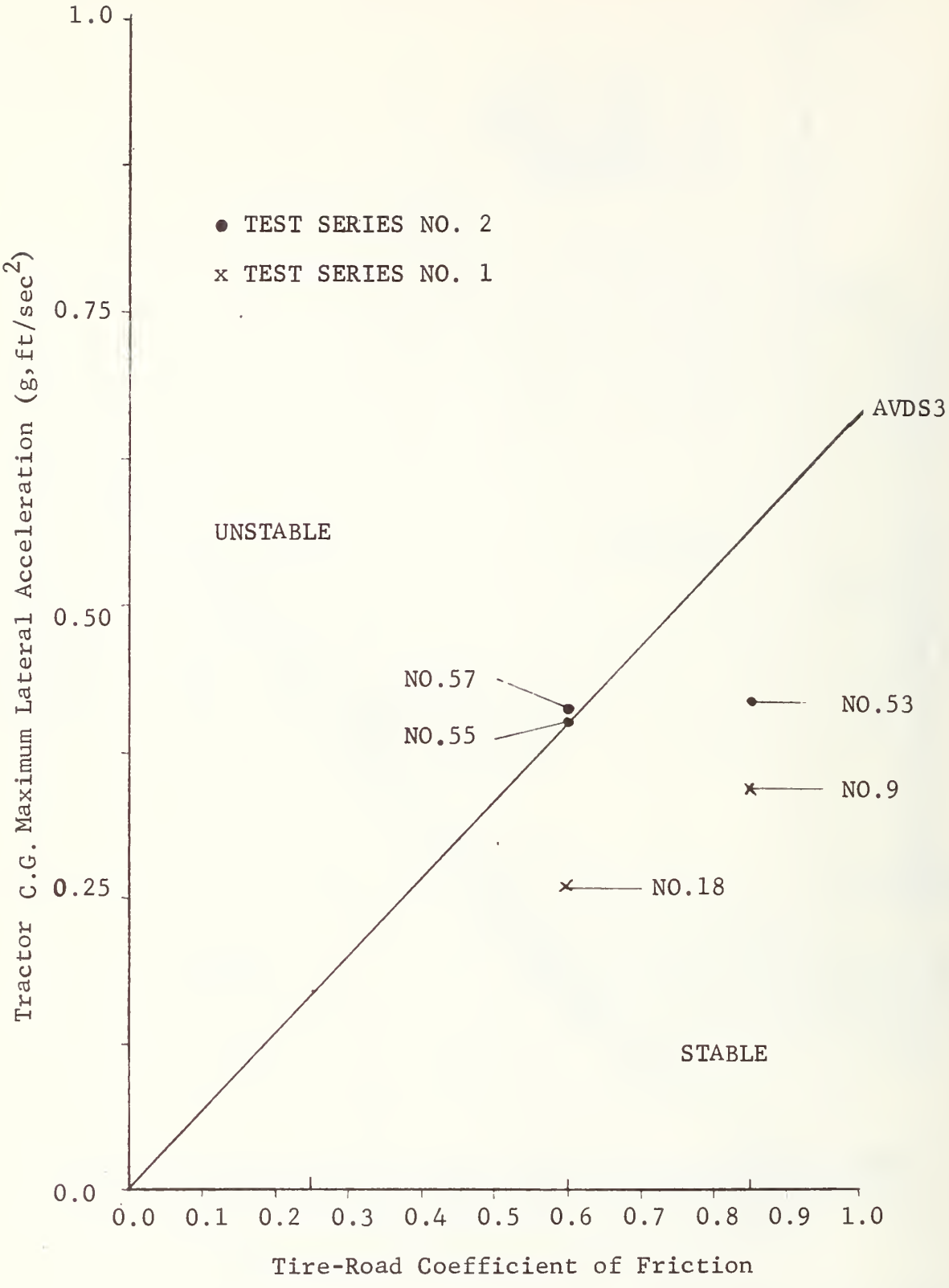


Figure 37. Stability limits for unloaded tractor semitrailers performing 12 ft evasive and lane changing maneuvers without braking.

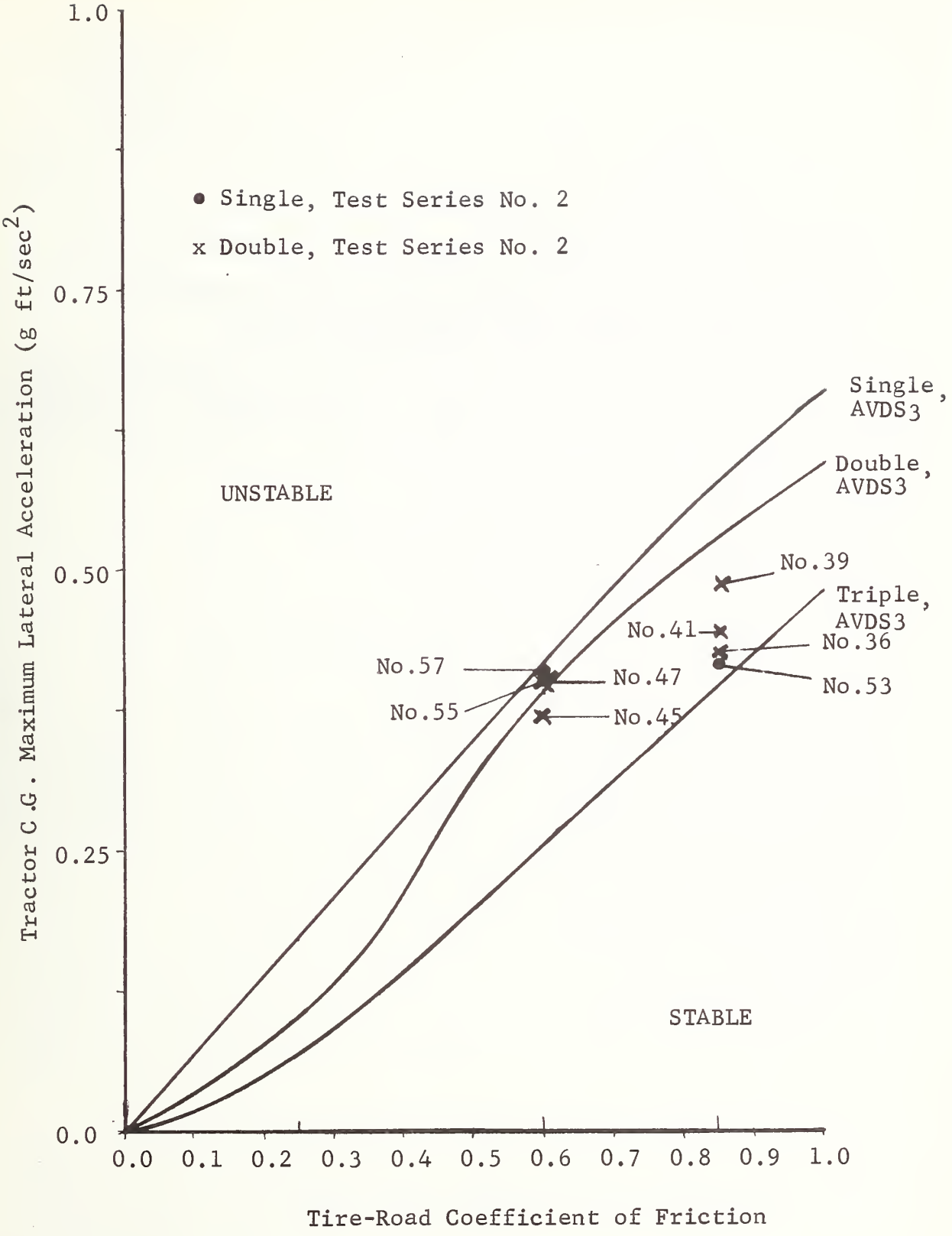


Figure 38. A comparison of stability limits for unloaded tractor single, double, and triple (27 ft) semitrailer systems performing 12 ft evasive and lane changing maneuvers without braking.

Table 9

STABILITY LIMITS FOR SINGLE- AND DOUBLE- TRAILER (27 ft) ARTICULATED VEHICLES PERFORMING LANE CHANGE MANEUVERS WITH BALANCED AND UNBALANCED BRAKING

Vehicle* Config.	Braking	Road Condition	Exp. No.	Acceleration (g)			
				Forward		Lateral	
				Exp.	Anal.	Exp.	Anal.
S	None	Wet	55	-	-	0.4	0.5
S	Bal.	Wet	56	-0.28	-0.46	0.4	0.5
S	Unbal.	Wet	-	-	-0.37	-	0.46
S	None	Dry	51	-	-	0.33	0.55
S	Bal.	Dry	52	-0.445	-0.55	0.31	0.55
S	Unbal.	Dry	-	-	-.46	-	0.55
D	None	Wet	45	-	-	0.37	0.37
D	Bal.	Wet	46	-0.39	-0.55	0.27	0.47
D	None	Dry	39	-	-	0.48	0.52
D	Bal.	Dry	40	-0.48	-0.6	0.31	0.52

*S = single
D = double

8. CONCLUSIONS

Because of the complexity of meaningful articulated vehicle models which must retain nonlinearities in their formulations, it can be concluded on the basis of work conducted in this study that methods of analytically obtaining Lyapunov functions are difficult to implement. Systematic numerical methods appear to be useful in the study of the stability of articulated vehicles in steady state (autonomous) cornering maneuvers. For nonautonomous systems, it appears that the Lyapunov concept of stability is too restrictive because it requires stability over infinite time and no momentary movement to another stable trajectory is allowed. A modified Lyapunov concept called finite time stability is applicable to the articulated vehicle problem because it considers the stability of the vehicle over a finite time interval. In this study the techniques of mathematical stability were studied and exhaustive attempts were made to apply these techniques to the generation of articulated vehicle limits. The limited success of this program leads to the conclusion that the universal generation of stability limits utilizing the concepts of mathematical stability appears to be at present beyond the state of the art. The major difficulty encountered is that the practical articulated vehicle problem is nonlinear and nonautonomous. Stability techniques available in the literature are oriented toward linear, autonomous systems with the more forward techniques being able to solve specific nonlinear autonomous problems; however, it was found that most of these are contrived problems not of practical value.

Looking on the more positive aspects of this investigation, we found that stability limits based on physical stability criteria can be generated for articulated vehicles performing road maneuvers in varied environments. The results of this study indicate that:

- Stability limits are insensitive to vehicle load (Figure 31 and 36) and trailer length (Figure 32).
- Stability limits are highly dependent on road-tire friction coefficient.
- Stability limits are relatively insensitive to added trailers (Figure 38). This result was reported in the final report on Articulated Vehicle Handling[23].
- Stability limits are sensitive to braking with decreased allowable maneuver velocity.
- Stability limits for evasive and lane change maneuvers are similar.

The stability criteria used to generate the stability limits through the digital computer program AVDS3 were verified by full-scale testing. It was shown that the computer simulation can perform successfully on multitrailer vehicles. The modeling of the hitch point between a trailer and dolly was verified. Experimental modeling studies on roll and pitch angle responses showed them to be small compared to yaw angles and therefore gives credibility to the use of load transfer assumptions. Finally discrepancies in analytical-experimental response phase indicate that vehicle lateral damping and flexibility may be important and should be included in the simulation model.

From the results of this study,the conclusion of the previous NHTSA study[23] on articulated vehicle handling concerning the importance of tire modeling is reinforced. Although much progress was made on this study in tire modeling, full-scale on-site tire parameter measurements which consider both cornering and tractive/braking forces are required. The friction ellipse concept needs experimental development.

From the experimental studies it can be concluded that articulated vehicles can safely perform violent high g maneuvers if maneuver space is available and professional drivers are used. The weakest link in the multitrailer systems appears to be the dolly. In violent maneuvers, especially involving braking, dolly jackknifing was frequently encountered. It was found that loading of multitrailer articulated vehicles is important for control in road maneuvering situations. Finally the analytical-experimental records indicate that professional drivers follow the most efficient driving modes acting with the proficiency of a good automatic control system.

It can be concluded that further study on the factors that affect articulated vehicle stability and handling is required; particularly on the tires, which were shown to be most important factors, in order to establish meaningful safety standards.

9. RECOMMENDATIONS

Recommendations for further work on articulated vehicle modeling, simulation, experimentation and stability criteria result from the findings of this program. It is recommended that in-depth, stability and response studies in articulated vehicle dynamics be continued to quantify the stability and maneuverability of articulated vehicles in normal traffic flow. The effect of parameters of road design including surfaces, curves, grades and banks should be evaluated thoroughly in a parameter study. Stability criteria involving trailer swing -- particularly with double- and triple-trailer units -- should be developed. In addition, an experimental-analytical program should be conducted to develop standard analytical-experimental procedures of articulated vehicles through stability and response criteria, the driver skill involved in safe vehicle operation, and the road design and surface condition requirements.

Although the models of articulated vehicles are sufficiently comprehensive for the prediction of gross vehicle response, effort should be expanded to determine the sensitivity of the response to modeling assumptions. This would involve formulating the problem in terms of more degrees of freedom and determining the effect of governing parameters on the response. Areas most likely to show fruitful investigation are the suspension system, wheels, tandem axles, and the tires. The vertical suspension system properties involve the forward or lateral load transfer from wheel to wheel caused by maneuvering, aerodynamic forces and braking forces. The lateral suspension system properties include lateral, forward and yawing motions of the vehicle, particularly in softly supported vehicles such as moving vans. In particular the problem of how lateral stiffness and damping affect computer simulated response should be thoroughly examined.

The assumptions made in fifth wheel connection and hitch point connections, particularly as they affect the transmission of forces and moments to other parts of the vehicle, should be critically examined. Wheel rotation degrees of freedom are involved in direct study of the dynamic implementation of braking systems, particularly antilock systems. Phenomena such as brake fade and how it affects articulated vehicle handling should be studied. Without a doubt, tires show the most pressing need for effort on modeling. A longitudinal/lateral force model that predicts the tractive acceleration/cornering force interaction for tractor semitrailer tires should be developed. This will in all probability involve curve fitting of empirical data. The model would be characterized for tires of typical size and construction with varied road surface, temperature and velocity effects included. Curve fitting or computer identification schemes may have to be utilized for individual tires under specified conditions. This work is necessary to evaluate the effect of tire properties on articulated vehicle handling and stability.

In view of the control and maneuverability problems confronting the driver of articulated vehicles, studies should be undertaken on the demands on driver skill, measured in terms of potential loss of control and in the context of the vehicle operating over its full range of environments and design configurations.

Experimental work should be utilized to verify the overall dynamic response and stability of single-, double- and triple-articulated vehicles. The experimental results of this program should be used with actual tire data to verify the Mikulcik[15] model for both trucks (as a special case of the tractor semitrailers) and tractor semitrailers. In addition this response model should be extended to triples. In this manner, both direct response and semi-inverse (AVDS) models would be available for vehicle evaluation. The experimental generation of tire characteristics should be pursued whether

it be accomplished indirectly with a special test fixture, directly from an articulated vehicle, or from analytical/experimental parameter identification studies. Actually, the test data obtained in the present program could be used with the AVDS3 or Mikulcik model to determine tire characteristics for the particular tires and operating conditions of this program.

Further work is required on the characterization of aerodynamic effects including the effects of steady wind and gusts on articulated vehicle response, and the interaction of these vehicles with other road vehicles. Aerodynamic constants associated with lateral and forward rectilinear motions and roll, pitch and yaw angular motions require further study. In addition the interaction effects of articulated vehicle units or the total characterization of the vehicle is required.

Roads should be characterized to facilitate study of the potential maneuverability conditions to be encountered by drivers of articulated vehicles. This characterization should include tire/road interaction for specific road surfaces, road conditions (oil, ice, water, dust) and geometry (hills, banks and curves).

Finally if the present work on the determination of mathematical stability limits is pursued further, it is recommended that the finite time stability concept be developed on a problem oriented basis. Techniques directly applicable to articulated vehicles will have to be developed.

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APPENDIX A

MATHEMATICAL FORMULATION

FOR

ARTICULATED VEHICLE HANDLING SIMULATION

APPENDIX A
LIST OF SYMBOLS

b_i	vehicle dimension
d_i	vehicle dimension
F_s	tire side force
F_z	tire normal load
h_i	vehicle dimension
IZZi	sprung mass moment of inertia-yaw
L_1, L_2	length of maneuver (lane change or evasive)
M_t	mass of tractor
MSTi	mass of semitrailer
MDi	mass of dolly
N_i	tire constant
P_i	tire constant
Q_i	tire constant
R	radius of circular arc
R_i	tire constant
S_1, S_2	width of maneuver (lane change or evasive)
S_i	tire constant
T_i	tire constant
V	tractor c.g. path velocity
$\dot{V}$X1	tractor c.g. forward acceleration
$\dot{V}$Y1	tractor c.g. lateral acceleration
ψi	yaw angle
$\dot{\psi} i$	yaw angular velocity
$\ddot{\psi} i$	yaw angular acceleration
δ	steering angle
$\Delta\delta$	incremental steering angle
ΔD	incremental total braking force
α_i	tire slip angle
α_{m_i}	tire maximum slip angle
μ	tire-road coefficient of friction (apparent/real)
θ_R	angle of circular arc
ρ	radius of curvature

APPENDIX A

MATHEMATICAL FORMULATION

FOR

ARTICULATED VEHICLE HANDLING SIMULATION

1. INTRODUCTION

In the previous work performed at IIT Research Institute (IITRI) under the Department of Transportation Contract No. DOT-HS-105-1-151, the nonlinear equations of motion were derived for a system consisting of a tractor, semitrailer, dolly, and second semitrailer. These equations were programmed on the digital computer to form the Articulated Vehicle Dynamic Simulation Model II (AVDSII) which is capable of predicting the dynamic response of single and double bottom tractor-trailers subject to acceleration, braking and side forces, aerodynamic forces and fifth wheel friction and damping. A semi-inverse computational method that determines the driver steering and braking demands and vehicle responses for a prescribed vehicle trajectory with specified vehicle, tire and aerodynamic parameters was utilized in AVDSII. The detailed developments of various mathematical relationships for AVDSII were presented in the final report entitled "Articulated Vehicle Handling"[23] in April 1972.

The nonlinear equations of motion, referred above, are further expanded in the present work to include one more unit of dolly and semitrailer (i.e. for a system consisting of a tractor, first semitrailer, first dolly, second semitrailer, second dolly, and third semitrailer). The simulation model AVDSII is modified and is called the Articulated Vehicle Dynamic Simulation Model (AVDS3).

AVDS3 thus features a transient solution of the nonlinear equations of motion of the single, double and triple tractor-trailers with the following independent degrees of freedom:

- Tractor forward motion
- Tractor lateral motion
- Tractor yaw angle ψ_1
- First semitrailer yaw angle ψ_2
- First dolly yaw angle ψ_3
- Second semitrailer yaw angle ψ_4
- Second dolly yaw angle ψ_5
- Third semitrailer yaw angle ψ_6

The formulation of various mathematical relationships for AVDS3 just necessitates a simple, straightforward extension of the approach utilized for AVDSII which was presented in much detail in the previous final report. As such, these relationships are not represented herein.

The following assumptions, techniques and models are used in both AVDSII and AVDS3.

- Roll and pitch angles are assumed to be small.
- Roll and pitch motions are excluded from the original problem formulation. Effects of these motions on tire forces through alterations in the vertical tire loads are reintroduced to the problem formulation using semistatic load transfer (forward and lateral accelerations used with suspension properties).

- Vertical and horizontal motions are uncoupled.

- Fifth wheels are assumed to have negligible weights and inertias. They are normally affixed to the respective frames through bearings and, hence, do not transmit any pitching moment when the relative yaw angle between corresponding units (tractor/first semitrailer, first dolly/second semitrailer, and second dolly/third semitrailer) is zero. Also, when the two units connected by the fifth wheel are directly in line, their roll angles are equal.

- The pin-joints between the dollies and semitrailers transmit no moment - only forces.

- A modified Ellis tire model (AVDSII, AVDS3) and an IITRI tire model (AVDS3) with friction ellipse concept describe the tire behavior.

- In expansion of the tire model, in terms of changes in steering angle and tire braking/accelerating forces, higher order terms in $\Delta\delta$ (incremental steering angle) and ΔD (incremental total braking force) are dropped.

- The fifth wheel yaw angular constraint is given in terms of a restoring spring force, viscous and coulomb damping.

2. THE NONLINEAR EQUATIONS OF MOTION

2.1 Triple Tractor-Trailers Vehicle

The equations of motion are written with reference to coordinate systems X_i Y_i Z_i (i=1 to 6) shown in Figure A-1. These reference axes are fixed in space and their origins coincide with the spring mass centers of gravity when the tractor, first semitrailer, first dolly, second semitrailer, second dolly, and third semitrailer are directly in line with pitch and roll angles and vertical displacement of all units equal to zero. Overall vehicle dimensions are shown in Figures A-2 through A-7.

In the first step, the equations of motion for each of the sprung masses are derived separately in terms of the general forces and moments (in x_i y_i z_i coordinate systems) at the suspension points, fifth wheels and pin-joints, and aerodynamic forces and couples (in X_i Y_i Z_i systems) applying Newton's second law. The coordinate systems x_i y_i z_i are fixed at the center of gravity of the corresponding sprung mass. The sets of equations for the six bodies are not independent and are related to each other by the constraints imposed by the fifth wheels and pin-joints. The constraint equations are developed next. This generates 96 independent scalar equations, made up of 36 equations of motion and 60 constraint equations.

The equations of motion and the constraint equations are combined, and pitch, roll, and vertical motions are uncoupled from the problem formulation to result in eight second-order differential equations in the eight independent coordinates.

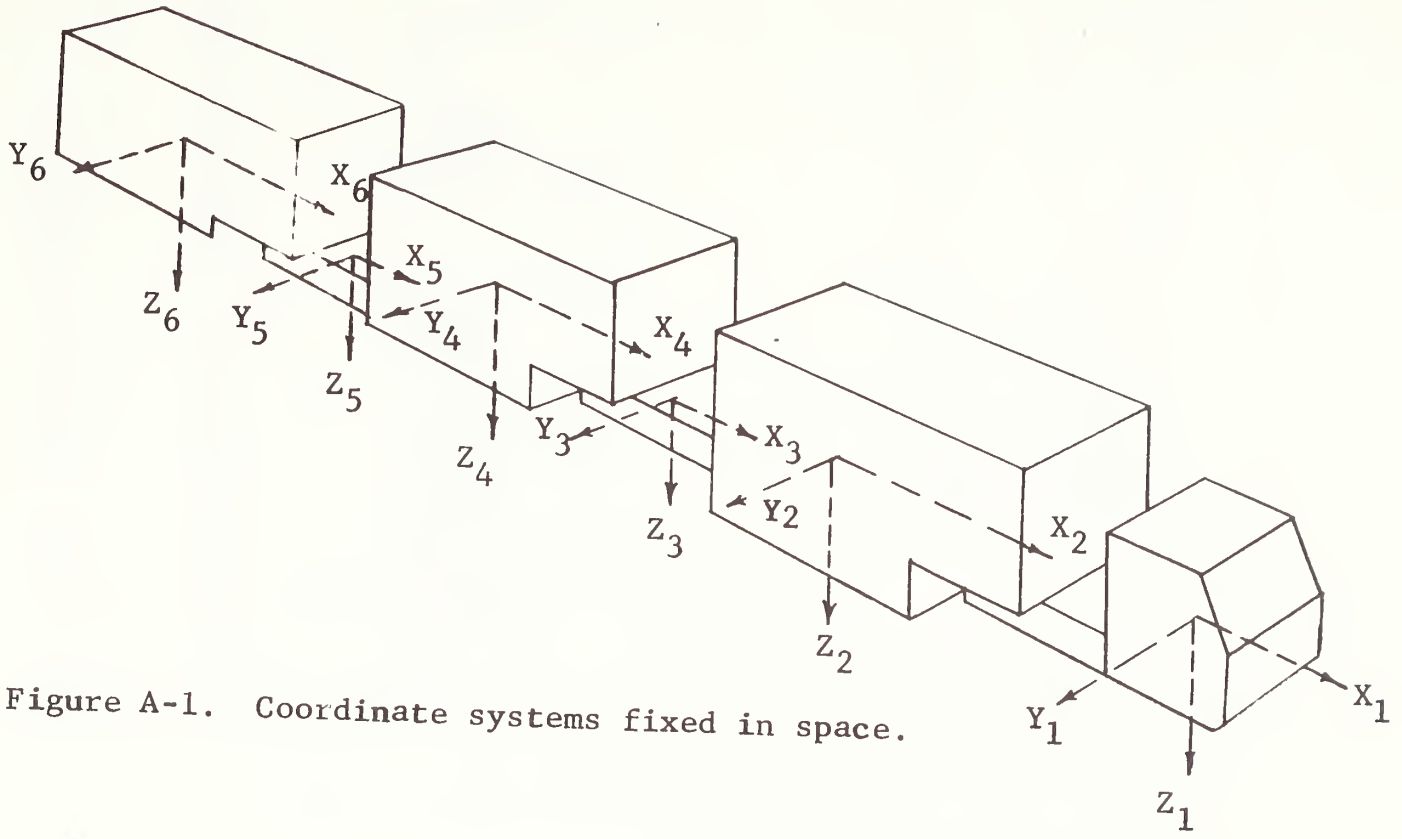


Figure A-1. Coordinate systems fixed in space.

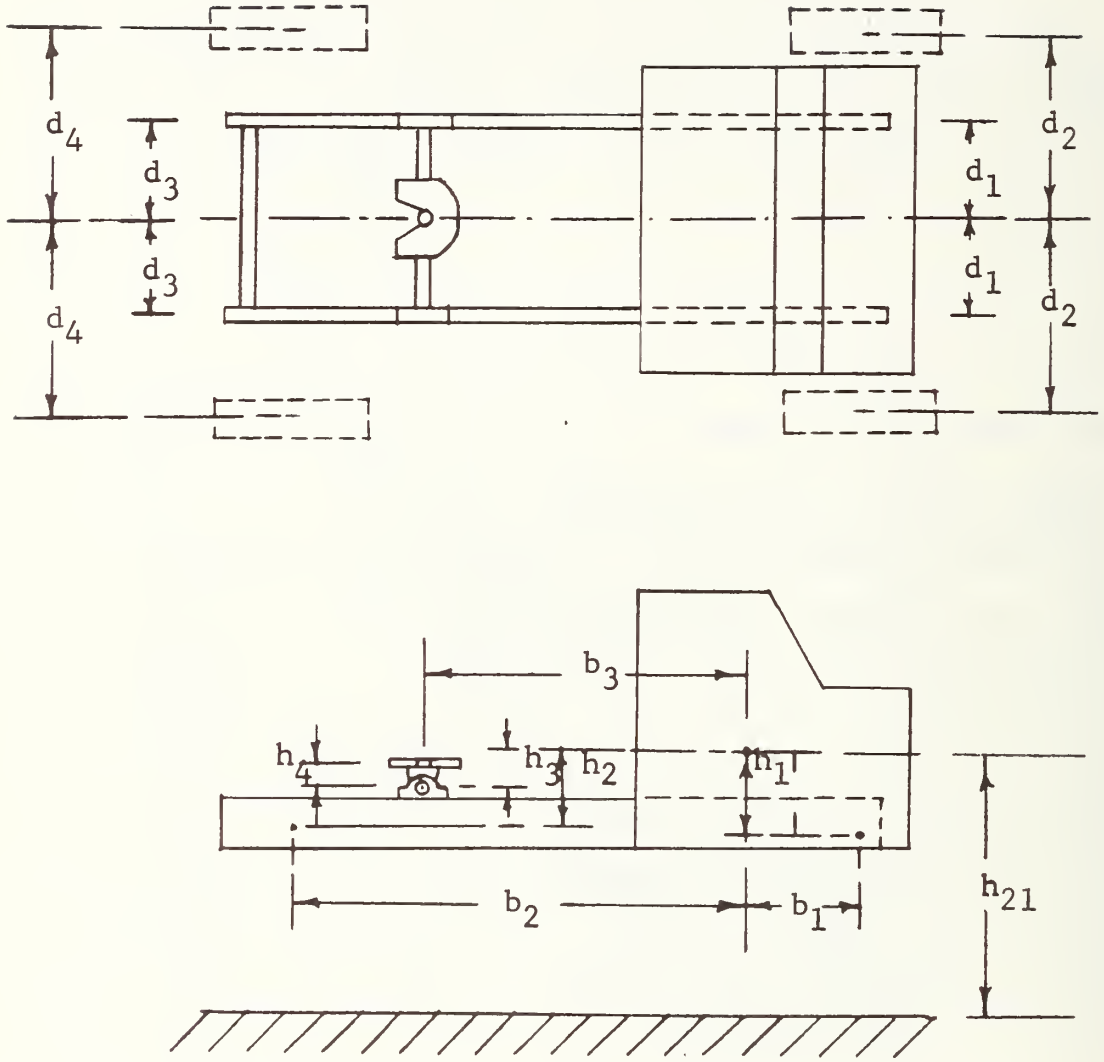


Figure A-2. Tractor dimensions.

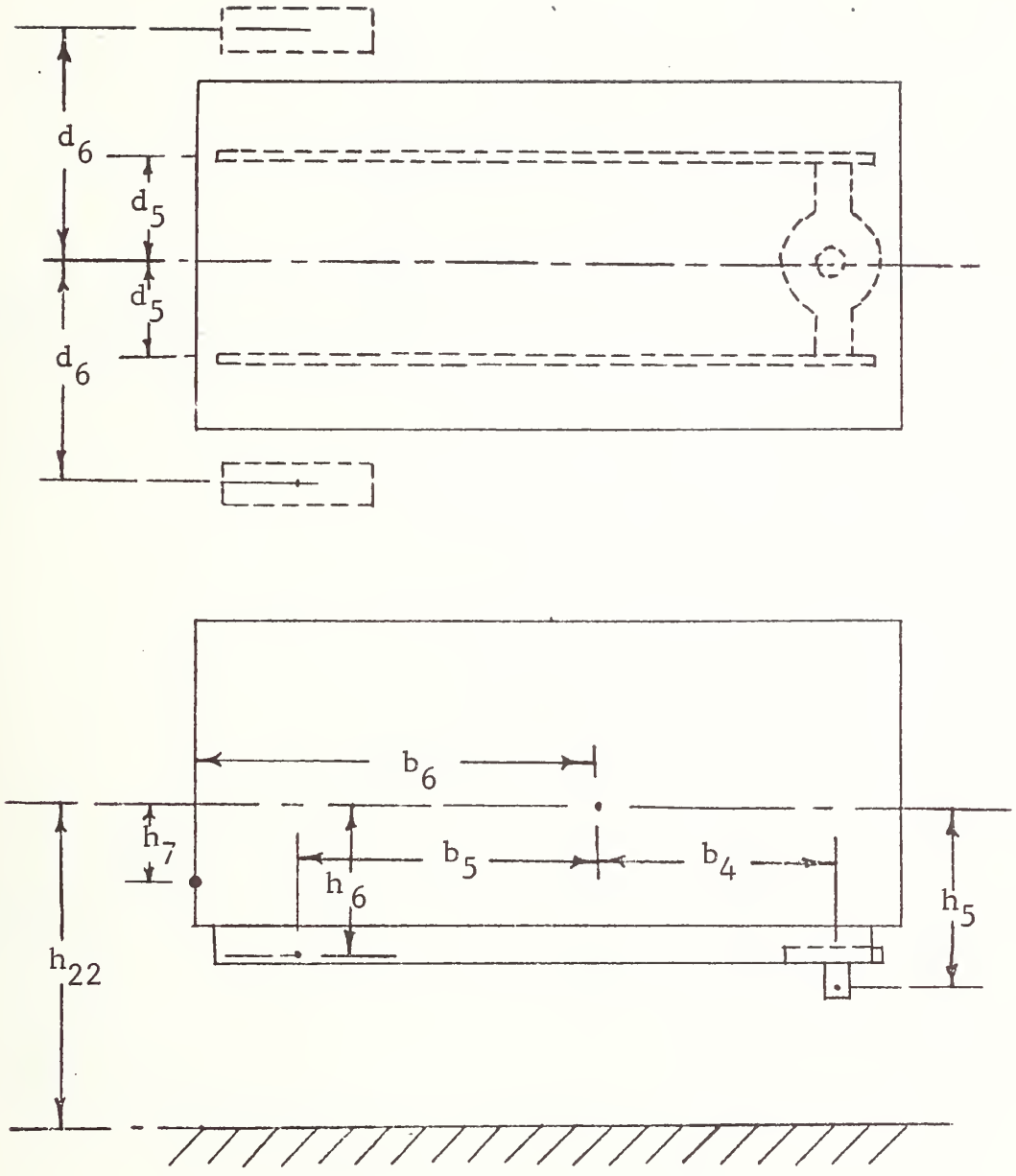


Figure A-3. First semitrailer dimensions.

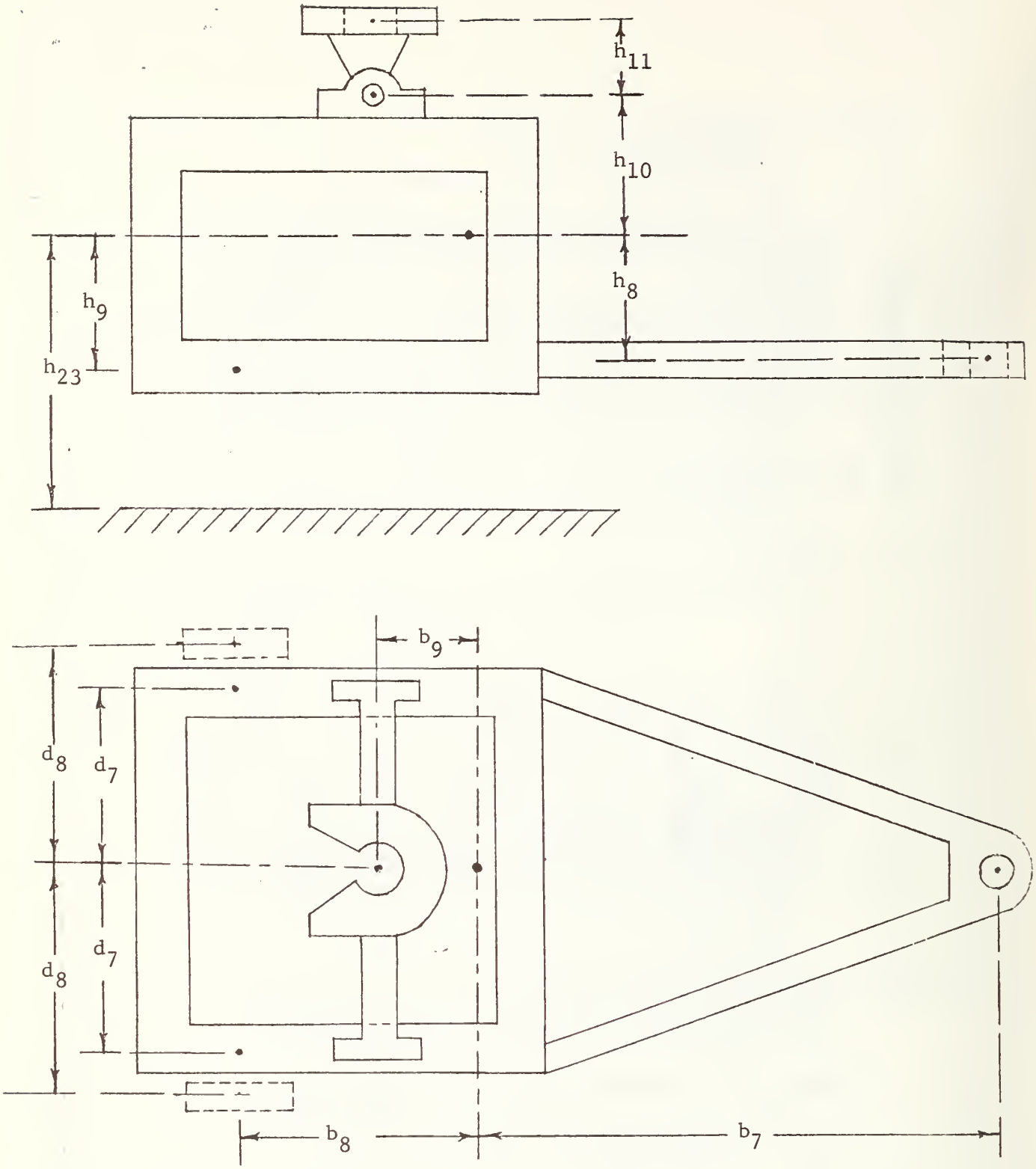


Figure A-4. First dolly dimensions.

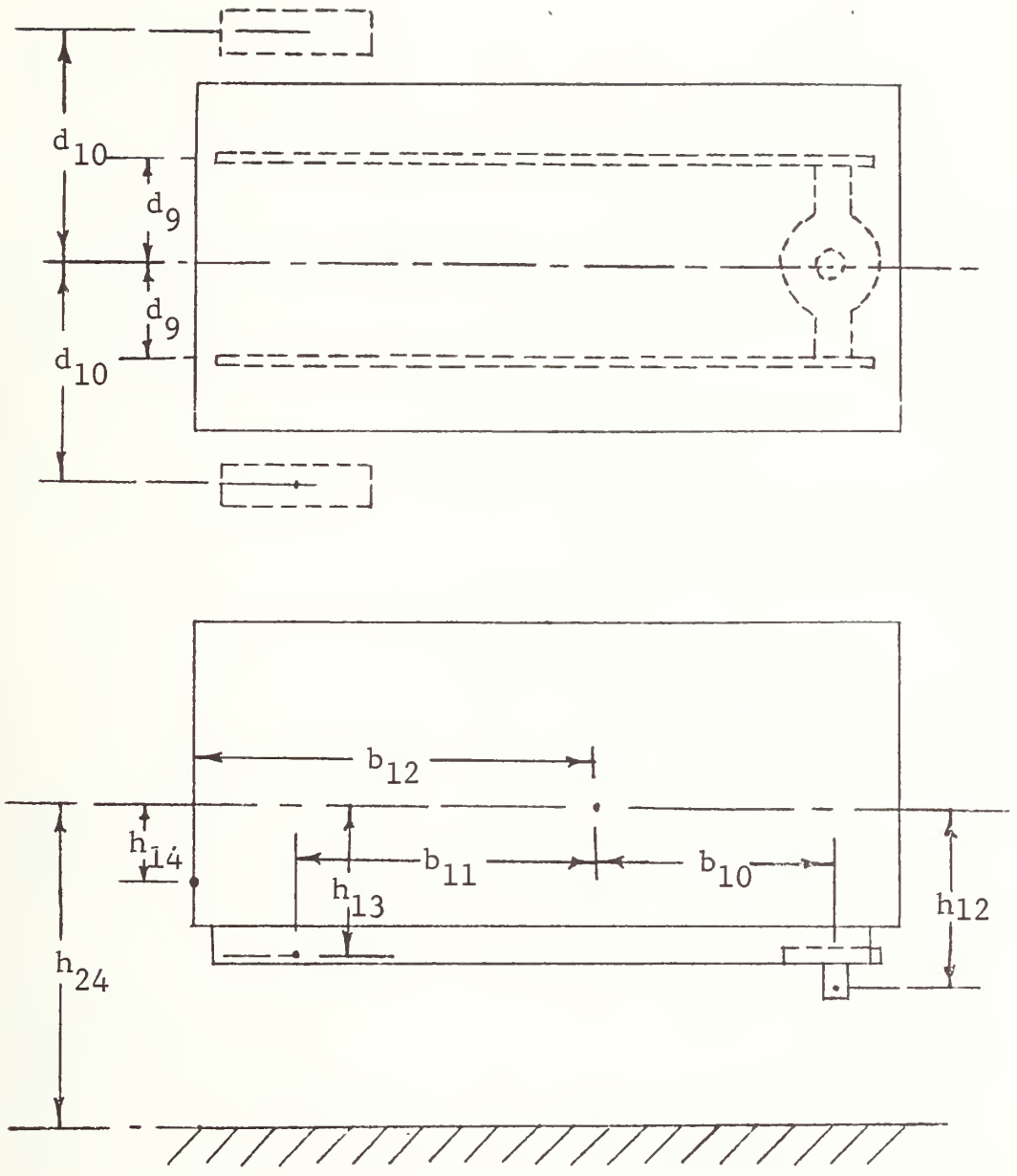


Figure A-5. Second semitrailer dimensions.

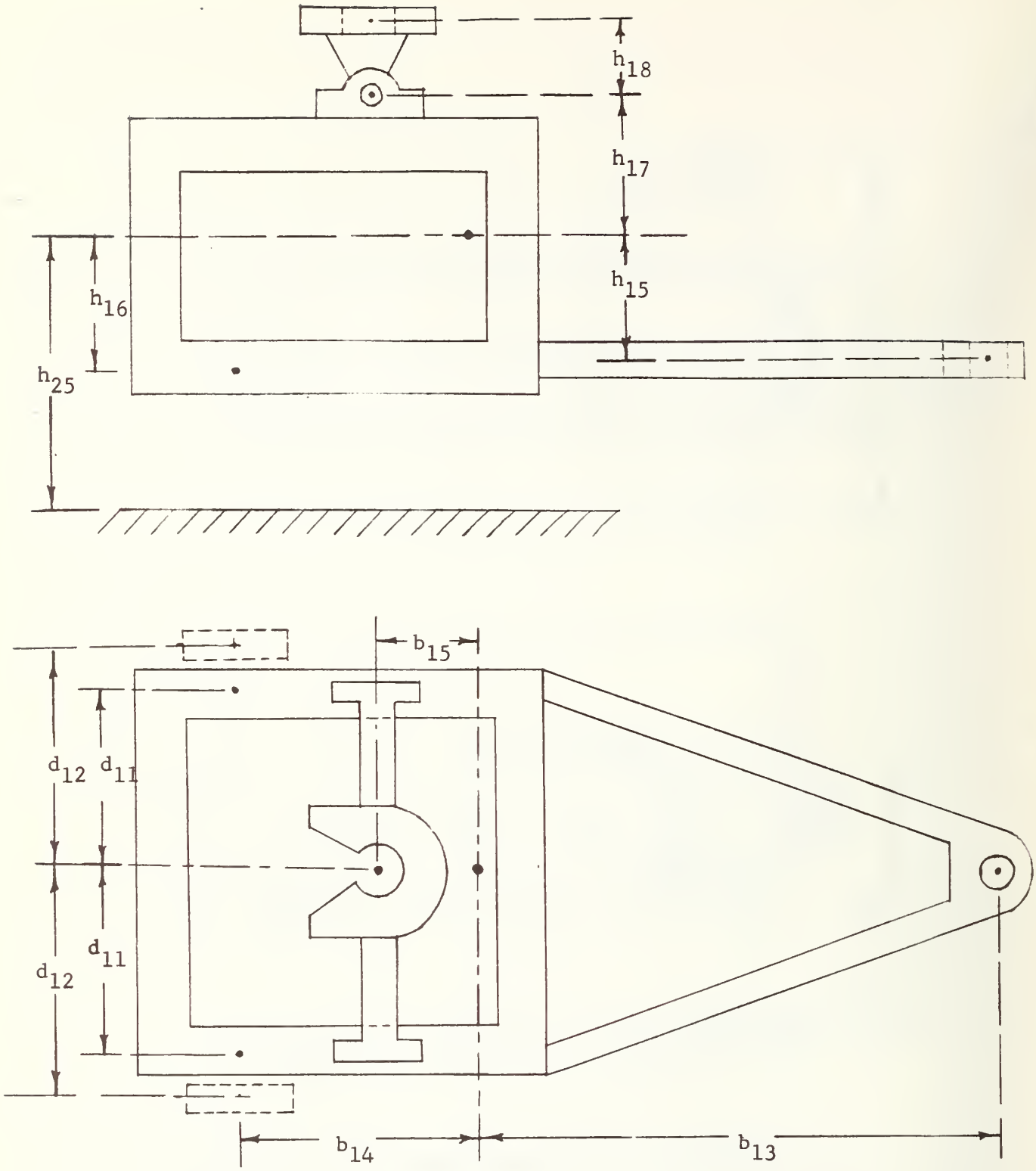


Figure A-6. Second dolly dimensions.

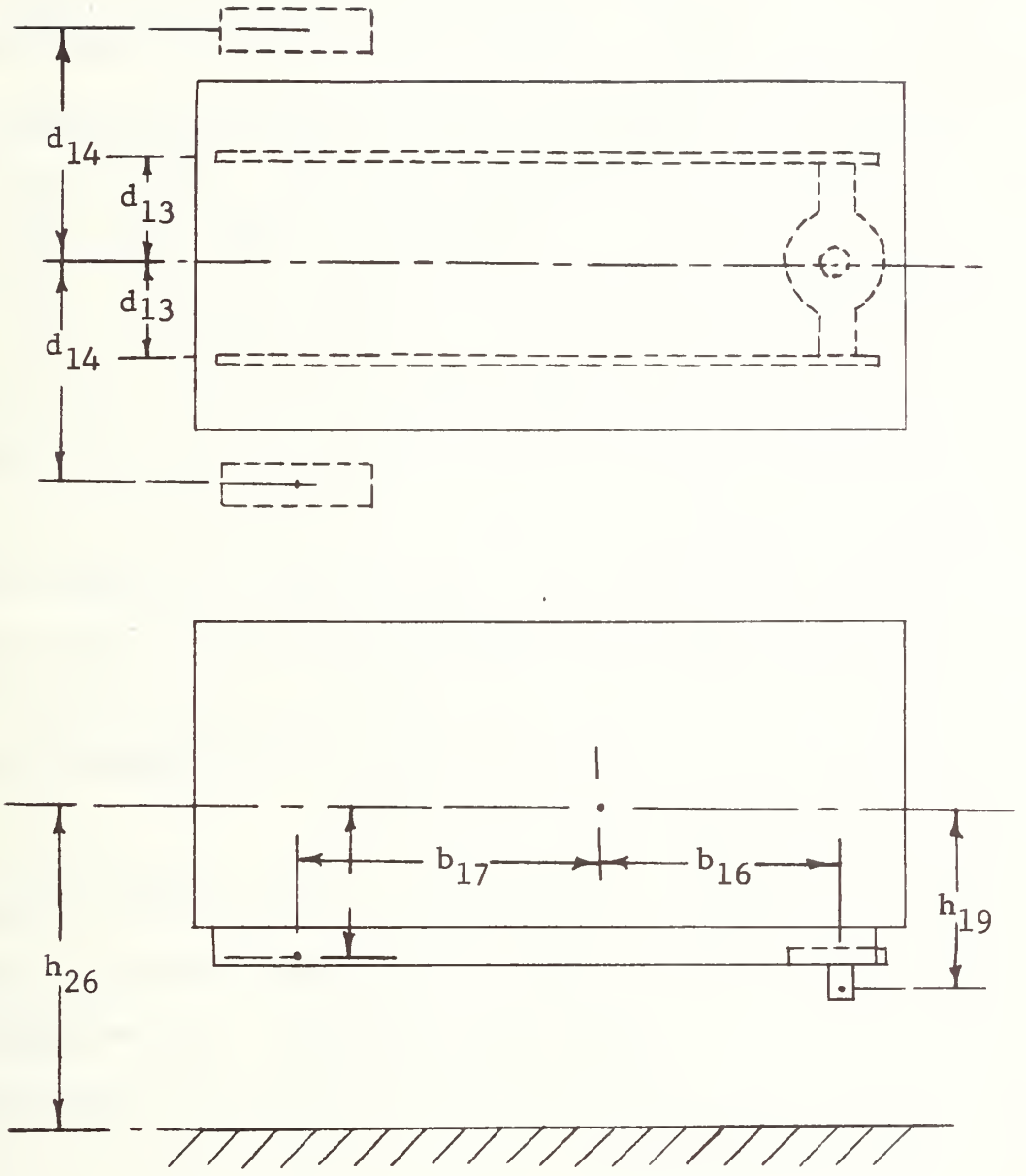


Figure A-7. Third semitrailer dimensions.

These coordinates are the forward and lateral translation of the tractor, and yaw angles for the tractor, three semitrailers, and two dollies. The effects of pitch and roll on the tire normal loads are introduced later through semistatic load transfer.

The forces and moments at the suspension points are written in terms of the tire forces (braking and side). On the front tires, relationships between the braking and side forces and the suspension point forces include the steering angle.

The computational strategy used is an inverse response calculation whereby the forward and lateral motion of the tractor is specified at small time increments and the required values of steering angle and braking forces are computed. Therefore, incremental steering angle and braking forces must be entered in the model. For this, the tire force equation and suspension-tire force relationships are expanded in terms of incremental steering, braking, and side forces.

The relationships for semistatic load transfer through roll and pitch variables are derived using the forward and lateral acceleration values with suspension properties. The suspension and tire springs are considered to be replaced by an equivalent spring placed at the tires.

The equations of motion and the suspension-tire relationships, when combined give the final set of eight equations relating the eight independent unknowns ($\psi 1$, $\psi 2$, $\psi 3$, $\psi 4$, $\psi 5$, $\psi 6$, $\Delta\delta$ and ΔD). These equations are programmed in the AVDS3 simulation model and are listed in the following pages.

$$\ddot{\psi}_1 \left[MST1 + MD1 + MST2 + MD2 + MST3 \right] b_3 \sin \psi_1$$

$$+ \ddot{\psi}_2 \left[MST1\ b_4 + \{ MD1 + MST2 + MD2 + MST3 \} \{ b_4 + b_6 \} \right] \sin \psi_2$$

$$+ \ddot{\psi}_3 \left[MD1\ b_7 + \{ MST2 + MD2 + MST3 \} \{ b_7 + b_9 \} \right] \sin \psi_3$$

$$+ \ddot{\psi}_4 \left[MST2\ b_{10} + \{ MD2 + MST3 \} \{ b_{10} + b_{12} \} \right] \sin \psi_4$$

$$+ \ddot{\psi}_5 \left[MD2\ b_{13} + MST3 \{ b_{13} + b_{15} \} \right] \sin \psi_5$$

$$+ \ddot{\psi}_6 \left[MST3\ b_{16} \right] \sin \psi_6$$

$$+ \Delta\delta \left[\left(\sum_{1}^{2} S_i \right) \cos \psi_1 - \left(\sum_{1}^{2} P_i \right) \sin \psi_1 \right]$$

$$+ \Delta D \left[\left(\sum_{1}^{4} T_i \right) \cos \psi_1 + \left(\sum_{5}^{6} T_i \right) \cos \psi_2 + \left(\sum_{7}^{8} T_i \right) \cos \psi_3 + \left(\sum_{9}^{10} T_i \right) \cos \psi_4 + \left(\sum_{11}^{12} T_i \right) \cos \psi_5 + \left(\sum_{13}^{14} T_i \right) \cos \psi_6 - \left(\sum_{1}^{2} Q_i \right) \sin \psi_1 \right]$$

$$= - \left(\sum_{1}^{4} R_i \right) \cos \psi_1 - \left(\sum_{5}^{6} R_i \right) \cos \psi_2 - \left(\sum_{7}^{8} R_i \right) \cos \psi_3 - \left(\sum_{9}^{10} R_i \right) \cos \psi_4 - \left(\sum_{11}^{12} R_i \right) \cos \psi_5 - \left(\sum_{13}^{14} R_i \right) \cos \psi_6 + \left(\sum_{1}^{4} N_i \right) \sin \psi_1 + \left(\sum_{5}^{6} N_i \right) \sin \psi_2 + \left(\sum_{7}^{8} N_i \right) \sin \psi_3 + \left(\sum_{9}^{10} N_i \right) \sin \psi_4 + \left(\sum_{11}^{12} N_i \right) \sin \psi_5 + \left(\sum_{13}^{14} N_i \right) \sin \psi_6 + \sum_{1}^{6} FaX_i$$

$$- \left[MT + MST1 + MD1 + MST2 + MD2 + MST3 \right] (\dot{VX1})$$

$$- \left[\{MST1 + MD1 + MST2 + MD2 + MST3\} \quad \{ b_3 \} \right] \quad (\dot{\psi}1)^2 \cos \psi 1$$

$$- \{MST1 + MD1 + MST2 + MD2 + MST3\} \quad \{ b_4 \} + \{ MD1 + MST2 + MD2 + MST3 \} \quad \{ b_6 \} \Big] \quad (\dot{\psi}2)^2 \cos \psi 2$$

$$- \left[\{ MD1 + MST2 + MD2 + MST3 \} \quad \{ b_7 \} + \{ MST2 + MD2 + MST3 \} \quad \{ b_9 \} \right] \quad (\dot{\psi}3)^2 \cos \psi 3$$

$$- \left[\{MST2 + MD2 + MST3\} \quad \{b_{10}\} + \{MD2 + MST3\} \quad \{b_{12}\} \right] \quad (\dot{\psi}4)^2 \cos \psi 4$$

$$- \left[\{ MD2 + MST3 \} \quad \{ b_{13} \} + \{ MST3 \} \quad \{ b_{15} \} \right] \quad (\dot{\psi}5)^2 \cos \psi 5$$

$$- \left[\{MST3\} \quad \{b_{16}\} \right] \quad (\dot{\psi}6)^2 \cos \psi 6 \qquad \text{(A-1)}$$

$$\ddot{\psi}1 \left[MST1 + MD1 + MST2 + MD2 + MST3 \right] \quad (-b_3) \cos \psi 1$$

$$+ \ddot{\psi}2 \left[-MST1 \; b_4 - \{MD1 + MST2 + MD2 + MST3\} \quad \{b_4 + b_6\} \right] \cos \psi 2$$

$$+ \ddot{\psi}3 \left[- MD1 \; b_7 - \{MST2 + MD2 + MST3\} \quad \{b_7 + b_9\} \right] \cos \psi 3$$

$$+ \ddot{\psi}4 \left[- MST2\ b_{10} - \{MD2 + MST3\} \{b_{10} + b_{12}\} \right] \cos \psi 4$$

$$+ \ddot{\psi}5 \left[- MD2\ b_{13} - \{MST3\} \{b_{13} + b_{15}\} \right] \cos \psi 5$$

$$+ \ddot{\psi}6 \left[- MST3\ b_{16} \right] \cos \psi 6$$

$$+ \Delta\delta \left[\left(\sum_{1}^{2} S_i \right) \sin \psi 1 + \left(\sum_{1}^{2} P_i \right) \cos \psi 1 \right]$$

$$+ \Delta D \left[\left(\sum_{1}^{4} T_i \right) \sin \psi 1 + \left(\sum_{5}^{6} T_i \right) \sin \psi 2 + \left(\sum_{7}^{8} T_i \right) \sin \psi 3 + \left(\sum_{9}^{10} T_i \right) \sin \psi 4 + \left(\sum_{11}^{12} T_i \right) \sin \psi 5 + \left(\sum_{13}^{14} T_i \right) \sin \psi 6 + \left(\sum_{1}^{2} Q_i \right) \sin \psi 1 \right]$$

$$= - \left(\sum_{1}^{4} R_i \right) \sin \psi 1 - \left(\sum_{5}^{6} R_i \right) \sin \psi 2 - \left(\sum_{7}^{8} R_i \right) \sin \psi 3 - \left(\sum_{9}^{10} R_i \right) \sin \psi 4 - \left(\sum_{11}^{12} R_i \right) \sin \psi 5 - \left(\sum_{13}^{14} R_i \right) \sin \psi 6$$

$$- \left(\sum_{1}^{4} N_i \right) \cos \psi 1 - \left(\sum_{5}^{6} N_i \right) \cos \psi 2 - \left(\sum_{7}^{8} N_i \right) \cos \psi 3 - \left(\sum_{9}^{10} N_i \right) \cos \psi 4 - \left(\sum_{11}^{12} N_i \right) \cos \psi 5 - \left(\sum_{13}^{14} N_i \right) \cos \psi 6$$

$$+ \sum_{1}^{6} Fa\ Y_i$$

$$- \left[MT + MST1 + MD1 + MST2 + MD2 + MST3 \right] (\dot{VY1})$$

$$- \left[\{MST1 + MD1 + MST2 + MD2 + MST3\} \quad \{b_3\} \quad (\dot{\psi}1)^2 \sin \psi 1 \right.$$

$$- \left[\{MST1 + MD1 + MST2 + MD2 + MST3\} \quad \{b_4\} + \{MD1 + MST2 + MD2 + MST3\} \quad \{b_6\} \right] (\dot{\psi}2)^2 \sin \psi 2$$

$$- \left[\{MD1 + MST2 + MD2 + MST3\} \quad \{b_7\} + \{MST2 + MD2 + MST3\} \quad \{b_9\} \right] (\dot{\psi}3)^2 \sin \psi 3$$

$$- \left[\{MST2 + MD2 + MST3\} \quad \{b_{10}\} + \{MD2 + MST3\} \quad \{b_{12}\} \right] (\dot{\psi}4)^2 \sin \psi 4$$

$$- \left[\{MD2 + MST3\} \quad \{b_{13}\} + \{MST3\} \quad \{b_{15}\} \right] (\dot{\psi}5)^2 \sin \psi 5$$

$$- \left[\{MST3\} \quad \{b_{16}\} \right] (\dot{\psi}6)^2 \sin \psi 6 \qquad \text{(A-2)}$$

$$\ddot{\psi}1 \left[IZZ1 \right]$$

$$+ \Delta\delta \left[\{S_1 - S_2\} d_2 + \{P_1 + P_2\} \{b_1 + b_3\} \right]$$

$$+ \Delta D \left[\{T_1 - T_2\} d_2 + \{T_3 - T_4\} d_3 + \{Q_1 + Q_2\} \{b_1 + b_3\} \right]$$

$$=$$

$$- \left[R_1 - R_2 \right] d_2 - \left[R_3 - R_4 \right] d_4 - \left[N_1 + N_2 \right] b_1 + \left[N_3 + N_4 \right] b_2$$

$$+ M_{15Z} + MaZ_1$$

$$- \left(\sum_1^4 N_i \right) b_3$$

$$+ \left[FaY_1 \cos \psi 1 - FaX_1 \sin \psi 1 \right] b_3$$

$$+ \left[(\dot{V}X1) \sin \psi 1 - (\dot{V}Y1) \cos \psi 1 \right] (MT) b_3 \qquad \text{(A-3)}$$

$$\ddot{\psi}1 \left[- MST1 \, b_3 \, b_6 \cos (\psi 1 - \psi 2) \right]$$

$$+ \ddot{\psi}2 \left[IZZ2 - MST1 \, b_4 \, b_6 \right]$$

$$+ \Delta\delta \left[\{b_4 + b_6\} \left\{ (\sum_1^2 S_i) \sin (\psi 1 - \psi 2) + (\sum_1^2 P_i) \cos (\psi 1 - \psi 2) \right\} \right]$$

$$+ \Delta D \left[\{T_5 - T_6\} d_6 + \{b_4 + b_6\} \left\{ \left(\sum_1^4 T_i\right) \sin(\psi 1 - \psi 2) + \left(\sum_1^2 Q_i\right) \cos(\psi 1 - \psi 2) \right\} \right]$$

$$=$$

$$- \{R_5 - R_6\} d_6 + \{N_5 + N_6\} \{b_5 - b_6\}$$

$$+ M_{16Z} + MaZ_2$$

$$+ b_6 \left[FaY_2 \cos \psi 2 - FaX_2 \sin \psi 2 \right]$$

$$- \left[b_4 + b_6 \right] \left[\left(\sum_1^4 R_i\right) \sin(\psi 1 - \psi 2) + \left(\sum_1^4 N_i\right) \cos(\psi 1 - \psi 2) \right]$$

$$+ \left[b_4 + b_6 \right] \left[FaY_1 \cos \psi 2 - FaX_1 \sin \psi 2 \right]$$

$$- \left[MST1 \; b_3 \; b_6 \right] \sin(\psi 1 - \psi 2) (\dot{\psi} 1)^2$$

$$+ \left[MST1 \; b_6 + MT \; \{b_4 + b_6\} \right] \left[(\dot{V}X1) \sin \psi 2 - (\dot{V}Y1) \cos \psi 2 \right]$$

(A-4)

$$\ddot{\psi}1 \left[-b_3 \{MD1\ b_9 + MST1\ (b_7 + b_9)\} \right] \cos(\psi 1 - \psi 3)$$

$$+ \ddot{\psi}2 \left[- \{MD1\ b_9\ (b_4 + b_6) + MST1\ b_4\ (b_7 + b_9)\} \right] \cos(\psi 2 - \psi 3)$$

$$+ \ddot{\psi}3 \left[IZZ3 - MD1\ b_7\ b_9 \right]$$

$$+ \Delta\delta \left[\{b_7 + b_9\} \left\{ \left(\sum_1^2 S_i\right) \sin(\psi 1 - \psi 3) + \left(\sum_1^2 P_i\right) \cos(\psi 1 - \psi 3) \right\} \right]$$

$$+ \Delta D \left[\{T_7 - T_8\}\ d_8 + \{b_7 + b_9\} \left\{ \left(\sum_5^6 T_i\right) \sin(\psi 2 - \psi 3) + \left(\sum_1^4 T_i\right) \sin(\psi 1 - \psi 3) + \left(\sum_1^2 Q_i\right) \cos(\psi 1 - \psi 3) \right\} \right]$$

$$=$$

$$- \left[R_7 - R_8 \right] d_8 + \left[N_7 + N_8 \right] \{b_8 - b_9\}$$

$$+ M_{19Z} + MaZ_3$$

$$+ b_9 \left[FaY_3 \cos\psi 3 - FaX_3 \sin\psi 3 \right]$$

$$- \{b_7 + b_9\} \left[\left(\sum_5^6 R_i\right) \sin(\psi 2 - \psi 3) + \left(\sum_5^6 N_i\right) \cos(\psi 2 - \psi 3) + \left(\sum_1^4 R_i\right) \sin(\psi 1 - \psi 3) + \left(\sum_1^4 N_i\right) \cos(\psi 1 - \psi 3) \right]$$

$$+ \quad \{b_7 + b_9\} \left[\left\{ \sum_{1}^{2} FaY_i \right\} \cos \psi 3 - \left\{ \sum_{1}^{2} FaX_i \right\} \sin \psi 3 \right]$$

$$- \; b_3 \left[MD1\, b_9 + MST1 \{b_7 + b_9\} \right] \sin (\psi 1 - \psi 3) (\dot{\psi} 1)^2$$

$$- \left[MD1\, b_9 \{b_4 + b_6\} + MST1\, b_4 \{b_7 + b_9\} \right] \sin (\psi 2 - \psi 3) (\dot{\psi} 2)^2$$

$$+ \left[MD1\, b_9 + MST1 \{b_7 + b_9\} + MT \{b_7 + b_9\} \right]$$

$$\left[(\dot{VX}1) \sin \psi 3 - (\dot{VY}1) \cos \psi 3 \right] \qquad \text{(A-5)}$$

$$\ddot{\psi} 1 \left[-b_3 \{MST2\, b_{12} + (MD1 + MST1)(b_{10} + b_{12})\} \right] \cos (\psi 1 - \psi 4)$$

$$+ \ddot{\psi} 2 \left[- \{MST2\, b_{12} (b_4 + b_6) + MD1\, b_4 (b_{10} + b_{12}) + MST1\, b_4 (b_{10} + b_{12})\} \right] \cos (\psi 2 - \psi 4)$$

$$+ \ddot{\psi} 3 \left[- \{MST2\, b_{12} (b_7 + b_9) + MD1\, b_7\, b_{10} + b_{12})\} \right] \cos (\psi 3 - \psi 4)$$

$$+ \ddot{\psi} 4 \left[IZZ4 - MST2\, b_{12}\, b_{10} \right]$$

$$+ \Delta\delta \left[\{b_{10} + b_{12}\} \left\{ \left(\sum_{1}^{2} S_i \right) \sin (\psi 1 - \psi 4) + \left(\sum_{1}^{2} P_i \right) \cos (\psi 1 - \psi 4) \right\} \right]$$

$$+ \quad \wedge D \quad \left[\quad \{T_9 - T_{10}\} \quad d_{10} + \quad \{b_{10} + b_{12}\} \quad \left\{ \sum_{7}^{8} T_i \right) \sin (\psi 3 - \psi 4) +$$

$$\left(\sum_{5}^{6} T_i \right) \sin (\psi 2 - \psi 4) + \left(\sum_{1}^{4} T_i \right) \sin (\psi 1 - \psi 4) + \left(\sum_{1}^{2} Q_i \right) \cos (\psi 1 - \psi 4) \right\} \Bigg]$$

$$=$$

$$- \quad \left[\quad R_9 - R_{10} \right] \quad d_{10} + \left[\quad N_9 + N_{10} \right] \quad \{b_{11} - b_{12}\}$$

$$+ \quad M_{20Z} + MaZ_4$$

$$+ \quad b_{12} \left[FaY_4 \cos \psi 4 - FaX_4 \sin \psi 4 \right]$$

$$- \quad \{b_{10} + b_{12}\} \quad \left[\quad \left(\sum_{7}^{8} R_i \right) \sin (\psi 3 - \psi 4 + \left(\sum_{5}^{6} R_i \right) \sin (\psi 2 - \psi 4) + \right.$$

$$\left(\sum_{1}^{4} R_i \right) \sin (\psi 1 - \psi 4) + \left(\sum_{7}^{8} N_i \right) \cos (\psi 3 - \psi 4) + \left(\sum_{5}^{6} N_i \right) \cos (\psi 2 - \psi 4) +$$

$$\left. \left(\sum_{1}^{4} N_i \right) \cos (\psi 1 - \psi 4) \right]$$

$$+ \quad \{b_{10} + b_{12}\} \quad \left[\quad \left\{ \sum_{1}^{3} FaY_i \right\} \cos \psi 3 - \left\{ \sum_{1}^{3} FaX_i \right\} \sin \psi 3 \right]$$

$$- \quad b_3 \left[\quad MST2 \; b_{12} + \quad \{MD1 + MST1\} \quad \{b_{10} + b_{12}\} \right] \quad \sin (\psi 1 - \psi 4) \; (\dot{\psi} 1)^2$$

$$- \quad \left[\quad MST2 \; b_{12} \quad \{b_4 + b_6\} \quad + MD1 \quad \{b_{10} + b_{12}\} \quad \{b_4 + b_6\} + \right.$$

$$\text{MST1}\ \{b_{10} + b_{12}\}\ b_4 \Big] \quad \sin(\psi 2 - \psi 4)\ (\dot{\psi} 2)^2$$

$$- \Big[\text{MST2}\ b_{12}\ \{b_7 + b_9\} + \text{MD1}\ \{b_{10} + b_{12}\}\ b_7 \Big] \sin(\psi 3 - \psi 4)\ (\dot{\psi} 3)^2$$

$$+ \Big[\text{MST2}\ b_{12} + \{\text{MD1} + \text{MST1} + \text{MT}\}\ \{b_{10} + b_{12}\} \Big]$$

$$\Big[(\dot{VX}1) \sin \psi 4 - (\dot{VY}1) \cos \psi 4 \Big] \qquad \text{(A-6)}$$

$$\ddot{\psi} 1 \Big[-b_3\ \{\text{MD2}\ b_{15} + (\text{MST2} + \text{MD1} + \text{MST1})\ (b_{13} + b_{15})\} \Big]$$

$$\cos(\psi 1 - \psi 5)$$

$$+ \ddot{\psi} 2 \Big[- \{\text{MD2}\ b_{15}\ (b_4 + b_6) + (\text{MST2} + \text{MD1})\ (b_4 + b_6)\ (b_{13} + b_{15}) +$$

$$\text{MST1}\ (b_{13} + b_{15})\ b_4\} \Big] \quad \cos(\psi 2 - \psi 5)$$

$$+ \ddot{\psi} 3 \Big[- \{\text{MD2}\ b_{15}\ (b_7 + b_9) + \text{MST2}\ (b_7 + b_9)\ (b_{13} + b_{15}) +$$

$$\text{MD1}\ (b_{13} + b_{15})\ b_7\} \Big] \quad \cos(\psi 3 - \psi 5)$$

$$+ \ddot{\psi} 4 \Big[- \{\text{MD2}\ b_{15}\ (b_{10} + b_{12}) + \text{MST2}\ b_{10}\ (b_{13} + b_{15})\} \Big] \quad \cos(\psi 4 - \psi 5)$$

$$+ \ddot{\psi} 5 \Big[\text{IZZ5} - \text{MD2}\ b_{13}\ b_{15} \Big]$$

$$+ \quad \Delta\delta \quad \left[\{ b_{13} + b_{15} \} \quad \left\{ \left(\sum_{1}^{2} S_i\right) \sin(\psi 1-\psi 5) + \left(\sum_{1}^{2} P_i\right) \cos(\psi 1-\psi 5) \right\} \right]$$

$$+ \quad \Delta D \quad \left[\{ T_{11}-T_{12} \} d_{12} + \{ b_{13} + b_{15} \} \quad \left\{ \left(\sum_{9}^{10} T_i\right) \sin(\psi 4-\psi 5) + \right.\right.$$

$$\left(\sum_{7}^{8} T_i\right) \sin(\psi 3-\psi 4) + \left(\sum_{5}^{6} T_i\right) \sin(\psi 2-\psi 5) + \left(\sum_{1}^{4} T_i\right) \sin(\psi 1-\psi 5) +$$

$$\left.\left. \left(\sum_{1}^{2} Q_i\right) \cos(\psi 1-\psi 5) \right\} \right]$$

$=$

$$- \quad \left[R_{11}-R_{12} \right] d_{12} + \left[N_{11} + N_{12} \right] \quad \{ b_{14}-b_{15} \}$$

$$+ \quad M_{23Z} + MaZ_5$$

$$+ \quad b_{15} \left[FaY_5 \cos \psi 5 - FaX_5 \sin \psi 5 \right]$$

$$- \quad (b_{13} + b_{15}) \left[\left(\sum_{9}^{10} R_i\right) \sin(\psi 4-\psi 5) + \left(\sum_{7}^{8} R_i\right) \sin(\psi 3-\psi 5) + \right.$$

$$\left(\sum_{5}^{6} R_i\right) \sin(\psi 2-\psi 5) + \left(\sum_{1}^{4} R_i\right) \sin(\psi 1-\psi 5) + \left(\sum_{9}^{10} N_i\right) \cos(\psi 4-\psi 5) +$$

$$\left. \left(\sum_{7}^{8} N_i\right) \cos(\psi 3-\psi 5) + \left(\sum_{5}^{6} N_i\right) \cos(\psi 2-\psi 5) + \left(\sum_{1}^{4} N_i\right) \cos(\psi 1-\psi 5) \right]$$

$$+ \quad \{ b_{13} + b_{15} \} \left[\left\{ \sum_{1}^{4} FaY_i \right\} \cos \psi 5 - \left\{ \sum_{1}^{4} FaX_i \right\} \sin \psi 5 \right]$$

$$-b_3 \left[MD2\ b_{15} + \{MST2 + MD1 + MST1\} \{b_{13} + b_{15}\} \right]$$

$$\sin(\psi 1-\psi 5)\ (\dot{\psi}1)^2$$

$$- \left[MD2\ b_{15}\ \{b_4 + b_6\} + \{MST2 + MD1\} \{b_4 + b_6\} \{b_{13} + b_{15}\} \right.$$

$$\left. + MST1\ \{b_{13} + b_{15}\}\ b_4 \right] \sin(\psi 2-\psi 5)\ (\dot{\psi}2)^2$$

$$- \left[MD2\ b_{15}\ \{b_7 + b_9\} + \{MST2\} \{b_7 + b_9\} \{b_{13} + b_{15}\} + \right.$$

$$\left. MD1\ \{b_{13} + b_{15}\}\ b_7 \right] \sin(\psi 3-\psi 5)\ (\dot{\psi}3)^2$$

$$- \left[MD2\ b_{15}\ \{b_{10} + b_{12}\} + MST2\ b_{10}\ \{b_{13} + b_{15}\} \right]$$

$$\sin(\psi 4-\psi 5)\ (\dot{\psi}4)^2$$

$$+ \left[MD2\ b_{15} + \{MST2 + MD1 + MST1 + MT\} b_{13} + b_{15} \right]$$

$$\left[(\dot{V}X1) \sin \psi 5 - (\dot{V}Y1) \cos \psi 5 \right] \qquad \text{(A-7)}$$

$$\ddot{\psi}1 \left[-b_3\ b_{16}\ \{MST1 + MD1 + MST2 + MD2\} \right] \cos(\psi 1-\psi 6)$$

$$+ \ddot{\psi}2 \left[-b_4\ b_{16}\ \{MST1 + MD1 + MST2 + MD2\} - b_6\ b_{16}\ \{MD1 + \right.$$

$$\left. MST2 + MD2\} \right] \cos(\psi 2-\psi 6)$$

$$+ \ddot{\psi}3 \left[-b_7 \, b_{16} \ \{MD1 + MST2 + MD2\} - b_9 \, b_{16} \ \{MST2 + MD2\} \right]$$

$$\cos (\psi 3 - \psi 6)$$

$$+ \ddot{\psi}4 \left[- b_{10} \, b_{16} \ \{MST2 + MD2\} - b_{12} \, b_{16} \ \{MD2\} \right] \cos (\psi 4 - \psi 6)$$

$$+ \ddot{\psi}5 \left[- b_{13} \, b_{16} \ \{MD2\} \right] \cos (\psi 5 - \psi 6)$$

$$+ \ddot{\psi}6 \left[IZZ6 \right]$$

$$+ \Delta\delta \left[\left(\sum_{1}^{2} S_i \right) \sin (\psi 1 - \psi 6) + \left(\sum_{1}^{2} P_i \right) \cos (\psi 1 - \psi 6) \right] b_{16}$$

$$+ \Delta D \left[\{T_{13} - T_{14}\} \ d_{14} + b_{16} \left\{ \left(\sum_{11}^{12} T_i \right) \sin (\psi 5 - \psi 6) + \left(\sum_{9}^{10} T_i \right) \sin \right. \right.$$

$$(\psi 4 - \psi 6) + \left(\sum_{7}^{8} T_i \right) \sin (\psi 3 - \psi 6) + \left(\sum_{5}^{6} T_i \right) \sin (\psi 2 - \psi 6) + \left(\sum_{1}^{4} T_i \right) \sin$$

$$\left. \left. (\psi 1 - \psi 6) + \left(\sum_{1}^{2} Q_i \right) \cos (\psi 1 - \psi 6) \right\} \right]$$

$$=$$

$$- \left[R_{13} - R_{14} \right] d_{14} + \left[N_{13} + N_{14} \right] b_{17}$$

$$+ M_{24Z} + MaZ_6$$

$$- b_{16} \left[\left(\sum_{11}^{12} R_i \right) \sin (\psi 5 - \psi 6) + \left(\sum_{9}^{10} R_i \right) \sin (\psi 4 - \psi 6) + \right.$$

$$\left(\sum_{7}^{8} R_i \right) \sin (\psi 3 - \psi 6) + \left(\sum_{5}^{6} R_i \right) \sin (\psi 2 - \psi 6) + \left(\sum_{1}^{4} R_i \right) \sin (\psi 1 - \psi 6)$$

$$+ \left(\sum_{11}^{12} N_i \right) \cos (\psi 5 - \psi 6) + \left(\sum_{9}^{10} N_i \right) \sin (\psi 4 - \psi 6) + \left(\sum_{7}^{8} N_i \right) \sin (\psi 3 - \psi 6)$$

$$\left. + \left(\sum_{5}^{6} N_i \right) \sin (\psi 2 - \psi 6) + \left(\sum_{1}^{4} N_i \right) \sin (\psi 1 - \psi 6) \right]$$

$$+ b_{16} \left[\left(\sum_{1}^{5} FaY_i \right) \cos \psi 6 - \left(\sum_{1}^{5} FaX_i \right) \sin \psi 6 \right]$$

$$- \left[MST1 + MD1 + MST2 + MD2 \right] b_3 \, b_{16} \sin (\psi 2 - \psi 6) \, (\dot{\psi} 1)^2$$

$$- \left[\{ MST1 + MD1 + MST2 + MD2 \} \, b_4 + \{ MD1 + MST2 + MD2 \} \, b_6 \right]$$

$$b_{16} \sin (\psi 2 - \psi 6) \, (\dot{\psi} 2)^2$$

$$- \left[\{ MD1 + MST2 + MD2 \} \, b_7 + \{ MST2 + MD2 \} \, b_9 \right] b_{16} \sin$$

$$(\psi 3 - \psi 6) \, (\dot{\psi} 3)^2$$

$$- \left[\{ MST2 + MD2 \} \, b_{10} + \{ MD2 \} \, b_{12} \right] b_{16} \sin (\psi 4 - \psi 6) \, (\dot{\psi} 4)^2$$

$$- \left[MD2 \, b_{13} \right] b_{16} \sin (\psi 5 - \psi 6) \, (\dot{\psi} 5)^2$$

$$+ \; b_{16} \left[MT + MST1 + MD1 + MST2 + MD2 \right] \left[(\dot{VX1}) \sin \psi 6 - (\dot{VY1}) \cos \psi 6 \right] \qquad \text{(A-8)}$$

The equations given herein are for a triple tractor-trailer vehicle. The following substitutions for some of the vehicle parameter values give the set of equations for double and single tractor-trailers and also for a truck (or automobile or bus).

2.2 Double Tractor-Trailers Vehicle

MD2=0.0	IZZ5=0.0	b_{12}=0.0	
MST3=0.0	IZZ6=0.0	h_{14}=0.0	(A-9)

These substitutions will yield a set of six equations relating the six independent unknowns:

$\ddot{\psi}_1$, $\ddot{\psi}_2$, $\ddot{\psi}_3$, $\ddot{\psi}_4$, $\Delta\delta$ and ΔD.

2.3 Single Tractor-Trailer Vehicle

MD2=0.0	IZZ5=0.0	b_{12}=0.0	
MST3=0.0	IZZ6=0.0	h_{14}=0.0	
MD1=0.0	IZZ3=0.0	b_6=0.0	(A-10)
MST2=0.0	IZZ4=0.0	h_7=0.0	

These substitutions will yield a set of four equations relating the four independent unknowns: $\ddot{\psi}1$, $\ddot{\psi}2$, $\Delta\delta$ and ΔD.

2.4 Truck (or automobile or bus)

MD2=0.0	IZZ5=0.0	b_{12}=0.0	
MST3=0.0	IZZ6=0.0	h_{14}=0.0	
MD1=0.0	IZZ3=0.0	b_6=0.0	(A-11)
MST2=0.0	IZZ4=0.0	h_7=0.0	
MST1=0.0	IZZ2=0.0	b_3=0.0	
		h_3=0.0	
		h_4=0.0	

These substitutions will yield a set of three equations relating the three independent unknowns: $\ddot{\psi}1$, $\Delta\delta$ and ΔD.

3. TIRES

The development of frictional (tire cornering and brake/tractive) forces between tires and the road control the dynamic behavior of articulated vehicles. These tire forces are nonlinear functions of both longitudinal and side slips. Increased brake/tractive force increases the longitudinal slip between the tire and the road. Also, there is a loss of cornering (side or lateral) force with increased brake/tractive force. For optimum cornering and brake/tractive forces, the longitudinal slip needs to be controlled at about 1 or 2 percent.

In the mathematical modeling of articulated vehicles, it is of utmost importance that a realistic tire mechanics model is used. The degree of correlation between actual and simulated dynamic behavior of these vehicles is largely dependent upon the tire model.

In lieu of a general quantitative model, the Ellis formulation of the tire side force phenomenon has been used intensively in analytical studies. The Ellis formula was modified in the previous study to allow variation of the maximum slip angle (α_m) with normal tire load (F_z).

In this study, a model based upon actual tests data on the side force versus slip angle is utilized. Tests data on the 10.00-20 Super Hi-Miler truck tires (Figure A-8) were obtained from the Goodyear Tire and Rubber Company, Ohio. These tires were also placed on the test vehicle.

A curve-fitting procedure has been utilized in derivation of the IITRI tire side force formula. In the first step of the procedure, approximate linear relationships for the maximum slip angle (α_m, slip angle at which maximum side force occurs) and the apparent coefficient of friction (μ, maximum side force value divided by the load) are established from Figure A-8 and are given below.

$$\alpha_m = \frac{9.0 - \frac{9.0F_z}{6800.0}}{57.27} \qquad \text{(A-12)}$$

$$\mu = 1.0 + \frac{0.35F_z}{6800.0} \qquad \text{(A-13)}$$

where F_z = tire normal load.

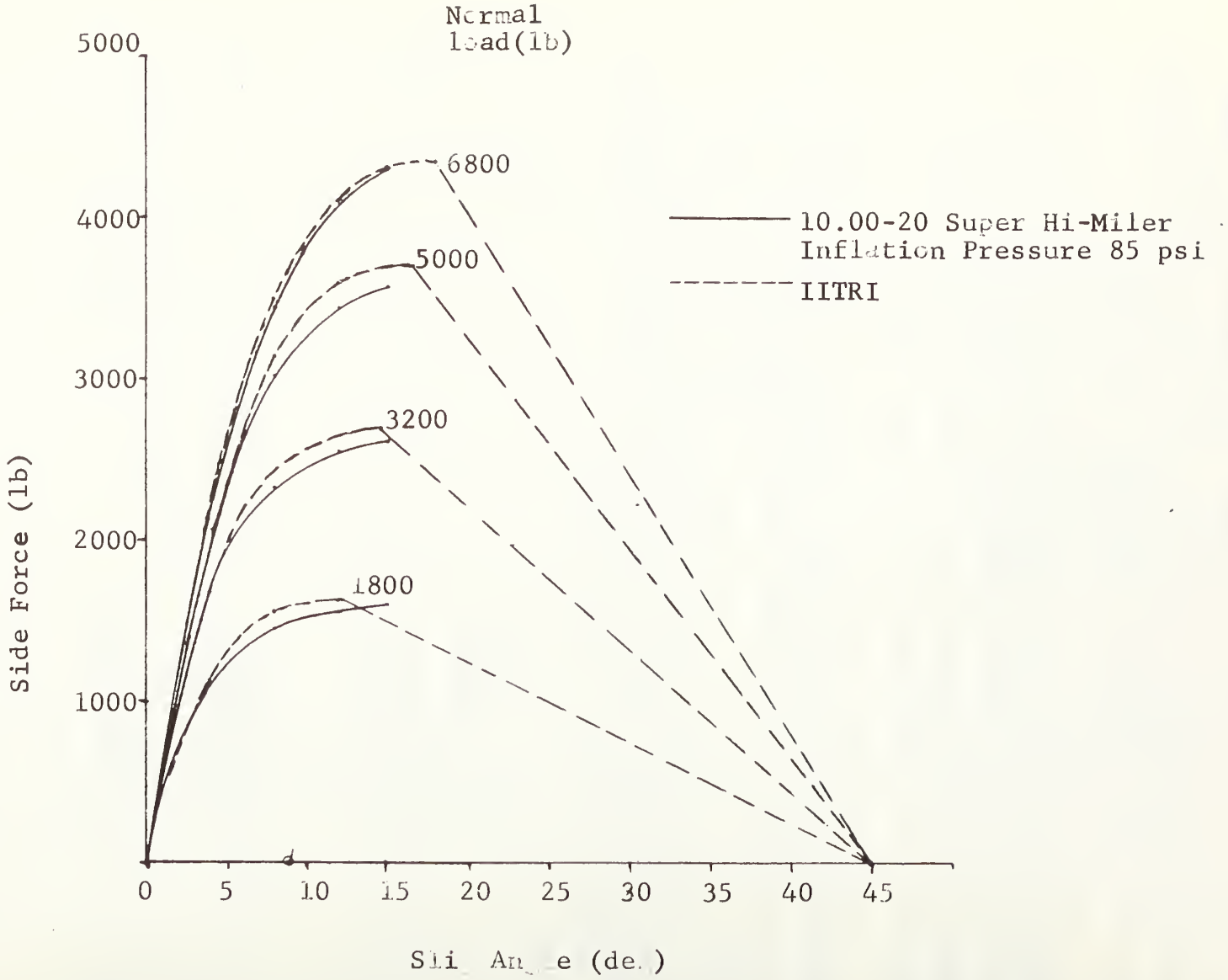


Figure A-8. Comparison of IITRI tire side force formula vs experimental data.

Next, the side force is assumed to be a third order polynomial function of slip angle ($F_s = A\alpha + B\alpha^2 + C\alpha^3$). The constants (A, B, C) of polynomial are evaluated from the conditions at α = 1/3 α_m, 2/3 α_m, and α_m for the 6800 lb load tests data (α_m=18°), giving

$$F_s = \frac{\mu F_z}{4.0} \left\{ 11.0 \frac{\alpha}{\alpha_m} - (\text{signum})\ 10.0 \frac{\alpha^2}{\alpha_m^2} + 3.0 \frac{\alpha^3}{\alpha_m^3} \right\} \qquad \text{(A-14)}$$

The above IITRI formula is good only for slip angle values not exceeding α_m. The side force, Fs, is assumed to drop linearly from its maximum value μF_z at $\alpha=\alpha_m$ to zero at $\alpha = \pm \frac{\pi}{4}$.

The side force values obtained from (A-14) and the linear relationship beyond $\alpha=\alpha_m$ are also plotted in Figure A-8.

4. VEHICLE MANEUVER MATHEMATICAL MODELS

The dynamic response of the articulated vehicle is evaluated during commonly encountered road maneuvers such as lane change, evasive (double lane change), and cornering. The maneuver trajectory for the tractor center of gravity is prescribed with a mathematical function. The tractor c.g. velocity during the maneuver is specified as a function of time. The velocity and trajectory functions are then differentiated for values of acceleration.

4.1 Lane Change Maneuver [23]

A seventh order polynomial function is used to prescribe the lane change maneuver trajectory (length (L) x width (S)). A typical trajectory is shown in Figure A-9. Applying the conditions at the start (X=0.0, Y=Y'=Y''=Y'''=0.0) and

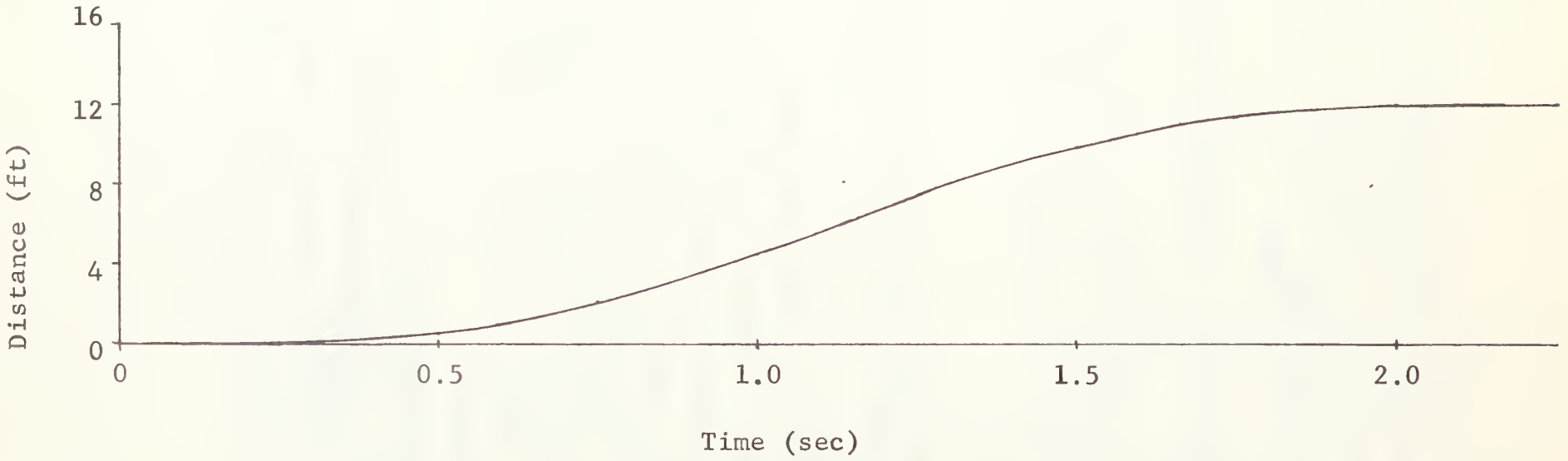


Figure A-9. Lane change maneuver (12.00 ft x 131.0 ft) at 40 mph.

at the end (X=L, Y=S, Y'=Y''=Y'''= 0.0) results in,

$$Y=f(X) = \frac{S}{L^7} (35L^3X^4-84L^2X^5+70LX^6 - 20X^7) \quad \text{(A-15)}$$

$$Y'= f'(X) = \frac{20S}{L^7} \{7L^3X^3 - 21L^2X^4 + 21LX^5 - 7X^6\} \quad \text{(A-16)}$$

$$Y'' = f''(X) = \frac{140S}{L^7} \{3L^3X^2 - 12L^2X^3 + 15LX^4 - 6X^5\} \quad \text{(A-17)}$$

The velocity and acceleration of the tractor center of gravity in X and Y directions are given by the following equations.

$$\dot{X} = \frac{V}{\left[1.0 + \left[f'(X)\right]^2\right]^{1/2}} \quad \text{(A-18)}$$

$$\dot{Y} = \frac{f(X)\ V}{\left[1.0 + \left[f'(X)\right]^2\right]^{1/2}} \quad \text{(A-19)}$$

$$\ddot{X} = - \frac{f'(X)\ f''(X)\ V^2}{\left[1.0 + \left[f'(X)\right]^2\right]^2} + \frac{\dot{V}}{\left[1.0 + \left[f'(X)\right]^2\right]^{1/2}} \quad \text{(A-20)}$$

$$\ddot{Y} = \frac{f''(X)\ V^2}{\left[1.0 + \left[f'(X)\right]^2\right]^2} + \frac{f'(X)\ \dot{V}}{\left[1.0 + \left[f'(X)\right]^2\right]^{1/2}} \quad \text{(A-21)}$$

4.2 Evasive Maneuver of Double Lane Change Maneuver

In evasive maneuver, a lane change is immediately followed by a reverse lane change. As such, the evasive maneuver trajectory is also specified by the polynomial function (A-15) for the lane change maneuver. The length ($L=L_1$, L_2) and the width ($S=+S_1,-S_2$) values for both lane changes are specified. A typical maneuver trajectory is shown in Figure A-10.

The velocity and acceleration values for the evasive maneuver are also given by Equations(A-15) to(A-21) with appropriate values of length (L_1 or L_2) and width ($+ S_1$ or $-S_2$).

4.3 Cornering Maneuver

The cornering maneuver trajectory is specified in three distinct parts. The approach to the curve and exit from the curve are specfied as partial lane change maneuvers and are prescribed with the polynomial function for lane change trajectory. The tractor c.g path around the curve is specified circular and a math-function for circle is used to presribe it.

A typical maneuver trajectory is shown in Figure A-11. The segments AB and CD are the partial lane changes respectively. BC is the circular path representing motion around the curve. (X_c, Y_c) are the center coordinates of BC. AB and CD are segments of the complete trajectories AA' and DD' respectively. The radius of curvature (ρ) at B on AA' (also at C on DD') is equal to the radius (R) of BC.

The radius of curvature at a point on curve is given by,

$$\rho = \frac{\left\{1.0 + \left|f'(X)\right|^2\right\}^{3/2}}{\left|f''(X)\right|} \simeq \frac{1.0}{\left|f''(X)\right|} \qquad \text{(A-22)}$$

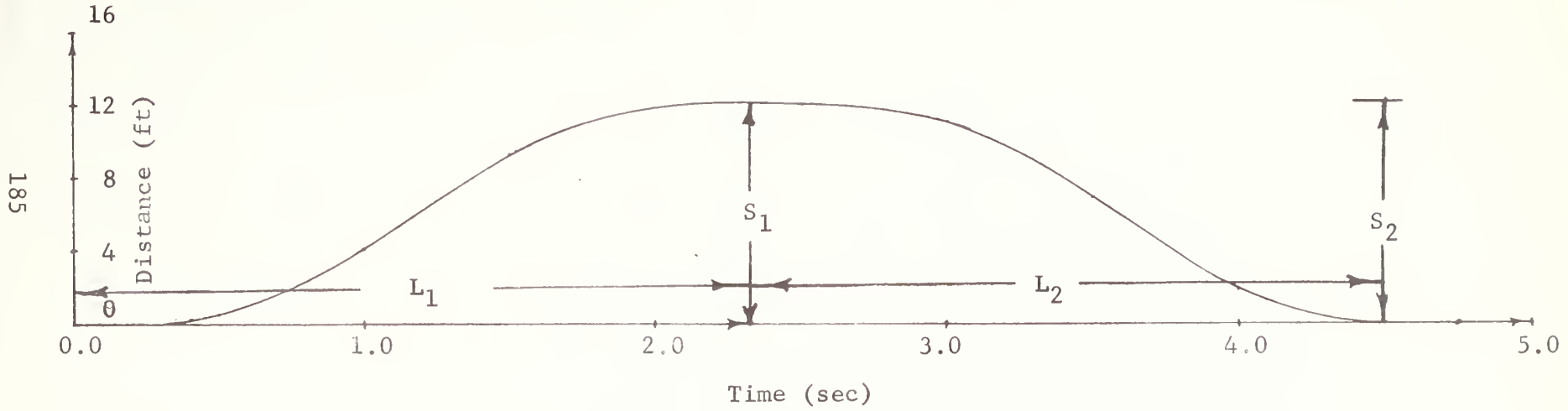


Figure A-10. Evasive (or double lane change) maneuver (275.0 ft long) at 40 mph.

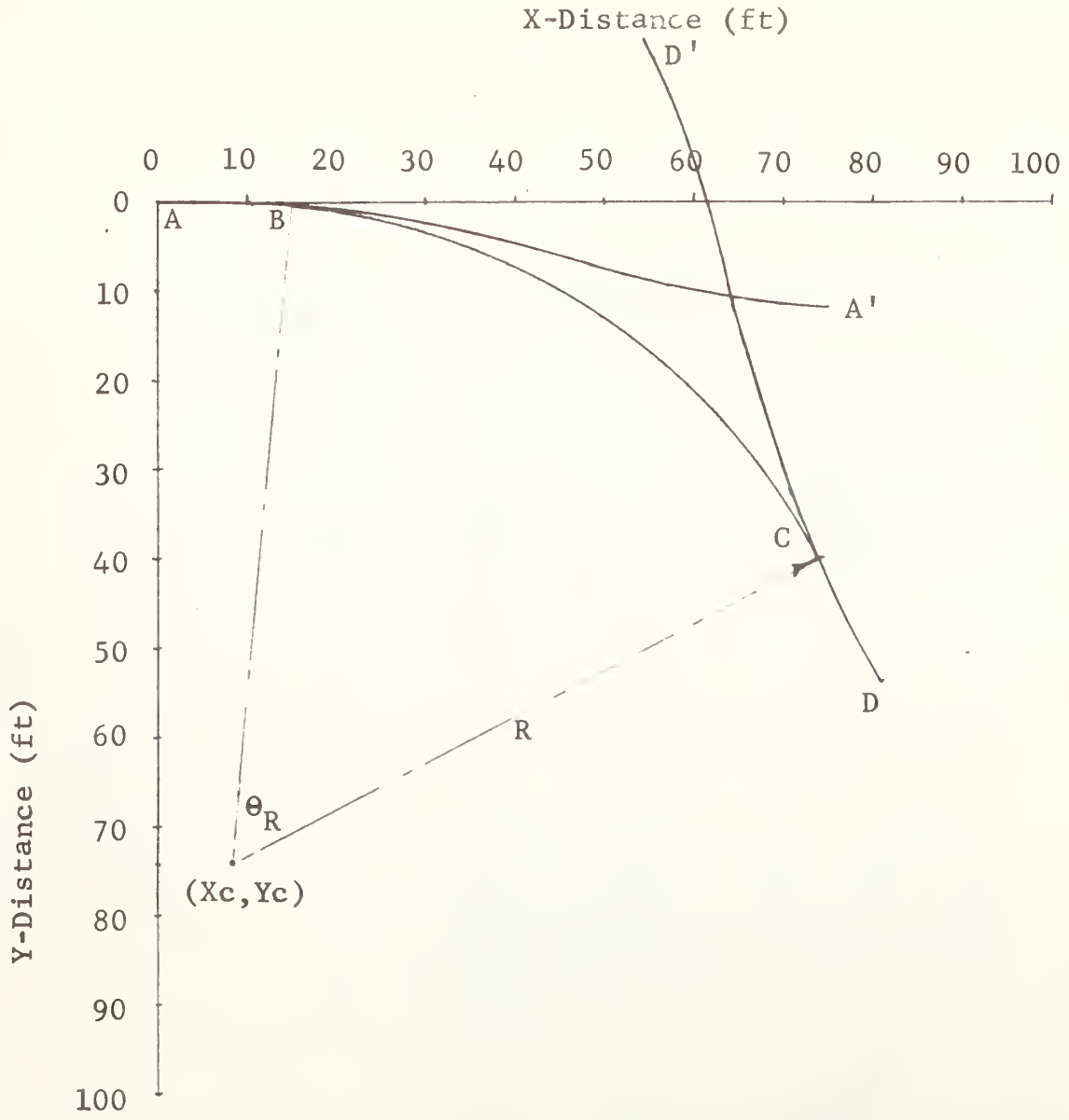


Figure A-11. Cornering maneuver (74.17 ft radius) at 20 mph.

The length (L) of AA' is assigned arbitrarily (usually five times the initial velocity). Assuming B at $X = \frac{L}{5}$, and combining (A-17) and (A-22) give,

$$S = \frac{625\ L^2}{4032\ R} \tag{A-23}$$

AA' [length (L) x width (S)]is given by (A-12). BC [with radius (R) and angle of arc (θ_R) specified]is given by,

$$[Y-Y_c]^2 + [X-X_c]^2 = R^2 \tag{A-24}$$

Neglecting rotation of axis, DD' is a mirror image of AA' with a change in end condition as

$$X = -\ L \qquad Y = S \tag{A-25}$$

This results in,

$$Y=f(X) = \frac{S}{L^7}\ (35L^3X^4 + 84L^2X^5 + 70LX^6 + 20X^7) \tag{A-26}$$

The velocity and acceleration values on segments AB and CD are given by Equations (A-18) to (A-21). On segment BC, these values are determined as follows. θ is the angle between velocity (V)-vector and X-axis.

$$\dot{X} = V\cos\theta \tag{A-27}$$

$$\dot{Y} = V\sin\theta \tag{A-28}$$

$$\ddot{X} = V\dot{\theta}\sin\theta + \dot{V}\cos\theta \tag{A-29}$$

$$\ddot{Y} = V\dot{\theta}\cos\theta + \dot{V}\sin\theta \tag{A-30}$$

For constant velocity,

$$\dot{V} = 0.0,\ \dot{\theta} = \frac{V}{R} \tag{A-31}$$

5. COMPUTER SIMULATION MODEL (AVDS3) MODIFICATIONS

The computer simulation model AVDS3 is an extension of the earlier model AVDS II. AVDS II is capable of predicting the dynamic response of single and double tractor-trailer vehicles. AVDS3 has the following additional capabilities and modifications incorporated in it.

* It is capable of simulating the dynamic behavior of a triple tractor-trailer vehicle.

* It can simulate cornering and evasive maneuvers.

* It features the new IITRI tire side force formula based upon tests data for truck tires along with the Ellis formula and the IITRI modified Ellis formula.

* Effects of tire wear, multiple wheels and tandem axles, and wheel lifting are modelled in AVDS3 as explained below.

5.1 Simulation of Tire Wear

The tire-road coefficient of friction is reduced as the tire wears. This, in turn, gives reduced cornering and brake/tractive forces from the tire for given values of normal load and slip angle. This effect of tire wear is modelled in AVDS3 with the provision for altering the tire-road friction value through multiplication with input constant values for each tire. Thus, any combination of tires on the vehicle can be studied.

5.2 Simulation of Multiple Wheels and Tandem Axles

Tractor rear, semitrailers and dollies more commonly have multiple wheels on an axle (generally two wheels on both left and right sides) or tandem axles (with or without multiple wheels on each).

In the mathematical modelling of AVDS3, multiple wheels under a given suspension point are assumed to share equally the suspension load and the brake force proportioned at that location.

Also, the wheels are assumed to have the same slip angle value calculated with the dimensions for the geometric center of all wheels. These values of normal load and slip angle are used in the calculation of side force at one wheel which, multiplied by the number of wheels, gives the total side force at that location. As the tire side force is a non-linear function of both the slip angle and the normal load, this method of calculating the cornering force at a location with multiple wheels gives a larger value than that calculated considering a single wheel in place of multiple wheels.

Tandem axles are considered replaced by a single axle located in between the two axles. The total number of wheels on both left and right sides are not altered. Then, the above procedure is utilized.

In AVDS3, it is possible to simulate either one, two or four wheels (same number on both left and right sides at a given location) at the tractor rear, on semitrailers, and on dollies.

5.3 Simulation of Wheel Lifting

During severe maneuvers, one or more wheels of the articulated vehicle can lift up from the ground. This results in zero normal load on these wheels and loss of both side and brake forces from them.

In AVDS3, roll and pitch motions are simulated using a quasi-static load transfer (forward and lateral accelerations used with suspension properties). The wheel normal loads arising from forward and lateral accelerations are called pitch load and roll load respectively.

In the computational procedure, the static load for all wheels is computed at the beginning of the simulation run. For each successive time step, first the pitch load and then the roll load are calculated. Finally, the total load on each wheel is obtained by summing up the static, pitch and roll loads and is checked for a sign change (negative to positive). If a sign change occurs on any wheel (or wheels), that wheel has lifted up. When all wheels are found in contact with the ground, further calculations are carried on.

At any time step, when wheel lifting occurs, the load transfer values are recalculated. In the process, it is assumed that the pitch load does not lift up the wheel. The available roll transfer load for the lifted wheel is then the negative sum of the static and pitch loads on it. Thus, roll load values for all the lifted wheels are fixed. With these fixed values, roll loads for the remaining wheels are recalculated.

APPENDIX B

MATHEMATICAL FORMULATION

FOR

ARTICULATED VEHICLE STABILITY ANALYSIS

APPENDIX B
LIST OF SYMBOLS

A	Jacobian matrix
ALP	matrix
B	Jacobian matrix
BLP	matrix
b_i	vehicle dimension
CLP	matrix
D_i	tire brake/tractive force
d_i	vehicle dimension
F_i	tire side force
F_{z_i}	tire normal load
F_{hx_i}, F_{hy_i}	fifth wheel forces
IZZi	sprung mass moment of inertia-yaw
MT	mass of tractor
MST1	mass of semitrailer
$\underline{p}$	controls input vector
P_{ij}	matrix element
$\underline{q}$	system state vector
Q_i	state variable
Q_{ij}	matrix element
t	time
V_{x_i}	tractor c.g. forward velocity
V_{y_i}	tractor c.g. lateral velocity
$\underline{x}$	perturbation vector
x_i	perturbation in state variable
ψi	yaw angle
$\dot{\psi} i$	yaw angular velocity
$\ddot{\psi}$	yaw angular acceleration
δ	steering angle
$\Delta\delta$	incremental steering angle
ΔD_i	incremental tire brake/tractive force
α_i	tire slip angle
α_{m_i}	tire maximum slip angle
μ	tire-road coefficient of friction

APPENDIX B

MATHEMATICAL FORMULATION FOR ARTICULATED VEHICLE STABILITY ANALYSIS

I. INTRODUCTION

A tractor single trailer vehicle model is used in this mathematical study on the stability of motion. The nonlinear equations of motion of the articulated vehicle are derived first in body-fixed coordinate systems. The roll, pitch and vertical degrees of motion of both the tractor and the semitrailer are excluded from the problem formulation. As in the handling simulation model AVDS3 (Appendix A), the effects of roll and pitch motions are included later by using semi-static load transfer (forward and lateral accelerations used with suspension properties). For this, the suspension and tire springs, at all locations, are considered replaced by an equivalent spring at the corresponding wheel location. The aerodynamic effects, the fifth wheel friction and damping, and the suspension damping are not considered.

As the roll, pitch, and vertical motions are not considered, a set of four simultaneous equations containing the following four variables are obtained.

- V_{x_1}, V_{y_1} — Tractor forward velocity (along x_1-axis) / Tractor lateral velocity (along y_1-axis)
- $\psi 1$ — Tractor yaw angle
- $\psi 2$ — Trailer yaw angle

The equations of motion are next expressed in state equation form. The normal form of the state equation is

$$\dot{\underline{q}} = f\ (\underline{q}\ \ p,\ t) \tag{B-1}$$

where,

$\underline{q}$ is the system state vector
$\underline{p}$ is the input controls vector
f is the vector function of $\underline{q}$, $\underline{p}$ and time.

In the present system, $\underline{q}$ and $\underline{p}$ are as follows.

$$\underline{q} = (\psi 1\text{-}\psi 2,\ \dot{\psi}1,\ \dot{\psi}1\text{-}\dot{\psi}2, V_{y_1},\ V_{x_1}) \tag{B-2}$$

$$\underline{p} = (\delta,\ D_1,\ D_2,\ D_3,\ D_4,\ D_5,\ D_6) \tag{B-3}$$

Next, the effect of perturbations in both the state variables and the control inputs are introduced in the equations with the substitutions

$$\underline{q}(t) = \underline{q}^*(t) + \underline{x}\ (t)$$

$$\delta(t) = \delta^*(t) + \Delta\delta \tag{B-4}$$

$$D_i(t) = D_i^*(t) + \Delta D_i$$

where the nominal motion values ($q^*(t)$, $\delta^*(t)$, $D_i^*(t)$) are obtained by employing the AVDS3 program.

The resulting equations are then expanded in Taylor series about the nominal motion and terms satisfying the nominal motion equation are dropped. The final set of differential equation for the perturbations in state variables is

$$\dot{\underline{x}} = A(t)\ \underline{x} + B(t) \begin{bmatrix} \Delta\delta \\ \Delta D_i \end{bmatrix} + h\ (\underline{x}, \Delta\delta, \Delta D_i, t) \tag{B-5}$$

where A(t) and B(t) are the Jacobian matrices, and $\underline{h}$ ($\underline{x}$, $\Delta\delta$, ΔD_i, t) contains terms that are higher than the first degree in $\underline{x}$, $\Delta\delta$ and ΔD_i.

The Jacobian matrices A(t) and B(t) are employed in the stability analysis of motion of the articulated vehicle as dictated by the theories presented in Appendix C.

2. THE NONLINEAR EQUATIONS OF MOTION FOR THE TRACTOR SEMITRAILER VEHICLE

2.1 Coordinate Systems

The choise of coordinate systems is of utmost importance in facilitating the writing and the subsequent manipulation of the equations of motion. As the roll, pitch, and vertical motions are not considered, only two dimensional coordinate systems are necessary. Coordinate systems fixed to the centers of mass of both the tractor and the semitrailer are used with a set of reference axes fixed in space to measure the yaw angle.

The reference axis fixed in space are X_i Y_i (i=1, tractor; i=2, semitrailer) shown in Figure B-1. The origins coincide with the centers of gravity when the tractor and the semitrailer are directly in line. The X_i axes point directly forward and the Y_i axes point to the right (from the driver's point of view) of the vehicle.

The coordinate systems X_i' Y_i' and $x_i y_i$, are fixed to the centers of mass (Figure B-2). X_i' Y_i' translate with the centers of mass and remain parrallel to X_i Y_i. $x_i y_i$ translate and yaw with the system. $\psi 1$ and $\psi 2$ are the yaw angles. Transformation of vectors between the system coordinates is given by the following relationships.

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} \cos\psi i & \sin\psi i \\ -\sin\psi i & \cos\psi i \end{bmatrix} \begin{bmatrix} X_i' \\ Y_i' \end{bmatrix} \qquad \text{(B-6)}$$

$$\begin{bmatrix} X_i \\ Y_i \end{bmatrix} = \begin{bmatrix} \cos\psi i & -\sin\psi i \\ \sin\psi i & \cos\psi i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} \qquad \text{(b-7)}$$

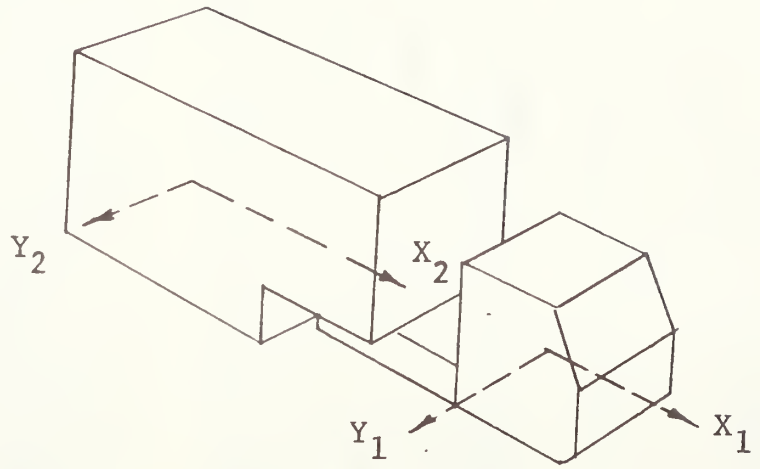


Figure B-1. Coordinate systems fixed in space.

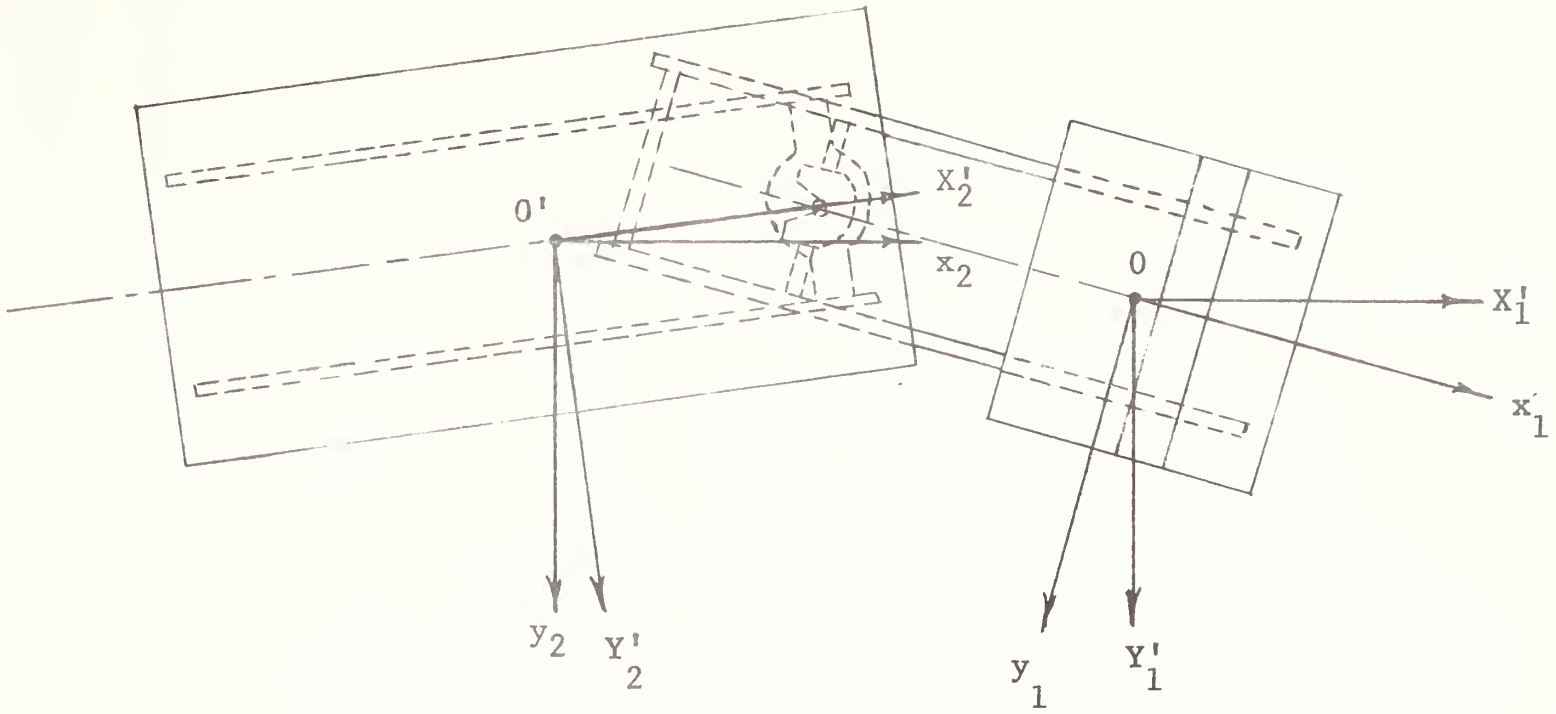


Figure B-2. Coordinate systems fixed at the c.g. of sprung masses.

2.2 The Equations of Motion in General Form

The equations of motion for both the tractor and the semitrailer are derived separately in terms of the tire side and brake/tractive forces, and the fifth wheel forces. The resulting sets of equations are, of course, not independent, and the fifth wheel constraint equations are established next. Finally, the equations of motion and the constraint equations are combined and the unknown fifth wheel forces are eliminated. This results in a set of four second-order differential equations in the four independent coordinates: forward and lateral translation of the tractor, yaw of the tractor, and yaw of the semitrailer.

The tire side and brake/tractive forces which act on both the tractor and the semitrailer are shown in Figures B-3 and B-4 respectively. The steering angle is denoted by δ. Overall dimensions of both the units are shown in Figures B-5 and B-6.

The tire side (F_i) and brake/tractive (D_i) forces are denoted by the following subscripts:

1 tractor, left front
2 tractor, right front
3 tractor, left rear
4 tractor, right rear
5 semitrailer, left rear
6 semitrailer, right rear

The following symbols are used for the fifth wheel forces.

F_{hx_1}, F_{hy_1} - fifth wheel forces on the tractor

F_{hx_2}, F_{hy_2} - fifth wheel forces on the semitrailer

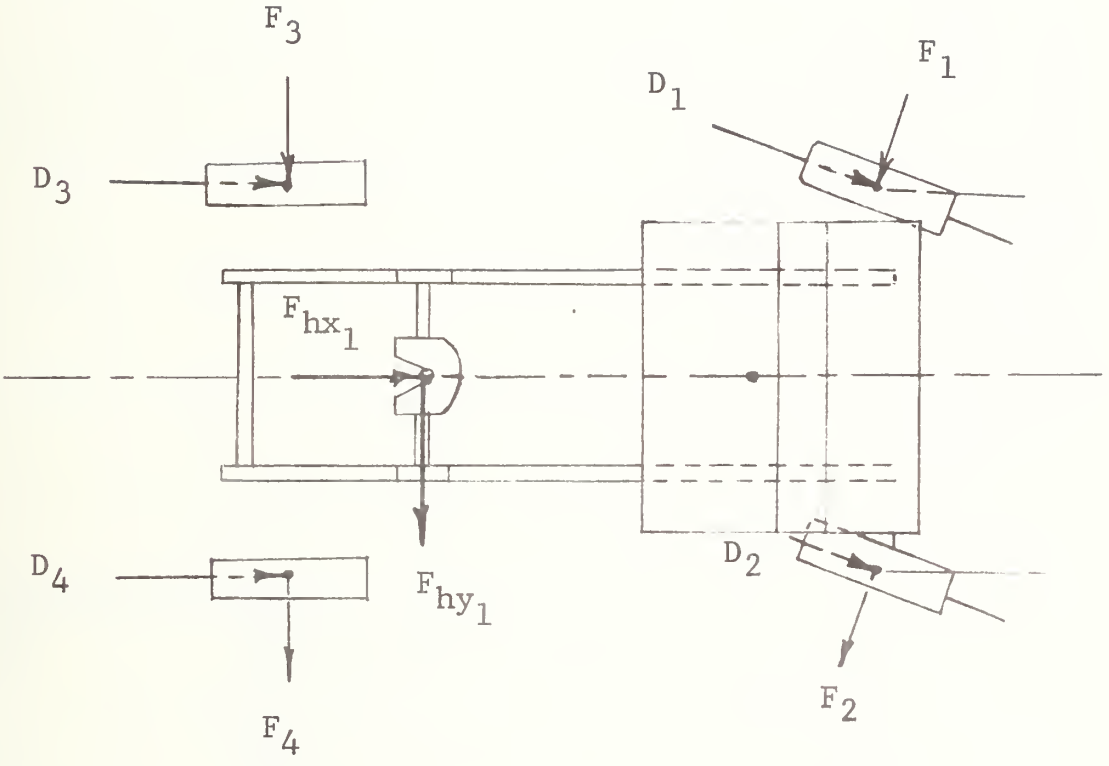


Figure B-3. Tire and fifth wheel forces on tractor.

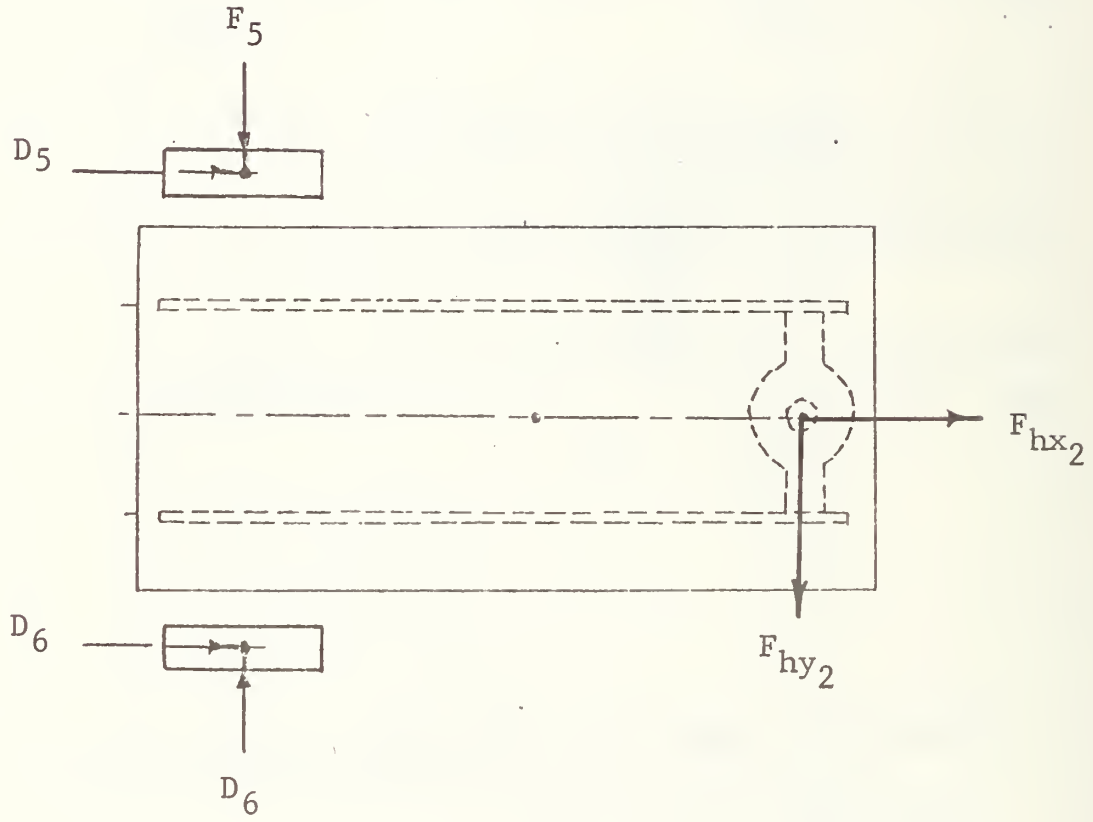


Figure B-4. Tire forces in semitrailer.

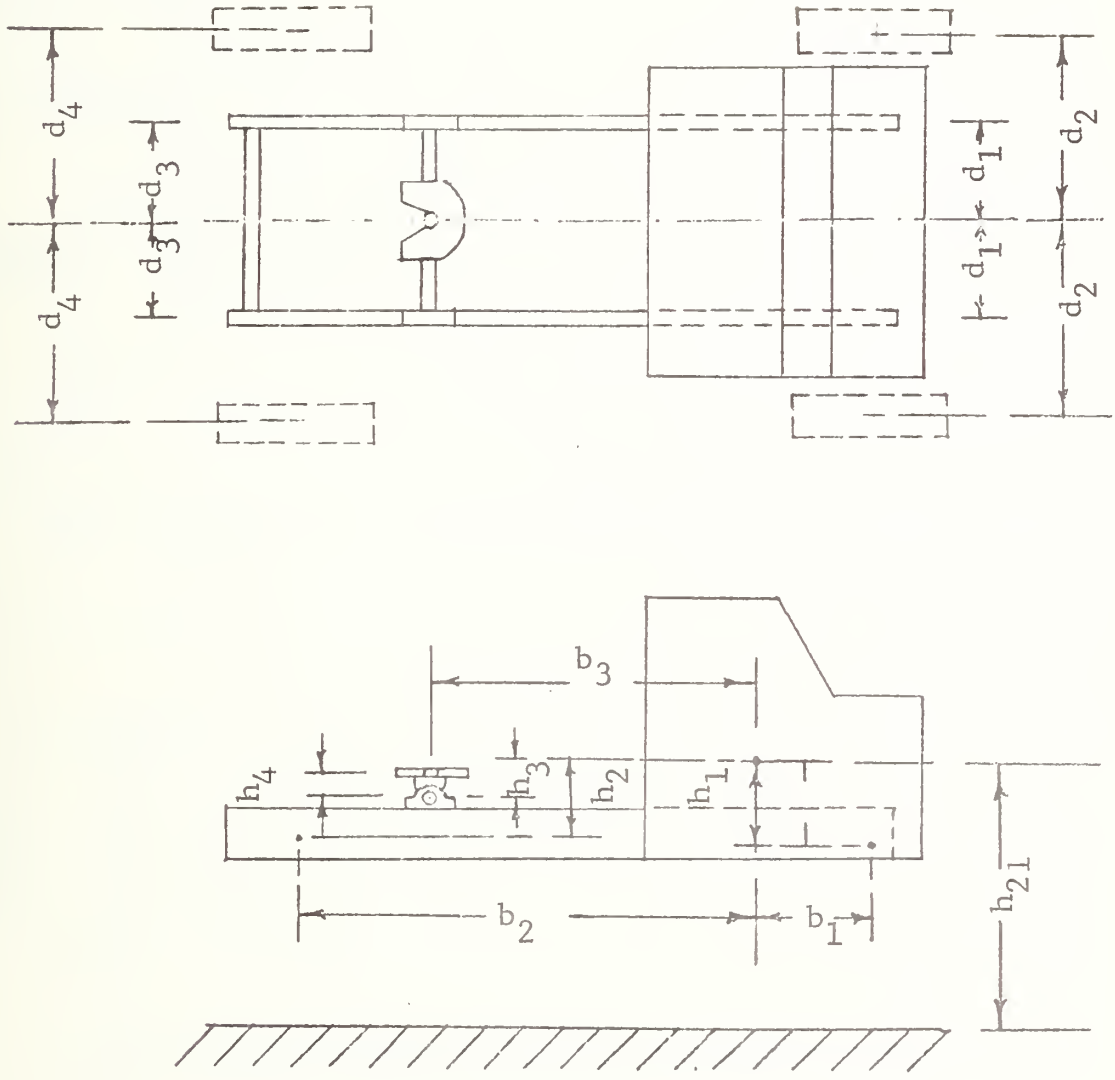


Figure B-5. Tractor dimensions.

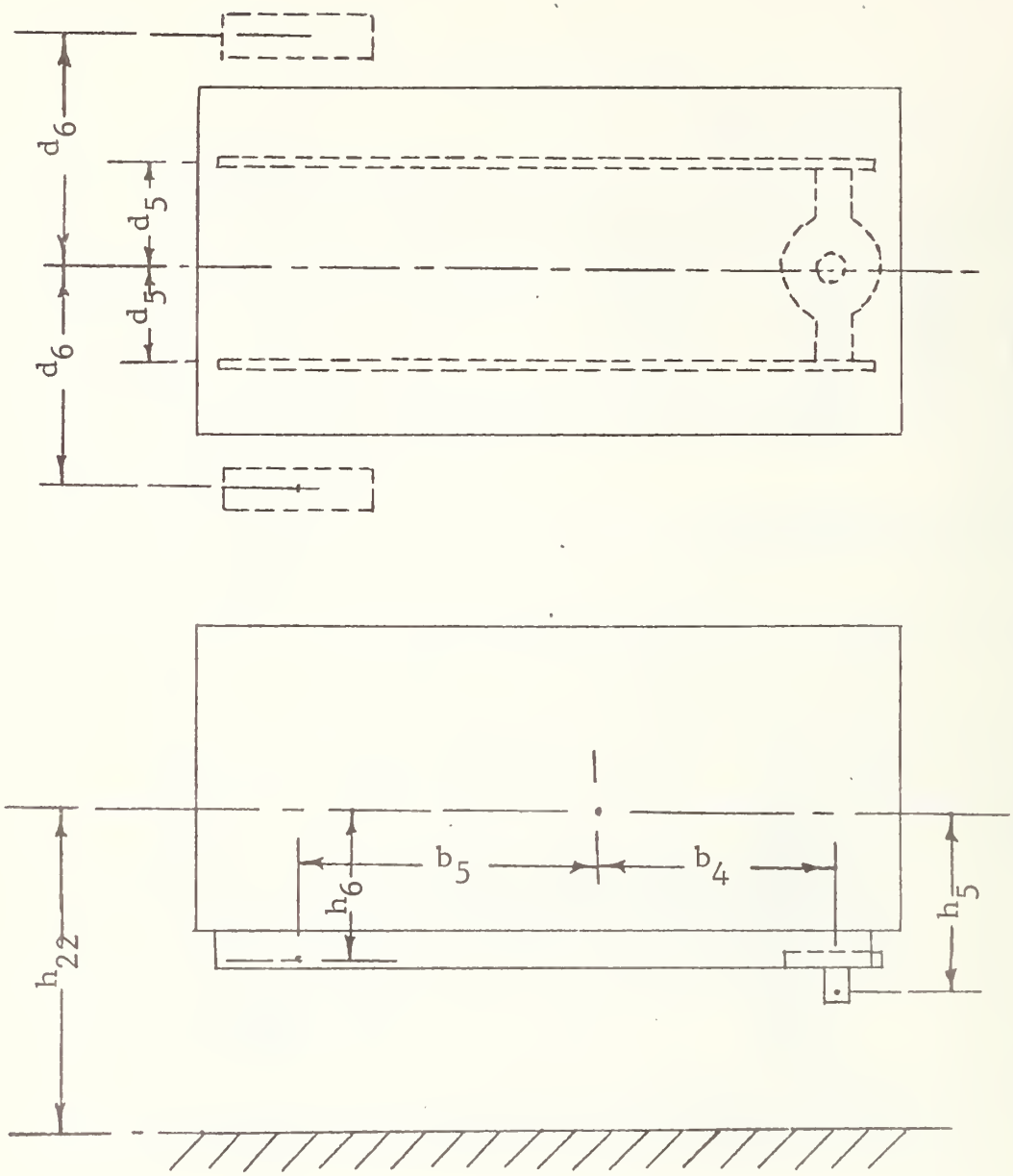


Figure B-6. Semitrailer dimensions.

The equations of motion for both the tractor and the semitrailer are written by applying Newton's second law. A force balance at the fifth wheel and the condition that the position of the hitch on the tractor must be coincident with the position of the hitch on the semitrailer lead to the constraint equations.

2.2.1 Tractor

Equating the rate of change of lineal momentum to the external forces yields:

$$MT \begin{bmatrix} \frac{d}{dt}(V_{x_1}) \\ \frac{d}{dt}(V_{y_1}) \end{bmatrix} = \begin{bmatrix} -(F_1+F_2)\sin\delta + (D_1+D_2)\cos\delta + (D_3+D_4) \\ (F_1+F_2)\cos\delta + (D_1+D_2)\sin\delta + (F_3+F_4) \end{bmatrix} + \begin{bmatrix} F_{hx_1} \\ F_{hy_1} \end{bmatrix} \quad \text{(B-8)}$$

where

MT is the mass of the tractor

V_{x_1} is the tractor velocity in x_1 direction

V_{y_1} is the tractor velocity in y_1 direction

Equating the time rate of change of the angular momentum of the tractor mass about its center of mass to the external moments gives,

$$IZZI \frac{d}{dt}(\dot{\psi}1) = \left\{(F_1+F_2)\cos\delta + (D_1+D_2)\sin\delta\right\} b_1 - \left\{F_3+F_4\right\} b_2$$

$$+ \left\{(D_1-D_2)\cos\delta - (F_1-F_2)\sin\delta\right\} d_2 + \left\{D_3-D_4\right\} d_4$$

$$- F_{hy_1} b_3 \qquad \text{(B-9)}$$

where IZZI is the tractor moment of inertia about its center of mass and $\dot{\psi}1$ is the tractor angular velocity.

2.2.2 Semitrailer

Equating the rate of change of lineal momentum to the external forces gives,

$$MST1 \begin{bmatrix} \frac{d}{dt}(V_{x_2}) \\ \\ \frac{d}{dt}(V_{y_2}) \end{bmatrix} = \begin{bmatrix} D_5 + D_6 \\ \\ F5 + F6 \end{bmatrix} + \begin{bmatrix} F_{hx_2} \\ \\ F_{hy_2} \end{bmatrix} \qquad \text{(B-10)}$$

where

MST1 is the mass of the semitrailer

V_{x_2} is the semitrailer velocity in x_2 direction

V_{y_2} is the semitrailer velocity in y_2 direction

Next, equating the time rate of change of the angular momentum of the semitrailer mass about its center of mass to the external moments yields,

$$IZZ2 \frac{d}{dt} (\dot{\psi}2) = -(F_5+F_6)b_5+(D5-D6)\ d_6 + F_{hy_2}\ b_4 \qquad \text{(B-11)}$$

where IZZ2 is the semitrailer moment of inertia about its center of mass, and $\dot{\psi}2$ is the semitrailer angular velocity.

2.3 Constraint Equations

The equations of motion which were written for the tractor and semitrailer in the preceding section are not independent of each other. They are related to each other by the constraints imposed by the hitch.

2.3.1 Position

The position of the hitch on the tractor must be coincident with the position of the hitch on the semitrailer. The first (velocity) and second (acceleration) derivatives with respect to time of the position constraint yield the following relationships.

The velocity constraint is,

$$\begin{bmatrix} V_{x_2} \\ V_{y_2}+b_4\{\dot{\psi}2\} \end{bmatrix} = \begin{bmatrix} \cos(\psi1-\psi2) & -\sin(\psi1-\psi2) \\ \sin(\psi1-\psi2) & \cos(\psi1-\psi2) \end{bmatrix} \begin{bmatrix} V_{x_1} \\ V_{y_1}-b_3\{\dot{\psi}1\} \end{bmatrix} \qquad \text{(B-12)}$$

The acceleration constraint is,

$$\begin{bmatrix} \frac{d}{dt}(V_{x_2}) - b_4\{\dot{\psi}2\}^2 \\ \\ \frac{d}{dt}(V_{y_2}) + b_4\{\ddot{\psi}2\} \end{bmatrix} = \begin{bmatrix} \cos\{\psi 1-\psi 2\} & -\sin\{\psi 1-\psi 2\} \\ \\ \sin\{\psi 1-\psi 2\} & \cos\{\psi 1-\psi 2\} \end{bmatrix} \begin{bmatrix} \frac{d}{dt}(V_{x_1}) + b_3\{\dot{\psi}1\}^2 \\ \\ \frac{d}{dt}(V_{y_1}) + b_3\{\ddot{\psi}1\} \end{bmatrix} \quad \text{(B-13)}$$

where $\{\psi 1 - \psi 2\}$ is the tractor semitrailer relative yaw angle.

2.3.2 Fifth Wheel Forces

A force balance leads to the following constraint equations.

$$\begin{bmatrix} F_{hx_2} \\ \\ F_{hy_2} \end{bmatrix} = \begin{bmatrix} \cos\{\psi 1-\psi 2\} & -\sin\{\psi 1-\psi 2\} \\ \\ \sin\{\psi 1-\psi 2\} & \cos\{\psi 1-\psi 2\} \end{bmatrix} \begin{bmatrix} F_{hx_1} \\ \\ F_{hy_1} \end{bmatrix} \quad \text{(B-14)}$$

2.4 Manipulation of the Equations

The equations of motion and the constraint equations are combined and the unknown fifth wheel forces are eliminated. This results in a set of four second-order differential equations which follows.

$$[MT + MST1][\frac{d}{dt}\{V_{x_1}\}] + MST1\left[b_3\{\dot{\psi}1\}^2 + b_4\{\dot{\psi}2\}^2\cos(\psi 1-\psi 2) - b_4\{\ddot{\psi}2\}\sin\{\psi 1-\psi 2\}\right]$$

$$= -[F_1 + F_2]\sin\delta + [D_1 + D_2]\cos\delta + [D_3 + D_4] + [D_5 + D_6]\cos\{\psi 1-\psi 2\} + [F_5 + F_6]\sin\{\psi 1-\psi 2\} \quad \text{(B-15)}$$

$$[MT + MST1][\frac{d}{dt}\{V_{y_1}\}] + MST1[-b_3\{\ddot{\psi}1\} - b_4\{\dot{\psi}2\}^2\sin\{\psi 1-\psi 2\} - b_4\{\ddot{\psi}2\}\cos\{\psi 1-\psi 2\}]$$

$$= [F_1 + F_2]\cos\delta + [D_1 + D_2]\sin\delta + [F_3 + F_4] - [D_5 + D_6]\sin\{\psi 1-\psi 2\} + [F_5 + F_6]\cos\{\psi 1-\psi 2\} \quad \text{(B-16)}$$

$$IZZ1\{\ddot{\psi}1\} + MT\, b_3[\frac{d}{dt}\{V_{y_1}\}] = -[F_1-F_2]\, d_2\sin\delta + [D_1-D_2]d_2\cos\delta + [F_1 + F_2]\, b_1\cos\delta + [D_1 + D_2]\, b_1\sin\delta + [D_3-D_4]d_4 - [F_3 + F_4]\, b_2 + [F_1 + F_2]\, b_3\cos\delta + [D_1 + D_2]b_3\sin\delta + [F_3 + F_4]\, b_3 \quad \text{(B-17)}$$

$$IZZ2\{\ddot{\psi}2\} + MT\, b_4\left[\{\frac{d}{dt}(V_{x_1})\}\sin\{\psi 1-\psi 2\} + \{\frac{d}{dt}(V_{y_1})\}\cos\{\psi 1-\psi 2\}\right]$$

$$= [D_5-D_6]d_6 - [F_7 + F_8]\, b_5 + \left[-[F_1 + F_2]\, b_4\sin\delta + [D_1 + D_2]\, b_4\cos\delta + [D_3 + D_4]b_4\right]\sin\{\psi 1-\psi 2\} + \left[(F_1 + F_2)\, b_4\cos\delta + [D_1 + D_2]\, b_4\sin\delta + [F_3 + F_4]\, b_4\right]\cos\{\psi 1-\psi 2\} \quad \text{(B-18)}$$

where,

$$\frac{d}{dt}\{V_{x_1}\} = \dot{V}_{x_1} - V_{y_1}\{\dot{\psi}1\} \tag{B-19}$$

$$\frac{d}{dt}\{V_{y_1}\} = \dot{V}_{y_1} + V_{x_1}\{\dot{\psi}1\} \tag{B-20}$$

2.5 Tire Side Forces (F_i)

The tire side force (F_i) is a nonlinear function of the normal load, the operating slip angle and the tire-road coefficient of friction. The IITRI modified Ellis formula is used here. The equation is

$$F_i = \frac{3\mu F_{z_i}}{\alpha_{m_i}}\left[\alpha_i - \frac{\alpha_i^3}{3\alpha_{m_i}^2}\right] \tag{B-21}$$

where,

F_{z_i} is the normal tire load

α_i is the tire slip angle

α_{m_i} is the maximum slip angle and is a function of the normal load (dropping from the maximum value of 20 deg at 6000.0 lb load to 0 deg at zero load)

μ is the tire-road coefficient of friction

2.6 Tire Normal Loads (F_{z_i})

The normal load on each wheel is made up of the static load from the vehicle weight and the dynamic load from the effects of roll and pitch motions. The static load is a function of the vehicle weight and geometry. A quasi-static load transfer (using forward and lateral acceleration values with suspension properties) is used to calculate the dynamic load on each wheel. The various mathematical relationships were developed for the handling simulation model AVDSII and are not presented herein.

2.7 Tire Slip Angles (α_i)

The slip angle of a tire is the angle between the vertical longitudinal plane of the wheel and the direction of motion of the wheel (Figure B-7). In executing a turn, the wheel velocities and slip angles are as shown in Figure B-8. Approximating the tangent of the slip angle by the angle itself, the following equations are obtained.

$$\alpha_1 = \frac{V_{y_1} + b_1 \{\dot{\psi}1\}}{V_{x_1} + d_2 \{\dot{\psi}1\}} - \delta \qquad \text{(B-22)}$$

$$\alpha_2 = \frac{V_{y_1} + b_1 \{\dot{\psi}1\}}{V_{x_1} - d_2 \{\dot{\psi}1\}} - \delta \qquad \text{(B-23)}$$

$$\alpha_3 = \frac{V_{y_1} - b_2 \{\dot{\psi}1\}}{V_{x_1} + d_4 \{\dot{\psi}1\}} \qquad \text{(B-24)}$$

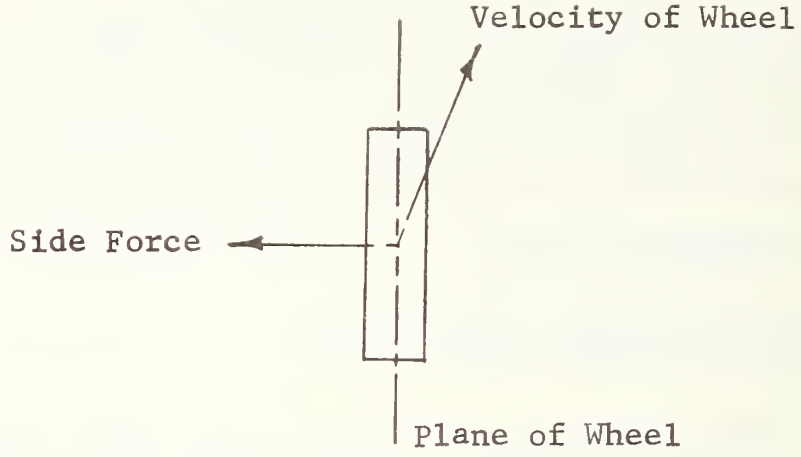


Figure B-7. Tire slip angle.

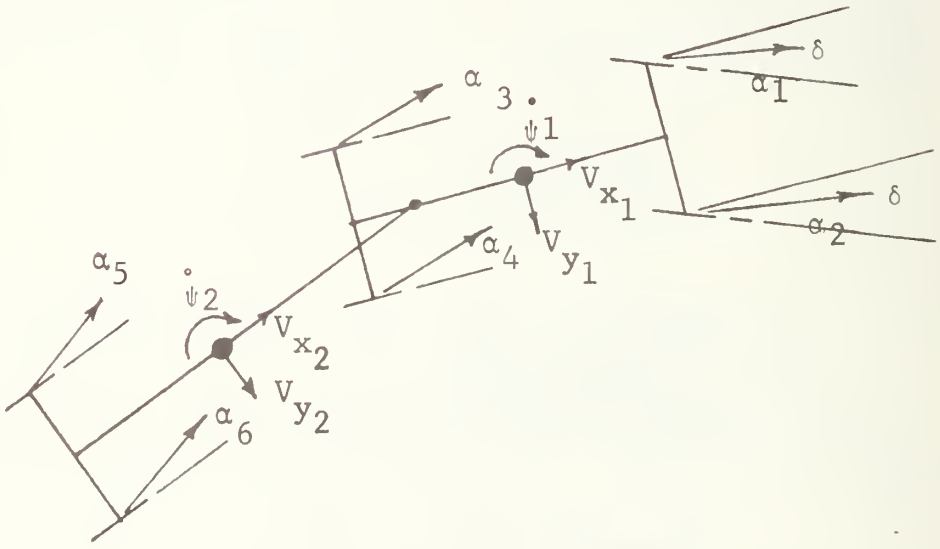


Figure B-8. Tractor and semitrailer slip angles during turning.

$$\alpha_4 = \frac{V_{y_1} - b_2 \{\dot{\psi}1\}}{V_{x_1} - d_4 \{\dot{\psi}1\}} \tag{B-25}$$

$$\alpha_5 = \frac{V_{y_2} - b_5 \{\dot{\psi}2\}}{V_{x_2} + d_6 \{\dot{\psi}2\}} \tag{B-26}$$

$$\alpha_6 = \frac{V_{y_2} - b_5 \{\dot{\psi}2\}}{V_{x_2} - d_6 \{\dot{\psi}2\}} \tag{B-27}$$

Substituting for V_{x_2} and V_{y_2} from (B-12),

$$\alpha_5 \quad \frac{V_{x_1} \sin\{\psi 1-\psi 2\} + \{V_{y_1}-b_3\{\dot{\psi}1\}\}\cos\{\psi 1-\psi 2\} - \{b_4+b_5\}\{\dot{\psi}2\}}{V_{x_1} \cos\{\psi 1-\psi 2\} - \{V_{y_1}-b_3\{\dot{\psi}1\}\}\sin\{\psi 1-\psi 2 + d_6\{\dot{\psi}2\}}$$

$$\alpha_6 = \frac{V_{x_1} \sin\{\psi 1-\psi 2\}+\{V_{y_1}-b_3\{\dot{\psi}1\} \cos\{\psi 1-\psi 2\} - \{b_4+b_5\}\{\dot{\psi}2\}}{V_{x_1} \cos\{\psi 1-\psi 2\}-\{V_{y_1}-b_3\{\dot{\psi}1\}\}\sin\{\psi 1-\psi 2\} - d_6\{\dot{\psi}2\}} \tag{B-29}$$

As

$$V_{x_1} >>>> d_2 \{\dot{\psi}1\}, \text{ and}$$

$$V_{x_1} \cos\{\psi 1-\psi 2\} >>>> \{V_{y_1}-b_3\{\dot{\psi}1\}\} \sin\{\psi 1-\psi 2\}+d_6\{\dot{\psi}2\},$$

the following approximate equations are used.

$$\alpha_1 = \alpha_2 = \frac{V_{y_1} + b_1 \{\dot{\psi}1\}}{V_{x_1}} \tag{B-30}$$

$$\alpha_3 = \alpha_4 = \frac{V_{y_1} - b_2 \{\dot{\psi}1\}}{V_{x_1}} \tag{B-31}$$

$$\alpha_5 = \alpha_6 = \frac{V_{x_1}\sin\{\psi 1-\psi 2\}+\{V_{y_1}-b_3\{\dot{\psi}1\}\cos\{\psi 1-\psi 2\}-\{b_4+b_5\}\{\dot{\psi}2\}}{V_{x_1}\cos\{\psi 1-\psi 2\}} \tag{B-32}$$

3. THE EQUATIONS OF MOTION IN STATE FORM

The four differential equations of motion (Equations (B-15) through (B-18)) derived in the preceding section are re-written in state equation form. The state vector is chosen to be

$$\underline{q} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} = \begin{bmatrix} \psi 1-\psi 2 \\ \dot{\psi}1 \\ \dot{\psi}1-\dot{\psi}2 \\ V_{y_1} \\ V_{x_1} \end{bmatrix} \tag{B-33}$$

To write the state equation, it is required that the first derivatives of the state variables be written in terms of the state variables themselves, of the input variables, and of time.

The first derivative of the state variable Q_1 is easily seen from (B-33) as,

$$\dot{Q}_1 = Q_3 \tag{B-34}$$

The four independent equations of motion (Equations (B-15)through(B-18)) are used to complete the writing of the state. equation. In the process, the following substitutions are made into the equations.

$$
\begin{aligned}
\psi 1 - \psi 2 &= Q_1 \\
\dot{\psi} 1 &= Q_2 \\
\dot{\psi} 2 &= Q_2 - Q_3 \\
V_{y_1} &= Q_4 \\
V_{x_1} &= Q_5 \\
\ddot{\psi} 1 &= \dot{Q}_2 \\
\ddot{\psi} 2 &= \dot{Q}_2 - \dot{Q}_3 \\
\dot{V}_{y_1} &= \dot{Q}_4 \\
\dot{V}_{x_1} &= \dot{Q}_5
\end{aligned}
\qquad \text{(B-35)}
$$

The resulting equations of motion in state variables are as follows. Equation (B-15) becomes,

$$
\begin{aligned}
& [MT+MST1]\,[\dot{Q}_5 - Q_4 Q_2] \\
+\ & MST1\,[b_3\,Q_2^2 + b_4\,\{Q_2-Q_3\}^2 \cos Q_1 - b_4\{\dot{Q}_2-\dot{Q}_3\}\sin Q_1] \\
=\ & [D_1+D_2]\cos\delta + [D_3+D_4]+[D_5+D_6]\cos Q_1 \\
-\ & [F_1+F_2]\sin\delta + [F_5+F_6]\sin Q_1
\end{aligned}
\qquad \text{(B-36)}
$$

Equation (B-16) becomes,

$$[MT+MST1]\,[\dot{Q}_4+Q_5\,Q_2]$$

$$+ MST1\,[-b_3\,\dot{Q}_2 - b_4\,\{Q_2-Q_3\}^2 \sin Q_1 - b_4\{\dot{Q}_2-\dot{Q}_3\}\cos Q_1]$$

$$= [D_1+D_2]\,\sin\delta - [D_5+D_6]\sin Q_1$$

$$+ [F_1+F_2]\,\cos\delta + [F_3+F_4] + [F_5+F_6]\,\cos Q_1 \qquad \text{(B-37)}$$

Equation (B-17) becomes,

$$IZZ1\,Q_2 + MT\,b_3\,[Q_4 + Q_5\,Q_2]$$

$$= d_2\,[D_1-D_2]\,\cos\delta + [b_1+b_3][D_1+D_2]\sin\delta + d_4\,[D_3-D_4]$$

$$- d_2\,[F_1-F_2]\,\sin\delta + [b_1+b_3]\,[F_1+F_2]\,\cos\delta$$

$$- [b_2-b_3]\,[F_3+F_4] \qquad \text{(B-38)}$$

Equation (B-18) becomes,

$$IZZ2\,[\ddot{Q}_2-\ddot{Q}_3] + MT\,b_4\,[\{\ddot{Q}_5-Q_4Q_2\}\sin Q_1 + \{\ddot{Q}_4+Q_5Q_2\}\cos Q_1]$$

$$= d_6\,[D_5-D_6] + b_4\,[\sin\delta\cos Q_1 + \cos\delta\sin Q_1][D_1+D_2]$$

$$+ b_4\,[D_3+D_4]\sin Q_1 - b_5\,[F_5+F_6]$$

$$+ b_4\,[\cos\delta\cos Q_1 - \sin\delta\sin Q_1]\,[F_1+F_2]$$

$$+ b_4\,[F_3+F_4]\cos Q_1 \qquad \text{(B-39)}$$

Combining Equations(B-21),(B-30),(B-31),(B-32) and(B-34), the following relationships are obtained.

$$[F_1+F_2] = \frac{3}{2}\mu\left\{\left[\frac{F_{z_1}}{\alpha_{m_1}} + \frac{F_{z_2}}{\alpha_{m_2}}\right]\alpha_1 - \left[\frac{F_{z_1}}{3\alpha_{m_1}^3} + \frac{F_{z_2}}{3\alpha_{m_2}^3}\right]\alpha_1^3\right\} \qquad \text{(B-40)}$$

$$[F_1-F_2] = \frac{3}{2}\mu\left\{\left[\frac{F_{z_1}}{\alpha_{m_1}} - \frac{F_{z_2}}{\alpha_{m_2}}\right]\alpha_1 - \left[\frac{F_{z_1}}{3\alpha_{m_1}^3} - \frac{F_{z_2}}{3\alpha_{m_2}^3}\right]\alpha_1^3\right\} \qquad \text{(B-41)}$$

$$[F_3+F_4] = \frac{3}{2}\mu\left\{\left[\frac{F_{z_3}}{\alpha_{m_3}} + \frac{F_{z_4}}{\alpha_{m_4}}\right]\alpha_3 - \left[\frac{F_{z_3}}{3\alpha_{m_3}^3} + \frac{F_{z_4}}{3\alpha_{m_4}^3}\right]\alpha_3^3\right\} \qquad \text{(B-42)}$$

$$[F_5+F_6] = \frac{3}{2}\mu\left\{\left[\frac{F_{z_5}}{\alpha_{m_5}} + \frac{F_{z_6}}{\alpha_{m_6}}\right]\alpha_5 - \left[\frac{F_{z_5}}{3\alpha_{m_5}^3} + \frac{F_{z_6}}{3\alpha_{m_6}^3}\right]\alpha_5^3\right\} \qquad \text{(B-43)}$$

where,

$$\alpha_1 = \alpha_2 = \frac{Q_4 + b_1 Q_2}{Q_5} - \delta \qquad \text{(B-44)}$$

$$\alpha_3 = \alpha_4 = \frac{Q_4 - b_2 Q_2}{Q_5} \qquad \text{(B-45)}$$

$$\alpha_5 = \alpha_6 = \frac{Q_5 \sin Q_1 + \{Q_4 - b_3 Q_2\} \cos Q_1 - \{b_4 + b_5\} \{Q_2 - Q_3\}}{Q_5 \cos Q_1} \qquad \text{(B-46)}$$

Equations (B-34), (B-36) through (B-46) together make up the overall state equation which may be written as,

$$\begin{bmatrix} Q_{11} & Q_{12} & - & - & Q_{15} \\ Q_{21} & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ Q_{51} & - & - & - & Q_{55} \end{bmatrix} \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \\ \dot{Q}_3 \\ \dot{Q}_4 \\ \dot{Q}_5 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & - & - & P_{15} \\ P_{21} & - & - & - & - \\ + & - & - & - & - \\ - & - & - & - & - \\ P_{51} & - & - & - & P_{55} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \end{bmatrix} \qquad \text{(B-47)}$$

4. LINEARIZED EQUATIONS OF MOTION FOR THE PERTURBATIONS IN BOTH THE STATE VARIABLES AND THE CONTROL INPUTS

In the preceding section, the equations of motion were presented in state equation form. With the goal of putting the equations in the form of Equation (B-5), perturbations on the initial system state and on the steering and braking inputs are introduced with the following substitutions in Equations (B-47).

$$
\begin{aligned}
Q_1 &= Q_1^* + x_1 \\
Q_2 &= Q_2^* + x_2 \\
Q_3 &= Q_3^* + x_3 \\
Q_4 &= Q_4^* + x_4 \\
Q_5 &= Q_5^* + x_5 \\
\delta &= \delta^* + \Delta\delta \\
D_i &= D_i^* + \Delta D_i
\end{aligned}
\qquad \text{(B-48)}
$$

where the nominal motion $\underline{q}^*$ (Q_i^*, δ^*, D_i^*) is obtained by employing the AVDS3 program. In the process, perturbations in the tire normal loads due to perturbed state variables and control inputs are ignored.

The resulting equations are then expanded in Taylor series about the nominal motion q^*, and terms satisfying the nominal motion equation $[\dot{q}^* = f(q^*, \delta^*, D_i^*)]$ and also higher than the first degree terms in $\underline{x}_i$, $\Delta\delta$ and ΔD_i are dropped. This gives the final set of linearized equations of motion for the perturbations in both the state variables and the control inputs which may be written as

$$
\underset{5\times5}{\begin{bmatrix} \\ \text{ALP} \\ \\ \end{bmatrix}}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix}
=
\underset{5\times5}{\begin{bmatrix} \\ \text{BLP} \\ \\ \end{bmatrix}}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}
$$

$$+ \quad \underset{5 \times 7}{\begin{bmatrix} & \text{CLP} & \end{bmatrix}} \begin{bmatrix} \Delta\delta \\ \Delta D_1 \\ \Delta D_2 \\ \Delta D_3 \\ \Delta D_4 \\ \Delta D_5 \\ \Delta D_6 \end{bmatrix} \tag{B-49}$$

where

$$\text{ALP}(1,1) = 1.0 \tag{B-50}$$
$$\text{ALP}(1,2) = 0.0 \tag{B-51}$$
$$\text{ALP}(1,3) = 0.0 \tag{B-52}$$
$$\text{ALP}(1,4) = 0.0 \tag{B-53}$$
$$\text{ALP}(1,5) = 0.0 \tag{B-54}$$
$$\text{ALP}(2,1) = 0.0 \tag{B-55}$$
$$\text{ALP}(2,2) = \text{IZZ1} \tag{B-56}$$
$$\text{ALP}(2,3) = 0.0 \tag{B-57}$$
$$\text{ALP}(2,4) = \text{MT } b_3 \tag{B-58}$$
$$\text{ALP}(2,5) = 0.0 \tag{B-59}$$
$$\text{ALP}(3,1) = 0.0 \tag{B-60}$$
$$\text{ALP}(3,2) = \text{IZZ2} \tag{B-61}$$
$$\text{ALP}(3,3) = -\text{IZZ2} \tag{B-62}$$
$$\text{ALP}(3,4) = \text{MT } b_4 \cos Q_1 \tag{B-63}$$
$$\text{ALP}(3,5) = \text{MT } b_4 \sin Q_1 \tag{B-64}$$
$$\text{ALP}(4,1) = 0.0 \tag{B-65}$$
$$\text{ALP}(4,2) = -b_3 \text{ MST1} - b_4 \text{ MST1} \cos Q_1 \tag{B-66}$$
$$\text{ALP}(4,3) = b_4 \text{ MST1} \cos Q_1 \tag{B-67}$$
$$\text{ALP}(4,4) = \text{MT} + \text{MST1} \tag{B-68}$$
$$\text{ALP}(4,5) = 0.0 \tag{B-69}$$
$$\text{ALP}(5,1) = 0.0 \tag{B-70}$$

$$ALP(5,2) = -b_4\ MST1\ \sin Q_1 \tag{B-71}$$

$$ALP(5,3) = b_4\ MST1\ \sin Q_1 \tag{B-72}$$

$$ALP(5,4) = 0.0 \tag{B-73}$$

$$ALP(5,5) = MT + MST1 \tag{B-74}$$

$$BLP(1,1) = 0.0 \tag{B-75}$$

$$BLP(1,2) = 0.0 \tag{B-76}$$

$$BLP(1,3) = 1.0 \tag{B-77}$$

$$BLP(1,4) = 0.0 \tag{B-78}$$

$$BLP(1,5) = 0.0 \tag{B-79}$$

$$BLP(2,1) = 0.0 \tag{B-80}$$

$$BLP(2,2) = -MT\ b_3\ Q_5 - d_2 \sin\delta\ [\{C1\}-3\{C2\}\alpha_1^2]\frac{b_1}{Q_5}$$

$$+ [b_1+b_3] \cos\delta\ [\{C3\}-3\{C4\}\alpha_1^2]\ \frac{b_1}{Q_5}$$

$$+ [b_2-b_3] \cos\delta\ [\{C5\}-3\ \{C6\}\ \alpha_3^2\ ]\ \frac{b_2}{Q_5} \tag{B-81}$$

$$BLP(2,3) = 0.0 \tag{B-82}$$

$$BLP(2,4) = -d_2 \sin\delta\ [\{C1\}-3\{C2\}\alpha_1^2\ ]\frac{1}{Q_5}$$

$$+ [b_1+b_3] \cos\delta\ [\{C3\}-\ 3\ \{C4\}\ \alpha_1^2]\ \frac{1}{Q_5}$$

$$- [b_2-b_3]\ [\{C5\}-3\{C6\}\alpha_3^2]\frac{1}{Q_5} \tag{B-83}$$

$$BLP(2,5) = -MT\ b_3\ Q_2$$

$$+ d_2 \sin\delta \left[\{C1\} - 3\{C2\}\ \alpha_1^2\right] \left[\frac{Q_4 + b_1\ Q_2}{Q_5^2}\right]$$

$$- [b_1+b_3] \cos\delta \; [\{C3\}-3\{C4\}\alpha_1^2]\left[\frac{Q_4 + b_1 Q_2^2}{Q_5}\right]$$

$$+ [b_2-b_3] \; [\{C5\}-3\{C6\}\alpha_3^2]\left[\frac{Q_4-b_2 Q_2^2}{Q_5}\right] \qquad \text{(B-84)}$$

$$BLP\ (3,1) = -MT\ b_4\ \dot{Q}_5 \cos Q_1 + MT\ b_4\ Q_4\ Q_2 \cos Q_1$$

$$+ MT\ b_4\ \dot{Q}_4\ \sin Q_1 + MT\ b_4\ Q_5\ Q_2\ \sin Q_1$$

$$- b_4\ \sin\delta\ [D_1+D_2]\sin Q_1 + b_4\ \cos\delta\ [D_1+D_2]\cos Q_1$$

$$+ b_4\ [D_3+D_4]\ \cos Q_1$$

$$-b_5[\{C7\}-3\{C8\}\alpha_5^2]\left[\sec^2 Q_1-[b_4+b_5]\left[\frac{Q_2-Q_3}{Q_5}\right]\sec Q_1 \tan Q_1\right]$$

$$-b_4[\cos\delta\ \sin Q_1 + \sin\delta \cos Q_1]\ [F_1+F_2]$$

$$-b_4\ \sin Q_1\ [F_3+ F_4] \qquad \text{(B-85)}$$

$$BLP\ (3,2) = MT\ b_4\ Q_4\ \sin Q_1 - MT\ b_4\ Q_5\ \cos Q_1$$

$$+ b_5\ [\{C7\}-\ 3\ \{C8\}\ \alpha_5^2]\left[\frac{b_3}{Q_5} + \frac{b_4+ b_5}{Q_5}\ \sec Q_1\right]$$

$$+ b_4\ [\cos\delta\cos Q_1-\sin\delta\ \sin Q_1]\ \frac{b_1}{Q_5}\ [\{C3\}-3\{C4\}\alpha_1^2]$$

$$- b_4\ [\{C5\}-\ 3\ \{C6\}\ \alpha_3^2\ ]\ \frac{b_2}{Q_5}\ \cos Q_1 \qquad \text{(B-86)}$$

$$BLP\ (3,3) = -\ b_5\ [b_4+b_5]\ \frac{\sec Q_1}{Q_5}\left[\{C7\}-3\ \{C8\}\alpha_5^2\right] \qquad \text{(B-87)}$$

$$BLP\ (3,4) = MT\ b_4\ Q_2\ \sin Q_1 - \frac{b_5}{Q_5}\ [\{C7\}-3\{C8\}\alpha_5^2]$$

$$+ b_4\ [\cos\delta\cos Q_1 - \sin\delta\sin Q_1]\ [\{C3\}-3\{C4\}\alpha_1^2]\ \frac{1}{Q_5}$$

$$+ b_4\ [\{C5\}-3\ \{C6\}\ \alpha_3^2]\ \frac{\cos Q_1}{Q_5} \qquad \text{(B-88)}$$

$$BLP\ (3,5) = -MT\ b_4\ Q_2\ \cos Q_1$$

$$-b_5[\{C7\}-3\{C8\}\alpha_5^2]\left[\frac{-(Q_4-b_3Q_2)}{Q_5^2} + (b_4+b_5)\ \frac{(Q_2-Q_3)}{Q_5^2}\ \sec Q_1\right]$$

$$-b_4[\cos\delta\ \cos Q_1 - \sin\delta\ \sin Q_1]\ \{C3\}-3\{C4\}\alpha_1^2\ \frac{Q_4+b_1\ Q_2}{Q_5^2}$$

$$-\ b_4\ [\{C5\}-3\{C6\}\alpha_3^2\ ]\ \cos Q_1\ \left[\frac{Q_4-b_2\ Q_2}{Q_5^2}\right] \qquad \text{(B-89)}$$

$$BLP\ (4,1) = MST1\ b_4\ [Q_2-Q_3]^2\ \cos Q_1$$

$$-b_4\ MST1\ [\dot{Q}_2-\dot{Q}_3]\ \sin Q_1 - [D_5+D_6]\ \cos Q_1$$

$$+\ [\{C7\}-3\{C8\}\alpha_5^2]\ \cos Q_1\left[\sec^2 Q_1 - \{b_4+b_5\}\ \frac{Q_2-Q_3}{Q_5}\ \sec Q_1 \tan Q_1\right]$$

$$-\ [F_5+F_6]\ \sin Q_1 \qquad \text{(B-90)}$$

$$BLP\ (4,2) = -\ [MT+MST1]\ Q_5 + 2\ b_4\ MST1\ [Q_2-Q_3]\ \sin Q_1$$

$$+ \frac{b_1}{Q_5} \cos\delta\ [\{C3\}-3\{C4\}\alpha_1^2] - \frac{b_2}{Q_5}\ [\{C5\}-3\{C6\}\alpha_3^2]$$

$$- \cos Q_1 [\{C7\}-3\{C8\}\alpha_5^2]\ [b_3+\{b_4+b_5\}\ \sec Q_1]\ \frac{1}{Q_5} \qquad \text{(B-91)}$$

$$BLP\ (4,3) = -\ 2\ b_4\ MST1\ [Q_2-Q_3]\ \sin Q_1$$

$$+ \cos\ Q_1\ [b_4+b_5]\ \frac{\sec Q_1}{Q_5}\ \left[\{C7\}-3\{C8\}\alpha_5^2\right] \qquad \text{(B-92)}$$

$$BLP\ (4,4) = \frac{\cos\delta}{Q_5}\ [\{C3\}-3\{C4\}\alpha_1^2] + \frac{1}{Q_5}\ [\{C5\}-3\{C6\}\alpha_3^2]$$

$$+\ \frac{\cos Q_1}{Q_5}\ \left[\{C7\}-\ 3\{C8\}\alpha_5^2\right] \qquad \text{(B-93)}$$

$$BLP\ (4,5) = -\ [MT+MST1]\ Q_2 - \cos\delta\ [\{C3\}-3\{C4\}\alpha_1^2]\left[\frac{Q_4+b_1\ Q_2}{Q_5^2}\right]$$

$$-\ \left[\{C5\}\ -\ 3\{C6\}\alpha_3^2\right]\left[\frac{Q_4-\ b_2\ Q_2}{Q_5^2}\right]$$

$$+\ \cos Q_1\left[\{C7\}-3\{C8\}\alpha_5^2\right]\left[\frac{-(Q_4-b_3Q_2)}{Q_5^2} + (b_4+b_5)\ \frac{(Q_2-Q_3)}{Q_5^2}\ \sec Q_1\right] \qquad \text{(B-94)}$$

$$BLP\ (5,1) = b_4\ MST1\ [Q_2-Q_3]^2 \sin Q_1 + b_4\ MST1\ [\dot{Q}_2-\dot{Q}_3]\ \cos Q_1$$

$$- [D_5+D_6]\ \sin Q_1 + [F_5+F_6]\ \cos Q_1$$

$$+ [\{C7\}-3\{C8\}\alpha_5^2] \sin Q_1 \left[\sec Q_1 - (b_4+b_5) \frac{(Q_2-Q_3)}{Q_5} \sec Q_1 \tan Q_1 \right] \qquad (B\text{-}95)$$

$$BLP\ (5,2) = [MT+MST1]\ Q_4 - 2\ b_3\ MST1\ Q_2$$

$$- 2\ b_4\ MST1\ [Q_2-Q_3]\ \cos Q_1$$

$$- \sin\delta\ [\{C3\}-3\{C4\}\alpha_1^2]\ \frac{b_1}{Q_5}$$

$$- [\{C7\}-3\{C8\}\alpha_5^2]\ \sin Q_1\ \left[b_3+(b_4+b_5)\sec Q_1 \right] \frac{1}{Q_5} \qquad (B\text{-}96)$$

$$BLP\ (5,3) = 2\ b_4\ MST1\ [\ Q_2-Q_3]\ \cos Q_1$$

$$+ \left[\{C7\}-3\{C8\}\alpha_5^2 \right] \sin Q_1\ (b_4+b_5)\ \frac{\sec Q_1}{Q_5} \qquad (B\text{-}97)$$

$$BLP\ (5,4) = [MT+MST1]\ Q_2 - \frac{\sin\delta}{Q_5} \left[\{C3\}-3\ C4[\alpha_1^2] \right]$$

$$+ [\{C7\}-3\{C8\}\alpha_5^2]\ \frac{\sin Q_1}{Q_5} \qquad (B\text{-}98)$$

$$CLP\ (1,1) = 0.0 \tag{B-99}$$

$$CLP\ (1,2) = 0.0 \tag{B-100}$$

$$CLP\ (1,3) = 0.0 \tag{B-101}$$

$$CLP\ (1,4) = 0.0 \tag{B-102}$$

$$CLP\ (1,5) = 0.0 \tag{B-103}$$

$$CLP\ (1,6) = 0.0 \tag{B-104}$$

$$CLP\ (1,7) = 0.0 \tag{B-105}$$

$$CLP\ (2,1) = -d_2\ [D_1-D_2]\ \sin\delta + [b_1+b_3]\ \cos\delta\ [D_1+D_2]$$

$$+ d_2\ [\{C1\}-3\{C2\}\alpha_1^2]\ \sin\delta\ -d_2\ [F_1-F_2]\ \cos\delta$$

$$- [b_1+b_3]\ [\{C3\}-3\{C4\}\alpha_1^2]\ \cos\delta\ -\ [b_1+b_3]\ [F_1+F_2]\ \sin\delta \tag{B-106}$$

$$CLP\ (2,2) = d_2\ \cos\delta + (b_1+b_3)\ \sin\delta \tag{B-107}$$

$$CLP\ (2,3) = -d_2\ \cos\delta + (b_1+b_3)\ \sin\delta \tag{B-108}$$

$$CLP\ (2,4) = d_4 \tag{B-109}$$

$$CLP\ (2,5) = -d_4 \tag{B-110}$$

$$CLP\ (2,6) = 0.0 \tag{B-111}$$

$$CLP\ (2,7) = 0.0 \tag{B-112}$$

$$CLP(3,1) = b_4 [D_1+D_2] [\cos\delta \cos Q_1 - \sin\delta \sin Q_1]$$

$$+ b_4 [\{C_3\} - 3\{C_4\}\alpha_1^2] [\sin\delta \sin Q_1 + \cos\delta \cos Q_1]$$

$$- b_4 [F_1+F_2] [\sin\delta \cos Q_1 - \cos\delta \sin Q_1] \quad \text{(B-113)}$$

$$CLP(3,2) = b_4 [\sin\delta \cos Q_1 + \cos\delta \sin Q_1] \quad \text{(B-114)}$$

$$CLP(3,3) = b_4 [\sin\delta \cos Q_1 + \cos\delta \sin Q_1] \quad \text{(B-115)}$$

$$CLP(3,4) = b_4 \sin Q_1 \quad \text{(B-116)}$$

$$CLP(3,5) = b_4 \sin Q_1 \quad \text{(B-117)}$$

$$CLP(3,6) = d_6 \quad \text{(B-118)}$$

$$CLP(3,7) = - d_6 \quad \text{(B-119)}$$

$$CLP(4,1) = (D_1+D_2) \cos\delta - [\{C3\} - 3\{C4\}\alpha_1^2]\cos\delta$$

$$- (F_1+F_2) \sin\delta \quad \text{(B-120)}$$

$$CLP(4,2) = \sin\delta \quad \text{(B-121)}$$

$$CLP(4,3) = \sin\delta \quad \text{(B-122)}$$

$$CLP(4,4) = 0.0 \quad \text{(B-123)}$$

$$CLP(4,5) = 0.0 \quad \text{(B-124)}$$

$$CLP\ (4,6) = -\ \sin Q_1 \tag{B-125}$$

$$CLP\ (4,7) = -\ \sin Q_1 \tag{B-126}$$

$$CLP\ (5,1) = -\ (D_1+D_2)\ \sin\delta + [\{C3\} - 3\{C4\}\alpha_1^2]\ \sin\delta$$

$$-\ (F_1+F_2)\ \cos\delta \tag{B-127}$$

$$CLP\ (5,2) = \cos\delta \tag{B-128}$$

$$CLP\ (5,3) = \cos\delta \tag{B-129}$$

$$CLP\ (5,4) = 1.0 \tag{B-130}$$

$$CLP\ (5,5) = 1.0 \tag{B-131}$$

$$CLP\ (5,6) = \cos Q_1 \tag{B-132}$$

$$CLP\ (5,7) = \cos Q_2 \tag{B-133}$$

where

$$C1 = \frac{3}{2}\,\mu \left[\frac{F_{z_1}}{\alpha_{m_1}} - \frac{F_{z_2}}{\alpha_{m_2}} \right] \tag{B-134}$$

$$C2 = \frac{3}{2}\,\mu \left[\frac{F_{z_1}}{3\alpha_{m_1}^{3}} - \frac{F_{z_2}}{3\alpha_{m_2}^{3}} \right] \tag{B-135}$$

$$C3 = \frac{3}{2}\,\mu \left[\frac{F_{z_1}}{\alpha_{m_1}} + \frac{F_{z_2}}{\alpha_{m_2}} \right] \tag{B-136}$$

APPENDIX C

THEORETICAL CONSIDERATIONS OF THE STABILITY OF MOTION OF THE TRACTOR-SEMITRAILER SYSTEM

APPENDIX C

LIST OF SYMBOLS

$A(t)$	Jacobian matrix
$B(t)$	Jacobian matrix
D_i	tire brake/tractive force
t	time
$\underline{q}$	system state vector
V_{x_1}	tractor c.g. forward velocity
V_{y_1}	tractor c.g. lateral velocity
$\underline{x}$	perturbation vector
ψi	yaw angle
$\dot{\psi} i$	yaw angular velocity
δ	steering angle
$\Delta\delta$	incremental steering angle
ΔD_i	incremental brake/tractive force

APPENDIX C

THEORETICAL CONSIDERATIONS OF THE STABILITY OF MOTION OF THE TRACTOR-SEMITRAILER SYSTEM

1. INTRODUCTION

In this appendix, the stability of motion of the tractor-semitrailer is considered from a theoretical viewpoint both in the sense of Lyapunov and in the sense of finite time stability. The general definitions of stability are given and the appropriate theorems are stated without proof. In Appendix B the equations of motions of the tractor-semitrailer are obtained in the body coordinates and after choosing state variables, they are expressed in the form

$$\dot{\underline{q}} = \underline{f}\ (\underline{q}, \delta, \underline{D}) \qquad \text{(C-1)}$$

where the five dimensional state variable vector q(t) is defined by

$$q' = (\psi_1 - \psi_2, \dot{\psi}_1, \dot{\psi}_1 - \dot{\psi}_2, V_{y_1}, V_{x_1}) \qquad \text{(C-2)}$$

and the steering angle δ and the braking forces $\underline{D}$ are the control inputs. Three types of nominal maneuvers are considered, namely, circular trajectory at constant forward velocity, lane change with and without braking, and cornering with and without braking. A prescribed maneuver whose stability is to be investigated is selected and the nominal motion $\underline{q}^*(t)$ and the steering angle $\delta^*(t)$ demanded by the maneuver and the distribution of braking force $D^*(t)$ at each wheel are computed by employing AVDS3.

The motion predicted by the mathematical model may not be identical with that of a practical maneuver because the latter is subjected to disturbances caused by variations in the aerodynamic loading, inexact realization of the control inputs $\delta^*(t)$ and $\underline{D}^*(t)$ and similar causes. A nominal motion that is unstable is not realizable in practice because if it is slightly perturbed by disturbances, it diverges from the nominal motion. The question to be answered is how does the motion $\underline{q}(t)$ which starts near a nominal motion $\underline{q}^*(t)$ behave with respect to $\underline{q}^*$? Does it remain very near $\underline{q}^*$ as time increases, which would imply that $\underline{q}^*$ is stable, or does it diverge from $\underline{q}^*$, indicating that $\underline{q}^*$ is unstable? In order to answer this question, a particular maneuver $\underline{q}^*(\delta^*, \underline{D}^*, t, \underline{q}_o, t_o)$ starting at time t_o with initial conditions $\underline{q}_o$ and control inputs δ^* and D^* is considered. This nominal motion is obtained by numerical integration of Equation (C-1) by employing the AVDS3 program.

Consider the effect of perturbations $\Delta\underline{q}_o$ on the initial state and $\Delta\delta$ and ΔD on the steering and braking inputs. Let $\underline{q}(\delta^* + \Delta\delta, D^* + \Delta D, t; \underline{q}_o + \Delta\underline{q}_o, t_o)$ be the resultant perturbed motion. The vector $\underline{x}(t)$ of perturbed state variables is defined by

$$\underline{x}(t) = \underline{q}(t) - \underline{q}^*(t) \quad \text{i.e.} \quad \underline{q}(t) = \underline{q}^*(t) + \underline{x}(t) \qquad (C\text{-}3)$$

Now, $\underline{q}(t)$ from Equation (C-3) is substituted into Equation (C-1) and the resulting equation is expanded in Taylor series about the nominal motion $\underline{q}^*$ to obtain

$$\dot{\underline{q}}^* + \dot{\underline{x}} = \underline{f}(\underline{q}^*,\delta^*,\underline{D}^*) + \left.\frac{\partial \underline{f}}{\partial \underline{q}}\right|_{\substack{\underline{q}^* \\ \underline{u}}} \underline{x} + \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\substack{\underline{q}^* \\ \underline{u}}} \Delta\underline{u} + \underline{h}(\underline{x},\Delta\underline{u},t) \qquad \text{(C-4)}$$

where $\underline{u} = \begin{bmatrix}\delta \\ \underline{D}\end{bmatrix}$ and $\Delta\underline{u} = \begin{bmatrix}\Delta\delta \\ \Delta\underline{D}\end{bmatrix}$ and $\underline{h}(\underline{x},\Delta\underline{u},t)$ contain terms that are higher than the first degree in $\underline{x}$ and $\Delta\underline{u}$ resulting from the Taylor series expansion about $\underline{q}^*$. Since the nominal motion satisfies the equation

$$\dot{\underline{q}}^* = \underline{f}(\underline{q}^*,\delta^*,\underline{D}) \qquad \text{(C-5)}$$

it follows from Equations (C-4) and (C-5) that the differential equation for the perturbations in state variables becomes

$$\dot{\underline{x}} = A(t)\ \underline{x} + B(t)\ \Delta\underline{u} + \underline{h}(\underline{x},\Delta\underline{u},t) \qquad \text{(C-6)}$$

where A(t) and B(t) denote the Jacobian matrices defined by

$$A(t) = \left.\frac{\partial \underline{f}}{\partial \underline{q}}\right|_{\substack{\underline{q}^* \\ \underline{u}^*}} \quad \text{and} \quad B(t) = \left.\frac{\partial \underline{f}}{\partial \underline{u}}\right|_{\substack{\underline{q}^* \\ \underline{u}^*}} \qquad \text{(C-7)}$$

The original problem of determining the stability of a maneuver $\underline{q}^*(\delta^*, D^*, t; \underline{q}_o, t_o)$ is now equivalent to the problem of determining the stability of the null solution, i.e., trivial solution of Equation (C-7) Consider a special class of maneuvers where the translation of the vehicle is described by a circular arc at a constant forward velocity. For this class of maneuvers, all the state variables of the nominal motion defined by Equation (C-2) have constant values, and in particular $\dot{\psi}_1 - \dot{\psi}_2$ is zero. Also the control inputs δ^* and D^* are constant. Such steady state nominal motions can be represented by isolated equilibrium points (also called singular points) in the state space $\underline{q}(t)$ as shown in Figure C-1.

The steady state nominal motion $\underline{q}^*$ whose stability is to be investigated is denoted by $\underline{q}_e^*$ to indicate that it is an equilibrium point in the state space rather than a time-varying trajectory $\underline{q}^*(t)$. The transformation of Equation (C-3), namely,

$$\underline{x}(t) = \underline{q}(t) - \underline{q}_e^* \qquad \text{(C-8)}$$

is now merely a transformation of coordinates such that the origin of the state space of $\underline{x}(t)$ is equivalent to the equilibrium point $\underline{q}_e^*$ of the state space of $\underline{q}(t)$ as shown in Figure C-1. In this case, Equation (C-6) becomes autonomous, i.e., its right-hand side is not an explicit function of time, and is described by

$$\dot{\underline{x}} = A\,\underline{x} + B\Delta\underline{u} + \underline{h}(\underline{x}, \Delta\underline{u}) \qquad \text{(C-9)}$$

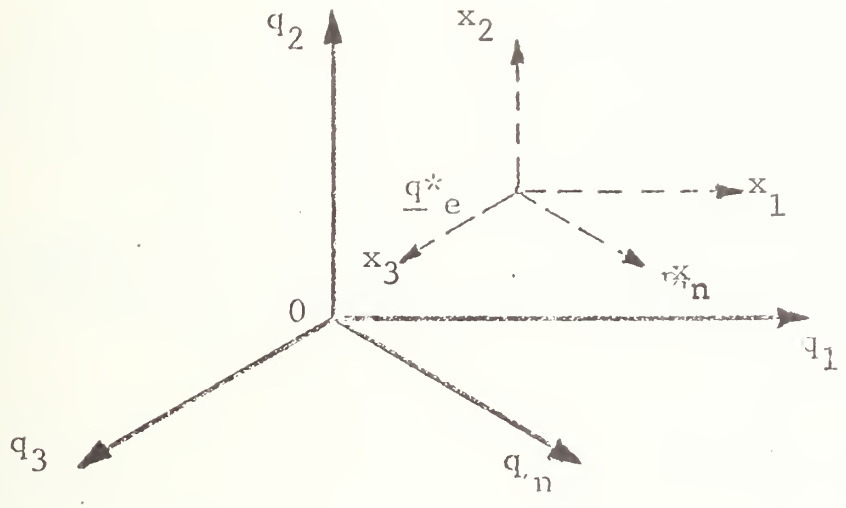


Figure C-1. Equilibrium point in state space for autonomous case.

Now, stability in the sense of Lyapunov is considered first and finite time stability is discussed later.

2. STABILITY IN THE SENSE OF LYAPUNOV

When considering the stability of a maneuver in the sense of Lyapunov, the control inputs δ^* and D^* of the nominal motion are not perturbed. This step is necessary since the concept of stability in the sense of Lyapunov does not admit perturbations in the inputs. The nominal motion is perturbed only by nonzero initial conditions which may be caused by impulsive changes in the control inputs at the initial time. Letting $\Delta \underline{u} = \underline{0}$ in Equation (C-6) the perturbation vector is described by the equation

$$\dot{\underline{x}} = A(t)\, x + \underline{h}(\underline{x},t) \qquad \text{(C-10)}$$

and in the autonomous case, we get

$$\dot{\underline{x}} = Ax + \underline{h}(\underline{x}) \qquad \text{(C-11)}$$

The definitions of stability of motion in the sense of Lyapunov are given next. The norms employed are Euclidian norms.

Definition 1 - The nominal motion $\underline{q}^*$ is stable in the sense of Lyapunov if for every $\epsilon > 0$ there exists $\eta > 0$ where η depends on ϵ and possibly on t_o such that $||\underline{x}(t_o)|| < \eta$ implies that $||\underline{x}(t)|| < \epsilon$ for all $t > t_o$.

Definition 2 - The nominal motion $\underline{q}^*$ is asymptotically stable if it is stable, and $\lim_{t \to \infty} ||\underline{x}(t)|| = 0$.

Definition 3 - The nominal motion $\underline{q}^*$ is uniformly stable if it is stable in such a way that η does not depend on t_o.

Definition 4 - The nominal motion $\underline{q}^*$ is uniformly asymptotically stable if: (1) it is uniformly stable and (2) all perturbed motions with $||\underline{x}(t_o)|| < \eta$ are such that $||\underline{x}(t)|| \to 0$ uniformly in both $\underline{x}(t_o)$ and t_o .

Definition 5 - The nominal motion $\underline{q}^*$ is unstable if there is an ϵ such that no η can be found to satisfy the conditions of Definition 1.

These concepts of stability are clarified in Figure C-2 but only for a one-dimensional case without limiting the generality of the conclusions to the n-dimensional problem. For the autonomous case of Equation (C-11), the definitions of stability are illustrated in Figure C-3 but only for a two-dimensional state space. For the n-dimensional space, the circles become hyperspheres of radius η and ϵ, respectively. For the nonautonomous system of Equation (C-10), Definitions 3 and 4 refine the notions of stability given by Definitions 1 and 2 concerning uniform stability. The concept of uniform stability can be explained as follows. For an asymptotically stable system, for every number γ, there always exists another number T, depending on γ, $\underline{x}_o$ and t_o such

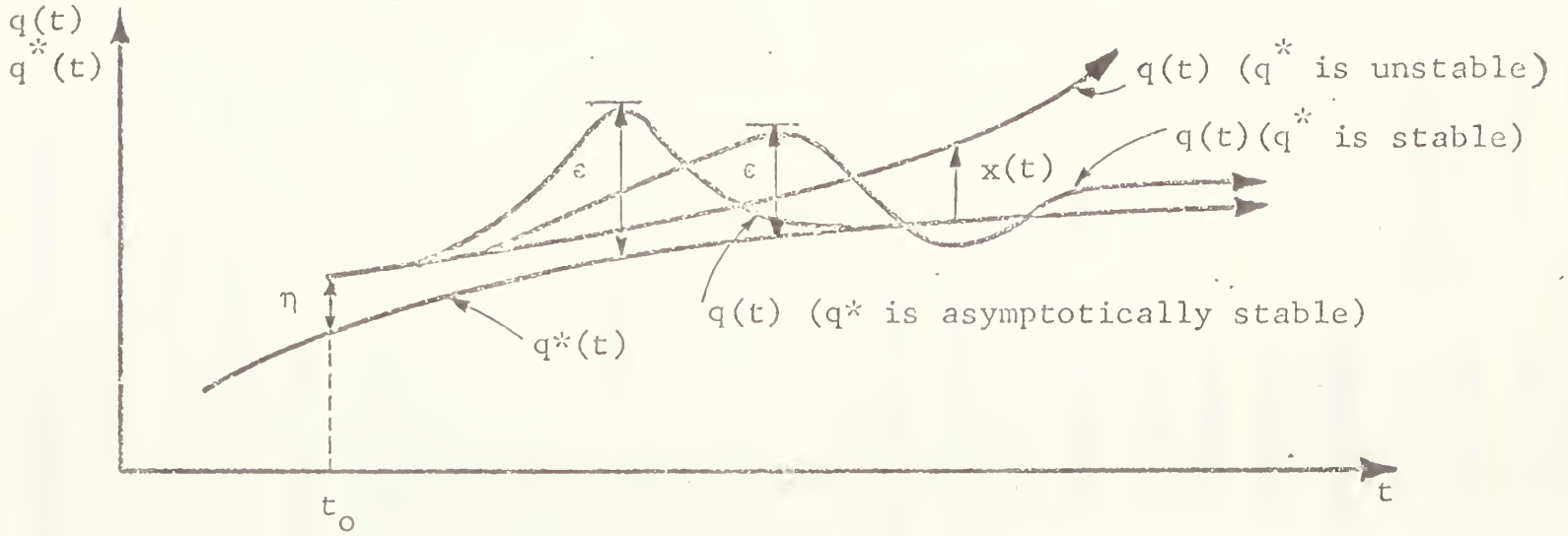


Figure C-2. Concept of stability.

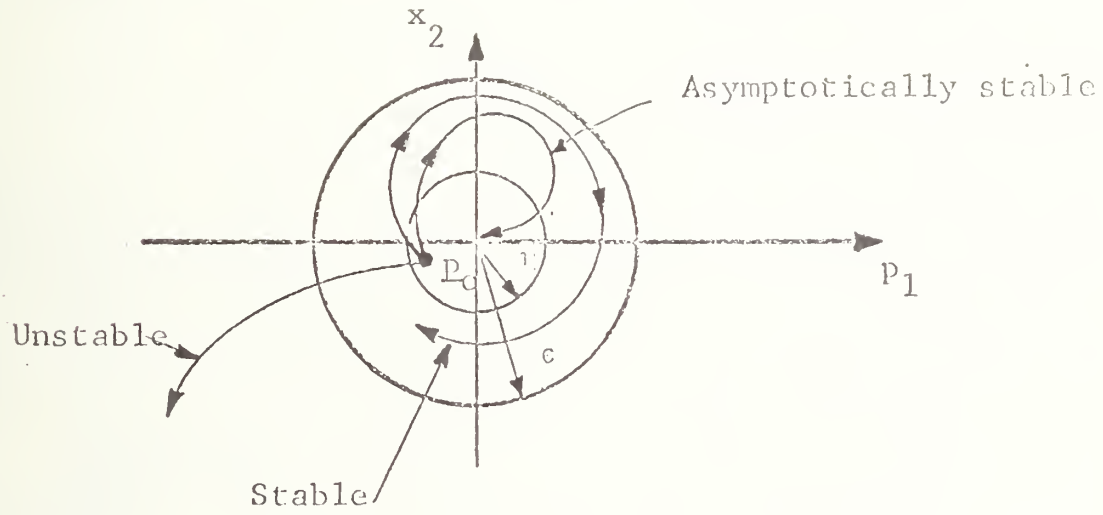


Figure C-3. Illustration of definitions of stability for autonomous case.

that when $||\underline{x}_o|| < \eta$, then $||\underline{x}(t)|| \leq \gamma$ for $t \geq t_o + T$. We say that $||\underline{x}(t)|| \rightarrow 0$ uniformly in t_o when the convergence time T is independent of t_o. For example, consider the scalar equation

$$\dot{x} = -\frac{1}{t} x \tag{C-12}$$

By elementary integration, the solution is given by $x(t) = \frac{t_o}{t} x(t_o)$. The null solution is asymptotically stable since $x(t) < x(t_o)$ for $t > t_o$ and $x(t) \rightarrow 0$ as $t \rightarrow \infty$. In Definition 1, given any $\epsilon > 0$ there exists $\eta > 0$ and in this case η can be chosen such that $\eta = \epsilon$. If $t_o = 1$ sec, then $x(t) \leq 0.1\ x(t_o)$ for $t \geq 1 + 9$, i.e., T = 9 sec. If $t_o = 1{,}000$ sec, then T = 9,000 sec for $x(t) \leq 0.1\ x(t_o)$. Hence, asymptotic stability here is not uniform in t_o. The practical implication of this conclusion is that in this case, asymptotic stability is no more useful than mere stability. It should be noted that in the case of the autonomous system of Equation (C-11), asymptotic stability implies uniform asymptotic stability.

Concerning Definition 4, it is stated that $||\underline{x}(t)|| \rightarrow 0$ uniformly in $\underline{x}_o$ when the convergence time T, as defined above, is only a function of η where $||\underline{x}_o|| < \eta$. In other words, the convergence time depends only on the norm or magnitude of the initial perturbation vector $\underline{x}_o$ and not on its direction. It safeguards against the existence of particular directions such that $T \rightarrow \infty$ when $\underline{x}_o$ approaches these preferred directions, in which case, asymptotic stability again would be no more useful than mere stability.

In the investigation of stability in the small, it is assumed that the perturbations are sufficiently small and the higher order terms in the Taylor series expansion, i.e., $\underline{h}(\underline{x},t)$ in Equation (C-10) or $\underline{h}(\underline{x})$ in Equation (C-11), are dropped. Hence, the stability of the linear approximation of the perturbation equations given by Equation (C-10) or (C-11) is investigated. The approach yields stability information in the small, i.e., the perturbations should be sufficiently small but there is no indication of the magnitude of perturbations that could be considered as small. Hence, when asymptotic stability exists, the size of the region of asymptotic stability is not known. To determine the size of this region, the stability in the large of the nonlinear Equation (C-10) or (C-11) is to be investigated. The pertinent theorems that have been employed for the stability investigation in the small are stated in the following.

Theorem 1. For the autonomous system, let $\dot{\underline{x}} = A\underline{x} + \underline{h}(\underline{x})$, i.e., Equation (C-11) be the equation of the perturbations about an equilibrium state $\underline{q}_e^*$. If

$$\lim_{||\underline{x}|| \to 0} \frac{||\underline{h}(\underline{x})||}{||\underline{x}||} = 0 \text{ , then} \qquad \text{(C-13)}$$

1. If the linearized system $\dot{\underline{x}} = A\underline{x}$ has only eigenvalues with negative real parts, $\underline{q}_e^*$ is asymptotically stable in the small.

2. If the linearized system $\dot{\underline{x}} = A\underline{x}$ has one or more eigenvalues with positive real parts, $\underline{q}_e^*$ is unstable in the small.
3. If the linearized $\dot{\underline{x}} = A\underline{x}$ has one or more eigenvalues with zero real parts and the remaining eigenvalues have negative real parts, the stability of $\underline{q}_e^*$ cannot be ascertained in the small by studying the linearized system alone.

This theorem is sometimes called the "Principle of the Stability in the First Approximation." It is also sometimes called "Lyapunov's First Method." The proof of this theorem is based on Lyapunov's Second or Direct Method and is given by Hahn [1], Hsu and Meyer [2], LaSalle and Lefschetz [3], and others. Since, in this case $\underline{h}(\underline{x})$ consists of higher order terms than the first in the Taylor series expansion, condition (C-13) is always satisfied. Theorem 1 has been employed for the stability investigation in the small of maneuvers where the translation of the vehicle is described by a circular arc at a constant forward velocity. The conditions for the matrix A to have eigenvalues with negative real parts have been determined by employing Routh's criterion. A computer program has been written up and utilized for this purpose.

The stability in the small of nonautonomous maneuvers is investigated by employing the following theorem which relates the stability in the small of a nominal maneuver $\underline{q}^*(t)$ to the stability of the null solution of the perturbation equation,

$$\dot{\underline{x}} = A(t)\underline{x} \tag{4.14}$$

which is the linear approximation of the nonlinear perturbation Equation (C-10), i.e., $\dot{\underline{x}} = A(t)\underline{x} + \underline{h}(\underline{x},t)$.

Theorem 2. Assuming that $||\underline{h}(\underline{x},t)||/||\underline{x}|| \rightarrow 0$ uniformly in t as $||\underline{x}|| \rightarrow 0$, uniform asymptotic stability of the origin (i.e., trivial or null solution) of the linearized system of Equation (C-14) implies that the nominal motion $\underline{q}^*(t)$ is also uniformly asymptotically stable in the small.

The proof of this theorem is given by Hsu and Meyer.[2] Theorem 2 for nonautonomous systems is analogous to Theorem 1 for autonomous systems. It should be noted that Theorem 2 states sufficient conditions for uniform asymptotic stability in the small. However, unlike autonomous systems, the problem is now to establish uniform asymptotic stability for the time-varying linear system of Equation (C-14). One approach is to employ a suitable Lyapunov function.

Another approach, which has been sometimes employed in the existing literature[4], is the "freezing time" method. At each instant of time $t = t_1, t_2, \ldots, t_k$, the time varying parameters are fixed at their current values and the matrix $A(t_i)$ is treated as a constant matrix for the interval t_i to t_{i+1}. The conditions for the constant matrix $A(t_i)$, with $t_i = t_1, t_2, \ldots, t_k$, to have eigenvalues with negative real parts can then be determined by employing Routh's criterion.

The following question then arises. If all eigenvalues of the matrix $A(t_i)$ have negative real parts for $t_i = t_1, t_2, \ldots, t_k$, does it mean that the origin of the linear system of Equation (C-14) is uniformly asymptotically stable? The answer to this question is not always in the affirmative as demonstrated by the following two counter-examples.

Example 1. This example is quoted by Aggarwal and Infante [5] and attributed to Marcus and Yamabe. [6] Consider the system $\dot{\underline{x}} = A(t)\underline{x}$ where $A(t)$ is given by

$$A(t) = \begin{bmatrix} -1 + a\cos^2 t & 1 - a\sin t\cos t \\ -1 - a\sin t\cos t & -1 + a\sin^2 t \end{bmatrix} \qquad \text{(C-15)}$$

with $a > 0$. When the determinant $|\lambda I - A|$ is considered, it is found that the eigenvalues of $A(t)$ are given by

$$\lambda_{1,2} = \frac{a - 2 + \sqrt{a^2 - 4}}{2} \qquad \text{(C-16)}$$

and are time invariant. They have negative real part for $a < 2$. A closed form solution of this equation is possible and is given by

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e^{(a-1)t}\cos t & e^{-t}\sin t \\ -e^{(a-1)t}\sin t & e^{-t}\cos t \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \qquad \text{(C-17)}$$

which shows that asymptotic stability requires that $a < 1$. Hence, if $a = 1.5$ then the freezing time method indicates that the origin of the system of Equation (C-15) is asymptotically stable whereas it is actually unstable.

Example 2. This example is quoted by Hsu and Meyer [2] and is attributed to Vinogradov. Consider the system $\dot{\underline{x}} = A(t)\underline{x}$ with A(t) given by

$$A(t) = \begin{bmatrix} -1 - 9\cos^2 6t + 12 \sin 6t \cos 6t & 12\cos^2 6t + 9 \sin 6t \cos 6t \\ -12 \sin^2 6t + 9 \sin 6t \cos 6t & -1 - 9\sin^2 6t - 12 \sin 6t \cos 6t \end{bmatrix} \tag{C-18}$$

This example is also so contrived that the eigenvalues of A(t) are

$$\lambda_{1,2} = -1, -10 \tag{C-19}$$

and are time-invariant. A closed form solution of this equation is available and is given by

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} e^{2t}(\cos 6t + 2 \sin 6t) & e^{-13t}(\sin 6t - 2 \cos 6t) \\ e^{2t}(2 \cos 6t - \sin 6t) & e^{-13t}(2 \sin 6t + \cos 6t) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \tag{C-20}$$

The presence of the term e^{2t} in Equation (C-20) shows that the origin of Equation (C-21) is actually unstable whereas the freezing time method indicates asymptotic stability. These

two examples show that the eigenvalues of A(t) do not carry a great deal of information regarding stability. Yet, the freezing time method has sometimes been successfully employed in the aerospace industry for the design of autopilots for aircraft and missiles. It appears that if the elements of A(t) are periodic in time or if its eigenvalues are near the imaginary axis, as in the foregoing examples, then the stability results based on the eigenvalues of A(t) may be invalid.

The "freezing time" method was employed in this investigation to study the stability of cornering maneuvers. The matrix $A(t_i)$ was obtained for $t_i = t_1,\ldots,t_k$ by employing the AVDS 3 simulation results and Routh's criterion was used to check the location of its eigenvalues. The results indicate that the maneuvers that are possible by AVDS 3 simulation are stable, i.e., the eigenvalues of A(t) have negative real parts.

The following definitions and theorems are now stated in order to discuss the use of suitable Lyapunov functions to investigate the stability of the nonautonomous, linearized system of Equation (C-14).

Definition 6. A time-varying scalar function $V(\underline{x},t)$ is positive definite in a region Ω containing the origin if $V(\underline{0},t) = 0$ and if a continuous and strictly increasing function ξ_1 exists such that $\xi_1(0) = 0$ and

$$V(\underline{x},t) \geq \xi_1 (||\underline{x}||) \text{ in } \Omega \qquad \text{(C-21)}$$

A strictly increasing function $\xi(z)$ obeys the property that if $z_2 > z_1$, $\xi(z_2) > \xi(z_1)$ for any z_1 and z_2. A function $V(\underline{x},t)$ is called negative definite if $-V(\underline{x},t)$ is positive definite.

Definition 7. A time-varying scalar function $V(\underline{x},t)$ is called decrescent in a region Ω containing the origin if a continuous and strictly increasing function ξ_2 exists such that $\xi_2(0) = 0$ and

$$V(\underline{x},t) \leq \xi_2 (||\underline{x}||) \text{ in } \Omega \qquad \text{(C-22)}$$

For example, consider the following functions defined in two dimensional space:

$$V(\underline{x},t) = x_1^2 (1 + \sin^2 t) + x_2^2 (1 + \cos^2 t) \qquad \text{(C-23)}$$

$$V(\underline{x},t) = x_1^2 + (1 + t) x_2^2 \quad t > 0 \qquad \text{(C-24)}$$

The function of Equation (C-23) is positive definite and decrescent whereas the V function of Equation (C-24) is positive definite but not decrescent since the function can become arbitrarily large for arbitrarily small $(x_1, x_2) \neq (0,0)$. When $V(\underline{x})$ is not an explicit function of time, then positive definiteness requires only that $V(\underline{0}) = 0$ and $V(\underline{x}) > 0$ for $\underline{x} \neq \underline{0}$. Now, the important Lyapunov stability theorems are introduced for the system of Equation (C-10).

Theorem 3. If there exists a continuous positive definite scalar function $V(\underline{x},t)$ in the neighborhood Ω of the origin such that $\dot{V}(\underline{x},t) < 0$ in Ω, then the origin of Equation (C-10) is stable and Ω is the region of stability.

Theorem 4. If there exists a continuous positive definite and decrescent function $V(\underline{x},t)$ in the neighborhood Ω of the origin such that

$$\dot{V}(\underline{x},t) \leq - \xi_3(||\underline{x}||) \text{ in } \Omega \qquad \text{(C-25)}$$

where $\xi_3(z)$ is a positive (but not necessarily strictly increasing) scalar function of one variable, then the origin of Equation (C-10) is uniformly asymptotically stable and Ω is the region of asymptotic stability.

The proofs of Theorems 3 and 4 are given by Hsu and Meyer [2] and by Hahn. [1] These theorems state the stability conditions in the large since the size of the region of stability Ω is known. However, when the perturbation equations are linearized and approximated by Equation (C-14) and Theorem 4 is employed to establish uniform asymptotic stability for the time-varying linear system, then according to Theorem 2 the origin of the nonlinear Equation (C-10) is uniformly asymptotically stable only in the small. It should be noted that for nonautonomous systems, it is perfectly legitimate to choose a V-function which is not an explicit function of time. In that case, $V(\underline{x})$ need only be positive definite in the time-invariant sense for uniform

asymptotic stability, provided that $\dot{V}$, which is an explicit function of time, satisfies the conditions of Equation (C-25).

The choice of a suitable Lyapunov function is not obvious for nonautonomous systems even in the case of linear problems. In the case of autonomous maneuvers where the linearized perturbation equations are described by

$$\dot{\underline{x}} = A\underline{x} \tag{C-26}$$

with A a constant matrix, a quadratic Lyapunov function is chosen as

$$V(\underline{x}) = \underline{x}' \, P\underline{x} \tag{C-27}$$

where P is a symmetric matrix and the prime denotes transpose. The time derivative of $V(\underline{x})$ is given by

$$\dot{V}(\underline{x}) = \dot{\underline{x}}' \, P\underline{x} + \underline{x}' \, P\dot{\underline{x}} \tag{C-28}$$

Substituting for $\dot{\underline{x}}$ in Equation (C-28) from Equation (C-26). it follows that

$$\begin{aligned} \dot{V}(\underline{x}) &= \underline{x}'(A'P + PA)\underline{x} \\ &= - \underline{x}' \, Q \, \underline{x} \end{aligned} \tag{C-29}$$

where Q is defined by

$$A'P + PA = -Q \tag{C-30}$$

Equation (C-30) is known as the matrix Lyapunov equation. It has been proved [2] that when all eigenvalues of A have negative

real parts, P is positive definite if and only if Q is positive definite. Therefore, by selecting Q positive definite, the positive definiteness of P can be guaranteed when matrix A is stable. The reverse is however not true, i.e., if P is chosen as positive definite, it does not follow that Q obtained from solution of Equation (C-30) will be positive definite when A is stable. It can also be shown that Equation (C-30) has a unique solution for P when the sum of any two eigenvalues of A is not zero and this condition is satisfied when A is a stable matrix, i.e., all its eigenvalues have negative real parts. Hence, it can be concluded that for linear autonomous Equation (C-26) the V function of Equation (C-27) is a Lyapunov function. A similar method for generating a Lyapunov function for linear nonautonomous system is however not available. The following quadratic functions were employed as trails in order to investigate stability in the small of nonautonomous maneuvers, where the linearized perturbation equation is described by Equation (C-14), namely $\dot{\underline{x}} = A(t)\underline{x}$.

Method 1. A quadratic function is chosen as

$$V(\underline{x}) = \underline{x}' P \underline{x} \qquad \text{(C-31)}$$

where P is a symmetric constant matrix. The time derivative of $V(\underline{x})$ along the trajectory of Equation (C-14) is given by

$$\dot{V}(\underline{x},t) = \underline{x}'(A'(t)P + P\,A(t)\underline{x} \qquad \text{(C-32)}$$

Choosing a positive definite Q matrix, the solution of P is obtained from the Lyapunov matrix equation

$$A'(0)P + P\,A(0) = -Q \tag{C-33}$$

Assuming that A(0) is a stable matrix, then P matrix obtained from solution of Equation (C-32) is positive definite. Now, setting

$$G(t) = A(t) - A(0) \tag{C-34}$$

Equation (C-32) can be expressed as

$$\begin{aligned}\dot{V}(\underline{x},t) &= \underline{x}'[(G(t) + A(0))'P + P(G(t) + A(0))]\underline{x} \\ &= \underline{x}'(-Q + G'(t)P + P\,G(t))\underline{x}\end{aligned} \tag{C-35}$$

If $\left[-Q + G'(t)P + P\,G(t)\right]$ is negative definite for $0 \leq t \leq T$, where 0 is the starting and T the terminal time of the maneuver, then the origin of Equation (C-14) is asymptotically stable according to Theorem 4 and the maneuver is asymptotically stable in the small. This method would yield good results if A(t) does not change much from A(0). This is not the case for the lane change and cornering maneuvers and the method did not yield good results. The results were very conservative.

Method 2. A quadratic function is chosen as

$$V(\underline{x},t) = \underline{x}'P(t)\underline{x} \tag{C-36}$$

where $P(t)$ is a symmetric time-varying matrix. The time derivative of $V(\underline{x},t)$ along the trajectory of Equation (C-14) is given by

$$\dot{V}(\underline{x},t) = \underline{x}'(A'(t)P(t) + P(t)A(t) + \dot{P}(t))\underline{x} \qquad \text{(C-37)}$$

Let $$A'(t)P(t) + P(t)A(t) = -Q \qquad \text{(C-38)}$$

where Q is selected as a positive definite constant matrix and $P(t)$ is obtained from the solution of Equation (C-38). If $P(t)$ thus obtained is positive definite and $-Q + \dot{P}(t)$ is negative definite for $0 \leq t \leq T$, then the origin of Equation (C-14) is asymptotically stable and the maneuver is asymptotically stable in the small. This method was employed and $\dot{P}(t)$ was approximated by

$$\dot{P}(t_k) = \frac{1}{\Delta t}\left[P(t_{k+1}) - P(t_k)\right] \qquad \text{(C-39)}$$

The results obtained are very conservative and it is believed that good results were not obtained because the finite difference approximation of Equation (C-39) is a very poor approximation of $\dot{P}(t)$. A great amount of computational labor is required to get a better approximation for $\dot{P}(t)$.

Method 3. This method is closely related to Method 2 and is based on a general idea used by Infante.[7] Here, $V(\underline{x},t)$ is chosen as in Method 2 from the solution of Equation (C-38). From Equations (C-36) and (C-37), it follows that

$$\frac{\dot{V}(\underline{x},t)}{V(\underline{x},t)} = \frac{\underline{x}'(-Q + \dot{P}(t))\underline{x}}{\underline{x}'P(t)\underline{x}} \qquad \text{(C-40)}$$

It can be shown that

$$\frac{\underline{x}'(-Q + \dot{P}(t))\underline{x}}{\underline{x}'P(t)\underline{x}} \leq \lambda_{max}(t) \qquad \text{(C-41)}$$

where $\lambda_{max}(t)$ is the maximum eigenvalue of the Equation

$$| -Q + \dot{P}(t) - \lambda P(t) | = 0 \qquad \text{(C-42)}$$

Then after integrating Equation (C-40), we obtain

$$\ln \frac{V(\underline{x},t)}{V(\underline{x},0)} \leq \int_0^t \lambda_{max}(t')dt'$$

$$\text{i.e.,} \quad V(\underline{x},t) \leq V(\underline{x},0) \exp \left[\int_0^t \lambda_{max}(t')dt' \right] \qquad \text{(C-43)}$$

If P(t) obtained from the solution of Equation (C-38) is positive definite and $\exp \left[\int_0^t \lambda_{max}(t')dt' \right] \leq 1$ for $0 \leq t \leq T$, then the origin of Equation (C-14) is stable and in addition if $\lambda_{max}(t) < 0$, then it is asymptotically stable. This method also suffers the same disadvantage of Method 2, i.e., it requires accurate value of $\dot{P}(t)$.

Method 4. This method avoids the determination of $\dot{P}(t)$ by employing discrete time approximation of Equation (C-14). The equation, $\dot{\underline{x}} = A(t)\underline{x}$, is approximated by

$$\frac{\underline{x}(t_{k+1}) - \underline{x}(t_k)}{t_{k+1} - t_k} = A(t_k)\ \underline{x}(t_k) \qquad \text{(C-44)}$$

$$\text{Then,} \quad \underline{x}(t_{k+1}) = (t_{k+1} - t_k)\ A(t_k)\ \underline{x}(t_k) + \underline{x}(t_k) \qquad \text{(C-45)}$$

$$\text{Let} \quad C(t_k) = (t_{k+1}) - t_k)\ A(t_k) + I \qquad \text{(C-46)}$$

The discrete time equation (C-45) is then written as

$$\underline{x}(t_{k+1}) = C(t_k)\ \underline{x}(t_k) \tag{C-47}$$

A stability theorem for discrete time system (C-47), analogous to Theorem 4 for continuous time system, is stated next.

Theorem 5. The trivial solution of Equation (C-47) is asymptotically stable if there exists a decrescent positive definite function $V(\underline{x},t_k)$ possessing a negative definite forward difference $\Delta V(\underline{x},t_k)$, where

$$\Delta V(\underline{x},t_k) = \frac{V(\underline{x}(t_{k+1}),\ t_{k+1}) - V(\underline{x}(t_k),t_k)}{t_{k+1} - t_k} \tag{C-48}$$

A proof of this theorem is given by Freeman.[8] This theorem also suffers the same disadvantage of Theorem 4 as there is no systematic method for choosing $V(x,t_k)$. However, the nonstationary discrete time Equation (C-47) may be reduced to a stationary discrete time equation as follows. Let $S(t_k)$ be a n x n matrix with bounded coefficients that is nonsingular for all t_k in the interval (0,T). Consider the transofrmation of state variables

$$\hat{\underline{x}}(t_k) = S(t_k)\ \underline{x}(t_k) \tag{C-49}$$

Transforming Equation (C-47) by employing this transformation, we get

$$\hat{\underline{x}}(t_{k+1}) = S(t_{k+1})\ C(t_k)\ S^{-1}(t_k)\ \hat{\underline{x}}(t_k) \tag{C-50}$$

Suppose that for the nonstationary system (C-47), a bounded, nonsingular transformation exists such that

$$S(t_{k+1})\ C(t_k)\ S^{-1}(t_k) = C(0) = \text{constant} \tag{C-51}$$

for all t_k in the interval (0,T). Equation (C-50) then becomes

$$\hat{\underline{x}}(t_{k+1}) = C(0)\ \hat{\underline{x}}(t_k) \tag{C-52}$$

If the nature of $S(t_k)$ is as specified, Equations (C-47) and (C-52) have the same stability properties. The stability of the nonstationary system (C-47) can then be determined by studying the stability of the transformed stationary system (C-52). For Equation (C-52), a quadratic V function is chosen as

$$V(\hat{\underline{x}}) = \hat{\underline{x}}' P \underline{x} \tag{C-53}$$

Its forward difference is given by

$$\Delta V(\hat{\underline{x}}) = \frac{1}{t_{k+1} - t_k} \left[V(\hat{\underline{x}}(t_{k+1})) - V(\hat{\underline{x}}(t_k)) \right] \tag{C-54}$$

Employing the expression (C-53) in Equation (C-54) and substituting for $\hat{\underline{x}}(t_{k+1})$ from Equation (C-52), it follows that

$$\begin{aligned} \Delta V(\hat{\underline{x}}) &= \frac{1}{t_{k+1} - t_k} \left[\hat{\underline{x}}' (C'(0)\ P\ C(0) - P) \hat{\underline{x}} \right] \\ &= \frac{1}{t_{k+1} - t_k} \left[- \hat{\underline{x}}'\ Q\ \hat{\underline{x}} \right] \end{aligned} \tag{C-55}$$

where Q is defined by

$$C'(0)\ P\ C(0) - P = - Q \tag{C-56}$$

This equation is the discrete time counterpart of the Lyapunov matrix Equation (C-30) which holds for continuous time systems. It can be proved [8] that when matrix C(0) is stable, i.e., all its eigenvalues are of magnitude less than unity, P is positive definite if and only if Q is positive definite. Therefore, by selecting Q positive definite, the positive definiteness of P can be guaranteed when matrix C(0) is stable. It can also be shown [8] that Equation (C-56) has a unique solution for P when the product of any two eigenvalues of C(0) is not unity and this condition is satisfied when C(0) is stable, i.e., all its eigenvalues are of magnitude less than unity.

The procedure can now be summarized as follows. Choose Q as an identity matrix and solve for P from Equation (C-56). Check positive definiteness of P by employing Sylvester's theorem (Bellman [9]) according to which a matrix is positive definite if and only if all the principal minors of its determinant are positive. If P is positive definite, all that is now required is to solve for $S(t_{k+1})$ from Equation (C-51) and to show that $S(t_k)$ is a matrix with bounded coefficients that is nonsingular for all t_k in the interval (0,T). It then implies that the origin of (C-47) is asymptotically stable and that the maneuver is asymptotically stable in the small.

3. FINITE TIME STABILITY

Articulated vehicles execute the commonly encountered maneuvers such as cornering and lane change in a short and finite interval of time. However, the classical definition of stability in the sense of Lyapunov involves an infinite interval of time. Stability in the sense of Lyapunov is then appropriate for maneuvers that are executed over a relatively long interval of time, as for example a maneuver where the translation of the vehicle is described by a long circular arc, and it may not be very meaningful for the study of stability of lane change and cornering. Also, the classical definition of stability in the sense of Lyapunov does not admit persisting perturbations in the inputs, whereas the actual steering and braking inputs by the driver are nearby to the nominal inputs. Hence, it may be preferable to employ a more practical concept of stability called finite time stability. A motion is called finite time stable if it lies within prescribed bounds over a prespecified finite interval of time and is unstable if it does not.

It appears that Kamenkov [10] and Lebedev [11] were among the pioneers in introducing the concept of finite time stability in 1953 and 1954, respectively. Their papers appeared in Prikladaya Matematica i Mekhanica, whose English translation as Journal of Applied Mathematics and Mechanics dates back only to 1959. However, some description of their work is given by Hahn. [12]

The ideas were later extended by Chzhan-Sy-In [13,14] and by Dorato [15] for linear systems. Weiss and Infante [16,17,18] generalized the concepts and proved several new theorems. They employ Lyapunov-like functions whose properties differ significantly from those of classical Lyapunov functions. In particular, there is no requirement of sign definiteness on such functions or their time derivatives. Definitions of finite time stability are given in the following. It is recalled that the perturbations in the state variables and inputs about a nominal motion $\underline{q}^*(t)$ were obtained earlier and are given by Equation (C-6), namely $\dot{\underline{x}} = A(t)\underline{x} + B(t)\,\Delta\underline{u} + \underline{h}(\underline{x},\Delta\underline{u},t)$.

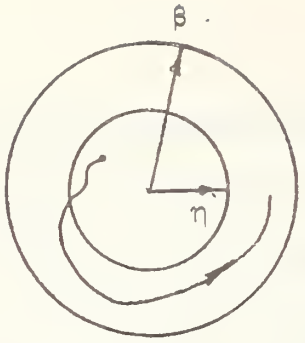
Definition 8. The nominal motion $\underline{q}^*(t)$ is said to be finite time stable under perturbed inputs with respect to $(\eta,\beta,\varepsilon,t_o,T)$, $\eta < \beta$, if, for any trajectory $\underline{x}(t)$ of Equation (C-6), the conditions $||\underline{x}(t_o)|| < \eta$ and $||\Delta\underline{u}(t)|| \leq \varepsilon$ for all $t_o \leq t \leq T$ imply $||\underline{x}(t)|| < \beta$ for all $t_o \leq t \leq T$.

The symbol $||\cdot||$ in general need not indicate a true norm but here it is used to represent the Euclidean norm. Time T is the terminal time of the maneuver. In case the inputs are not perturbed, then we let $\varepsilon = 0$ in this definition. The numbers η, β, ε, t_o, T are all specified a priori from physical considerations. A motion that is stable in the sense of Lyapunov may be unstable in the sense of Definition 8 and vice versa. The following two definitions are finite time counterparts of asymptotic stability under perturbed inputs. Since the word asymptotic is not meaningful in finite time stability, the word contractive is used instead.

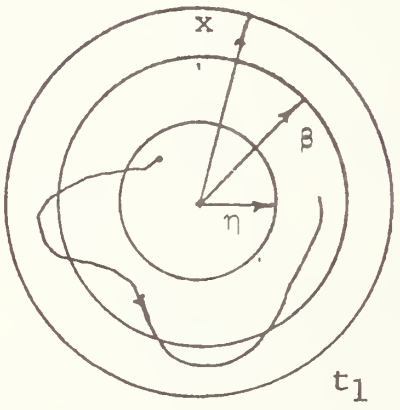
Definition 9. The nominal motion $\underline{q}^*(t)$ is quasi-contractively stable under perturbed inputs with respect to $(\eta,\beta,\gamma,\epsilon,t_o,T)$, $\eta \leq \beta < \gamma$, if, for any trajectory $\underline{x}(T)$ of Equation (C-6), the conditions $\left\| \underline{x}(to) \right\| < \eta$ and $||\Delta u(t)|| \leq \epsilon$ for all $t_o \leq t \leq T$ imply (1) stability under perturbed inputs with respect to $(\eta,\gamma,\epsilon,t_o,T)$; and (2) there exists t_1 where $t_o < t_1 < T$ such that $||\underline{x}(t)|| < \beta$ for all $t_1 < t < T$.

Definition 10. The nominal motion $\underline{q}^*(t)$ is contractively stable under perturbed inputs with respect to $(\eta,\beta,\gamma,\epsilon,t_oT)$, $\eta < \beta < \gamma$, if, for any trajectory $\underline{x}(t)$ of Equation (C-6), the conditions $||\underline{x}(t_o)|| < \eta$ and $||\Delta u(t)|| \leq \epsilon$ for all $t_o \leq t \leq T$ imply (1) stability under perturbed inputs with respect to $(\eta,\gamma,\epsilon,t_o,T)$ and (2) there exists t_1 where $t_o < t_1 < T$ such that $||\underline{x}(t)|| < \beta$ for all $t_1 < t < T$.

The three types of finite time stability definitions are illustrated in Figure C-4 in terms of trajectory behavior. Weiss and Infante [17] have presented three theorems, each dealing with Definitions 8, 9, and 10, respectively, where in Equation (C-6) the input enters the system equations linearly, i.e., in Equation (C-6), $\underline{h}(\underline{x},t)$ is not a function of Δu but only of $\underline{x}$ and t. Here, only their theorem dealing with Definition 8 is stated and employed.

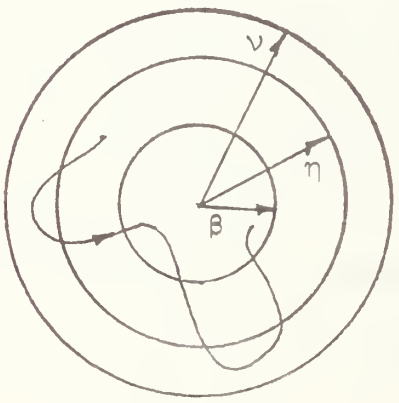


Stability definition 1.



Stability definition 2. $\left\| \underline{u}(t) \right\| \leq \epsilon$

$to \leq t \leq T$



Stability definition 3.

Figure C-4. Illustration of finite time stability definitions.

Theorem 6. Consider a special case of Equation (C-6) where the perturbation in the input enters the system equations linearly and the perturbation equations are described by

$$\dot{\underline{x}} = \underline{f}(\underline{x},t) + \Delta u \tag{C-57}$$

The nominal motion is finite time stable under perturbed inputs with respect to $(\eta,\beta,\varepsilon,t_o,T)$, $\eta < \beta$, if there exists a real valued function $V(\underline{x},t)$ and real valued functions $\varphi(t)$ and $\rho(t)$ integrable on $[t_o,T]$ such that

1. $||\text{grad } V(\underline{x},t)|| \leq \rho(t)$

2. $\dot{V}|_{\Delta \underline{u}} = 0$

and

3. $$\int_{t_1}^{t_2} [\varphi(t) + \varepsilon\rho(t)]\, dt \leq \min_{||\underline{x}|| = \beta} V(\underline{x},t_2) - \max_{||\underline{x}|| = \eta} V(\underline{x},t)$$

where $t_o \leq t_1 < t_2 \leq T$.

Conditions (1) and (2) are to hold for all $\underline{x}$ in the region $\eta \leq ||\underline{x}|| \leq \beta$ and $t_o \leq t \leq T$.

It should be noted that, unlike the case of Lyapunov stability, there is no requirement of sign definiteness or semidefiniteness on either V or $\dot{V}$. A slightly more general form of this theorem has been presented by Weiss [18] who shows that the property of uniform stability can be concluded from the

theorem. This point is also discussed by Heinen and Wu[19] It should be noted that Theorem 6 yields only sufficient conditions for finite time stability. Now consider the case where during a maneuver, the nominal inputs δ^* and D^* are not perturbed and hence $\Delta\underline{u} = \underline{0}$. In this case, the following theorem yields sufficient conditions for finite time stability.

Theorem 7. The nominal motion is finite time stable with respect to (η,β,t_o,T), $\eta < \beta$, if there exists a real valued function $V(\underline{x},t)$ and a real valued function $\varphi(t)$ integrable on $t_o \leq t \leq T$ such that

1. $\dot{V}(\underline{x},t) < \varphi(t)$ for all $\dot{\underline{x}}$ in the region $\eta \leq ||\underline{x}|| \leq \beta$ for all time $t_o \leq t \leq T$ and

2. $$\int_{t_1}^{t_2} \varphi(t)\, dt \leq \min_{||\underline{x}|| = \beta} V(\underline{x},t_2) - \max_{||\underline{x}|| = \eta} V(\underline{x},t_1)$$

where $t_o \leq t_1 < t_2 \leq T$.

In the investigation of finite time stability in the small, it is assumed that the perturbations about the nominal motion and in the nominal inputs are sufficiently small and the higher order terms in the Taylor series expansion, i.e., $\underline{h}(\underline{x},\Delta\underline{u},t)$ in Equation (C-6) are dropped. Hence, the stability of the linear approximation of the perturbation equations,

$$\dot{\underline{x}} = A(t)\underline{x} + B(t)\Delta\underline{u} \tag{C-58}$$

is investigated. In case the nominal inputs are not perturbed, then Δu is set equal to zero in Equation (C-58). The following quadratic V-functions were employed as trails in order to investigate finite time stability in the small of nonautonomous maneuvers.

Method 1. This method has been proposed by Dorato.[20] The V-function is chosen as

$$V = \underline{x}'\underline{x} = ||\underline{x}||^2 \tag{C-59}$$

Then $$\dot{V} = \dot{\underline{x}}'\underline{x} + \underline{x}'\dot{\underline{x}} \tag{C-60}$$

Substituting for $\dot{\underline{x}}$ in Equation (C-60) from Equation (C-58), it follows that

$$\begin{aligned}\dot{V} &= (A(t)\underline{x} + B(t)\Delta\underline{u})'\underline{x} + \underline{x}'(A(t)\underline{x} + B(t)\Delta\underline{u}) \\ &= \underline{x}'(A(t) + A'(t)\underline{x} + 2\ \Delta\underline{u}'\ B'(t)\underline{x}\end{aligned} \tag{C-61}$$

It can be shown that

$$\underline{x}'(A(t) + A'(t))\underline{x} \leq \Lambda_1(t)\ ||\underline{x}||^2 \tag{C-62}$$

where $\Lambda_1(t)$ is the largest eigenvalue of $[A(t) + A'(t)]$. Hence, we get

$$\dot{V}|_{\Delta\underline{u} = 0} \leq \Lambda_1(t)\ ||\underline{x}||^2 \leq \Lambda_1(t)\ \beta^2 \tag{C-63}$$

Then $\varphi(t)$ in Theorem 6 is chosen as

$$\varphi(t) = \Lambda_1(t)\ \beta^2 \tag{C-64}$$

It can also be shown that

$$||B'(t)\underline{x}|| \leq \Lambda_2^{1/2}(t)\ ||\underline{x}|| \tag{C-65}$$

where $\Lambda_2(t)$ is the largest eigenvalue of $B(t)B'(t)$. Also, for this V-function, we have

$$\min_{||\underline{x}|| = \beta} V(\underline{x},t_2) = \beta^2 \quad \text{and} \quad \max_{||\underline{x}|| = \eta} V(\underline{x},t_1) = \eta^2$$

Dorato suggests that $\rho(t)$ in Theorem 6 be chosen as

$$\rho(t) = \Lambda_1(t)\ \beta^2 + 2\varepsilon\Lambda_2^{1/2}(t)\ \beta \tag{C-66}$$

and then the condition for finite-time stability $(\eta,\beta,\varepsilon,t_o,T)$, $\eta < \beta$, becomes

$$\int_{t_1}^{t_2} [\Lambda_1(t)\ \beta^2 + \varepsilon\ \Lambda_2^{1/2}(t)\ \beta]\ dt \leq \frac{1}{2}\ (\beta^2 - \eta^2) \tag{C-67}$$

for $t_o \leq t_1 < t_2 \leq T$.

Consider the case where during the maneuver, the nominal inputs are not perturbed, i.e., $\Delta\underline{u} = 0$ in Equation (C-58). Choosing $\varphi(t)$ as in Equation (C-64), the condition for finite time stability (η,β,t_o,T), $\eta < \beta$, according to Theorem 7 is given by

$$\int_{t_1}^{t_2} \Lambda_1(t)\beta^2 \, dt \leq \beta^2 - \eta^2$$

$$\text{i.e., } \int_{t_1}^{t_2} \Lambda_1(t) \, dt \leq 1 - \frac{\eta^2}{\beta^2} \quad t_o \leq t_1 < t_2 \leq T \tag{C-68}$$

For the case $\Delta \underline{u} = 0$ in Equation (C-58), Dorato [15] has obtained a sufficient condition for finite time stability which appears to be less conservative than the condition of Equation (C-68). Choosing V-function as in Equation (C-59) and noting the inequality (C-63), we obtain

$$\frac{d}{dt}(\underline{x}'\underline{x}) \leq \Lambda_1(t) \, \underline{x}'\underline{x} \tag{C-69}$$

Integrating both sides of Equation (C-69), it follows that

$$\underline{x}'\underline{x} \leq \underline{x}'(t_o) \, \underline{x}(t_o) \, e^{\int_{to}^{t} \Lambda_1(t) \, dt} \leq \eta^2 e^{\int_{to}^{t} \Lambda_1(t) dt} \tag{C-70}$$

A sufficient condition to guarantee that $\underline{x}'(t) \, \underline{x}(t) = \|x(t)\|^2 < \beta^2$ is that $e^{\int_{to}^{t} \Lambda_1(t) \, dt} \leq \frac{\beta^2}{\eta^2}$ or equivalently

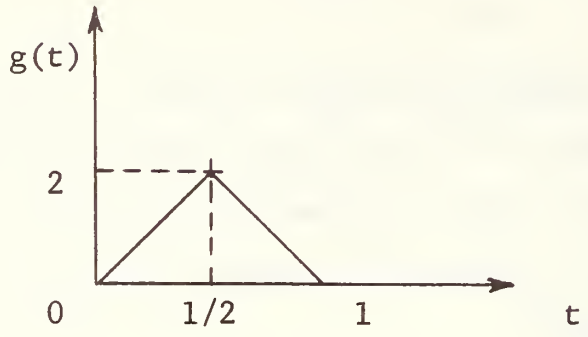
$$\int_{t_1}^{t_2} \Lambda_1(t) \, dt \leq \ln \frac{\beta^2}{\eta^2} \tag{C-71}$$

$t_o \leq t_1 < t_2 \leq T$.

Consider the example given by Dorato[15]. The perturbation vector is described by $\dot{\underline{x}} = A(t) \, \underline{x}$ where,

$$A(t) = \begin{bmatrix} 0 & 1 \\ -1-g(t) & 0 \end{bmatrix} \text{ and } \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \text{(C-72)}$$

Choose $\eta = 1$ and $\ln \frac{\beta^2}{\eta^2} = 1$, i.e., $\beta^2 = e$, $t_o = 0$, $T = 1$ and $g(t)$ as shown in the sketch below.



Now $A'(t) + A(t)$ is given by

$$A'(t) + A(t) = \begin{bmatrix} 0 & -g(t) \\ -g(t) & 0 \end{bmatrix} \qquad \text{(C-73)}$$

and its eigenvalues are $\lambda_1(t) = g(t)$ and $\lambda_2(t) = -g(t)$. Hence, $\Lambda_1 = |g(t)|$ and $\int_0^1 |g(t)|\, dt = 1$ whereas $\ln \frac{\beta^2}{\eta^2} = 1$.

Hence, it follows that condition (C-71) is satisfied and the system is finite time stable. However, inequality (C-68) is not satisifed in this example and therefore it appears that condition (C-71) is less conservative than condition (C-68). It should be noted that these conditions are only sufficient conditions for stability and when they are not satisfied, one cannot conclude that the motion is finite time unstable.

This method was employed to investigate finite time stability of nonautonomous maneuvers. The results obtained are very conservative and not useful. The choice of the square of the Euclidean norm as the V-function is not a good choice. This becomes apparent when autonomous maneuvers are considered where the nominal inputs are not perturbed and the perturbations in the state variables are described by Equation (C-26). The choice of the square of the Euclidean norm as the V-function implies that in Equation (C-27), P is chosen as the indentity matrix. Now, when P is chosen as the identity matrix in Equation (C-30), it does not follow that Q obtained from the solution of this equation will be positive definite when A is asymptotically stable in the sense of Lyapunov. Consider, for example, the case $\dot{\underline{x}} = A\underline{x}$

$$\text{where} \quad A = \begin{bmatrix} 0 & 1 \\ -48 & -14 \end{bmatrix} \qquad \text{and} \quad \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{C-74}$$

The eigenvalues of A are given by $\lambda_1 = -6$ and $\lambda_2 = -8$ and hence A is asymptotically stable in the sense of Lyapunov. Now, we get

$$A' + A = \begin{bmatrix} 0 & -47 \\ -47 & -28 \end{bmatrix} \tag{C-75}$$

which is an indefinite matrix with eigenvalues $\lambda_1 = -63$ and $\lambda_2 = 35$. Hence, $\Lambda_1 = 35$ and the results obtained either from inequality of Equation (C-68) or that of Equation (C-71) are

not good. Garrard [21] has suggested that the logarithm of the square of the Euclidean norm can be used as a V-function, i.e.,

$$V = \ln \left[\frac{||\underline{x}||}{\eta} \right]^2 \tag{C-76}$$

This choice of V-function, however, yields an inequality similar to that of Equation (C-71).

Method 2. A quadratic function is chosen as a V-function i.e.,

$$V(\underline{x},t) = \underline{x}' \, P(t) \underline{x} \tag{C-77}$$

where P(t) is a symmetric time-varying matrix. When the inputs are not perturbed, i.e., $\Delta \underline{u} = 0$ in Equation (C-58), $\dot{V}$ is given as in Equation (C-37) by

$$\dot{V}(\underline{x},t) = \underline{x}' \left[- Q + \dot{P}(t) \right] \underline{x} \tag{C-78}$$

where

$$A'(t) \, P(t) + P(t) \, A(t) = - Q \tag{C-79}$$

Now, Q is selected as a positive definite constant matrix and P(t) is obtained from the solution of Equation (C-79). Let $\Lambda_3(t)$ be the maximum eigenvalue of the matrix $(- Q + \dot{P}(t))$. It follows that

$$\dot{V}(\underline{x},t) \leq \Lambda_3(t) \, ||\underline{x}||^2 \tag{C-80}$$

Choosing $\varphi(t) = \Lambda_3(t)\ \beta^2$, condition of Theorem 7 becomes

$$\int_{t_1}^{t_2} \Lambda_3(t)\ \beta^2\ dt \leq \min_{||\underline{x}|| = \beta} V(\underline{x},t_2) - \max_{||\underline{x}|| = \eta} V(\underline{x},t_1) \qquad \text{(C-81)}$$

Since $V(\underline{x},t_2) \leq \lambda_{min}(t_2)\ ||\underline{x}||^2$ and $V(\underline{x},t_1) \geq \lambda_{max}(t_1)\ ||\underline{x}||^2$, where $\lambda_{min}(t_2)$ is the minimum eigenvalue of $P(t_2)$ and $\lambda_{max}(t_1)$ is the maximum eigenvalue of $P(t_1)$, inequality (C-81) can be expressed as

$$\int_{t_1}^{t_2} \Lambda_3(t)\ \beta^2\ dt \leq \lambda_{min}(t_2)\ \beta^2 - \lambda_{max}(t_1)\ \eta^2 \qquad \text{(C-82)}$$

The disadvantage of this method is that inequality (C-82) is very conservative. Furthermore, as discussed earlier, there are difficulties in obtaining a good approximation to $\dot{P}(t)$ without excessive computations.

4. LYAPUNOV STABILITY IN THE LARGE OF AUTONOMOUS MANEUVERS

Lyapunov stability in the small of autonomous maneuvers has been discussed in an earlier section by assuming that the perturbations in the state variables are sufficiently small during the nominal motion and that the nominal inputs are not perturbed.

In this section, Lyapunov stability in the large of autonomous maneuvers is discussed. The nominal inputs are not perturbed but it is not assumed that the perturbations in the state variables are small. Hence, the perturbations are described by the nonlinear autonomous Equation (C-11).

The objective is to determine the size of the region of asymptotic stability, also known as the domain of attraction, by application of the second method of Lyapunov. For this autonomous case, Theorem 4 can be simplified as shown below.

Theorem 8 . Let $V(\underline{x})$ be a scalar function with continuous first partial derivatives. Let Ω_k designate a region around the origin where $V(\underline{x}) < k$. Assume that region Ω_k is bounded and that within Ω_k

1. $V(\underline{x}) > 0$ for $\underline{x} \neq 0$, i.e., $V(\underline{x})$ is positive definite
2. $\dot{V}(\underline{x}) \leq 0$, i.e., $\dot{V}(\underline{x})$ is negative semidefinite.

If the system trajectory cannot stay forever at the points where $\dot{V} = 0$ within Ω_k other than at the origin, then the origin $\underline{x} = 0$ is asymptotically stable and every solution in Ω_k tends to the origin as $t \rightarrow \infty$.

A proof of this theorem is given by LaSalle and Lefschetz [3]. This theorem permits the use of a V-function that is positive definite with $\dot{V}$ being negative semidefinite, and yet conclusions can still be reached concerning the asymptotic stability of the origin and Ω_k is an estimate of the domain of attraction around the origin. Thus, it is clear that if an appropriate V-function

can be constructed, then Ω_k will be an estimate of the domain of attraction. It is also clear that this estimate is directly dependent on the choice of the Lyapunov function. A large number of methods have been investigated in the past for the generation of Lyapunov functions. Most of the methods are not very general and are restricted to very special system configurations such as a single nonlinearity of a special kind. The method of Lucre', which is discussed in detail by Lefschetz [22], is applicable to a very restricted class of feedback systems with a single nonlinearity in the loop. In the variable gradient method developed by Schultz and Gibson [23,24], the gradient of V is assumed, $\dot{V}$ is obtained and constrained to be at least negative semidefinite, and then the V-function is obtained by line integration.

The variable gradient method presents a great number of alternatives in choosing a V-function and it is not known a priori which alternative would yield a largest domain of attraction. Its serious disadvantage is that the V-function generated is not usually a quadratic form. Sylvester's theorem [9] is applicable to the determination of sign definiteness of quadratic functions but for general nonquadratic functions, such techniques are not available.

The stability of artificial satellites has led to a number of studies of the stability of steady motion of mechanical system of rigid bodies which may also include some flexible parts. Pringle [25], and Meirovitch [26,27] and others have studied this

problem. Firstly, an expression is obtained for the Hamiltonian function in generalized coordinates and generalized momenta coordinates. For a steady state nominal motion, the coordinates have constant values. The Hamiltonian is perturbed about this steady state motion and the Hamiltonian in the perturbations is used as a Lyapunov function. Stability conditions are obtained by application of Theorem 8 by restricting the Hamiltonian in the perturbations to be a positive definite function and its time rate of change to be a negative semidefinite function. In the case of articulated vehicles, the Hamiltonian and its time rate of change are not quadratic functions. The use of the Hamiltonian as a Lyapunov function does not yield any useful results due to the lack of a criterion to determine the sign definiteness of general nonquadratic functions.

The foregoing techniques and other methods reported in the literature for the construction of Lyapunov functions were examined in order to investigate their suitability for the study of stability of maneuvers of articulated vehicles. The conclusion was that the methods have various shortcomings for complex problems. For engineering problems, closed-form expressions defining the stability boundary can be rarely obtained. What is needed is an effective computational procedure for estimating the domain of attraction. For the study of stability in the large of maneuvers of articulated vehicles, estimation of the domain of attraction is the only suitable technique other than simulation. The Zubov method [28] and the optimal quadratic estimation method [29]

are the two methods which are amenable to machine computation for estimating the domain of attraction and they are discussed in the following.

4.1 Zubov's Method

Let $\varphi(x_1,\ldots,x_n)$ be a given positive definite quadratic function and $V(x_1,\ldots,x_n)$, be an as yet unknown function. Consider the equation

$$\dot{V}(x_1,\ldots,x_n) = -\varphi(x_1,\ldots,x_n)\,[1 - V(x_1,\ldots,x_n)] \qquad \text{(C-83)}$$

Since $\dot{V}$ evaluated along the trajectory of Equation (C-11) may be written as

$$\begin{aligned}\dot{V} &= \frac{\partial V}{\partial x_1}\dot{x}_1 + \ldots + \frac{\partial V}{\partial x_n}\dot{x}_n \\ &= \frac{\partial V}{\partial x_1}(a_{11}x_1 + \ldots + a_{1n}x_n + h_1(\underline{x})) + \ldots \\ &\quad + \frac{\partial V}{\partial x_n}(a_{n1}x_1 + \ldots + a_{nn}x_n + h_n(\underline{x}))\end{aligned} \qquad \text{(C-84)}$$

Equation (C-83) after substitution for $\dot{V}$ from Equation (C-84) yields the partial differential equation

$$\begin{aligned}&\frac{\partial V}{\partial x_1}(a_{11}x_1 + \ldots + a_{1n}x_n + h_1(\underline{x})) + \ldots + \frac{\partial V}{\partial x_n}(a_{n1}x_1 + \ldots \\ &\quad + a_{nn}x_n + h_n(\underline{x})) \\ &= -\varphi(x_1,\ldots,x_n)[1 - V(x_1,\ldots,x_n)]\end{aligned} \qquad \text{(C-85)}$$

In Equation (C-84), a_{ij} represents the i^{th} row of the A matrix. It is assumed that Equation (C-11) is asymptotically stable in the small, i.e., the linear approximation $\dot{\underline{x}} = A\underline{x}$ is asymptotically stable which implies that all eigenvalues of A have negative real parts. This assures that the equilibrium solution of Equation (C-11) has a nonempty domain of attraction. If $0 < V(\underline{x}) < 1$ for $\underline{x} = 0$, then $V(\underline{x})$ is positive definite and from Equation (C-85), $\dot{V}$ is negative definite. It is enough to choose $\varphi(x_1,\ldots,x_n)$ to be a positive semidefinite function and then $\dot{V}$ will be negative semidefinite for $0 < V(\underline{x}) < 1$. In this case, according to Theorem 8, for asymptotic stability one has to show that the system trajectory cannot stay at the points where $\dot{V} = 0$ within the domain of attraction other than at the origin. Now, $V(\underline{x})$ which is the solution of the partial differential Equation (C-85) is a Lyapunov function.

It is clear from the above discussion that the Lyapunov function $V(\underline{x})$ is obtained from the solution of the partial differential Equation (C-85) and $V(\underline{x}) = 1$ defines the boundary of the domain of attraction. It is also possible to reformulate the theory in terms of an alternate Lyapunov function $W(\underline{x})$ which is related to the Lyapunov function $V(\underline{x})$ by the substitution

$$W(\underline{x}) = - \ln(1 - V(\underline{x})) \qquad (C\text{-}86)$$

with the restriction that $0 \leq V(\underline{x}) < 1$. Since $\frac{\partial V}{\partial x_i} = e^{-W} \frac{\partial W}{\partial x_i}$, the partial differential Equation (C-85) is now transformed into

$$\frac{\partial W}{\partial x_1}(a_{11}x_1 + \ldots + a_{1n}x_n + h_1(\underline{x})) + \ldots + \frac{\partial W}{\partial x_n}(a_{n1}x_1 + \ldots + a_{nn}x_n + h_n(\underline{x})) = -\varphi(x_1,\ldots,x_n) \tag{C-87}$$

Equivalently, the Lyapunov function $W(\underline{x})$ is obtained from the solution of the partial differential Equation (C-87) and $W = \infty$ defines the boundary of the domain of attraction and depends on the choice of the function $\varphi(x_1,\ldots,x_n)$. Furthermore, with the exception of simple cases, Equation (C-85) or (C-87) cannot be solved for $V(\underline{x})$ or $W(\underline{x})$ exactly in closed form. Two simple examples are given in the following where the solution of the equations can be obtained easily.

Example (Zubov [28])

Let Equation (C-11) be described by

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 + + x_1(x_1^2 + x_2^2)\\ \dot{x}_2 &= -x_1 - x_2 + x_2(x_1^2 + x_2^2)\end{aligned} \tag{C-88}$$

Assume a function $\varphi(x_1,x_2) = 2(x_1^2 + x_2^2)$. Then, the partial differential Equation (C-85) becomes

$$\frac{\partial V}{\partial x_1}(-x_1 + x_2 + x_1^3 + x_1x_2^2) + \frac{\partial V}{\partial x_2}(-x_1 + x_2^3 - x_2 + x_1^2x_2) = -2(x_1^2 + x_2^2)[1 - V] \tag{C-89}$$

The solution of Equation (C-89) can be obtained by elementary integration technique as

$$V(x_1,x_2) = x_1^2 + x_2^2 \tag{C-90}$$

and the boundary of the domain of asymptotic stability is given by $x_1^2 + x_2^2 = 1$. This boundary is a circle as shown in Figure C-5.

Example (Hahn [12])

Let Equation (C-11) be described by

$$\begin{aligned} \dot{x}_1 &= - x_1 + 2x_1^2x_2 \\ \dot{x}_2 &= - x_2 \end{aligned} \tag{C-91}$$

Assuming a function $\varphi(x_1,x_2) = 2(x_1^2 + x_2^2)$, the modified Lyapunov function $W(\underline{x})$ is found from the solution of the partial differential Equation (C-87) which becomes

$$\frac{\partial W}{\partial x_1}(- x_1 + 2x_1^2x_2) + \frac{\partial W}{\partial x_2}(-x_2) = -2(x_1^2 + x_2^2) \tag{C-92}$$

Equation (C-92) can also be solved by elementary integration technique to yield

$$W(\underline{x}) = x_2^2 + \frac{x_1^2}{(1 - x_1x_2)} \tag{C-93}$$

The boundary of the domain of asymptotic stability is given by $W = \infty$, i.e., $x_1x_2 = 1$. This domain is shown in Figure C-6.

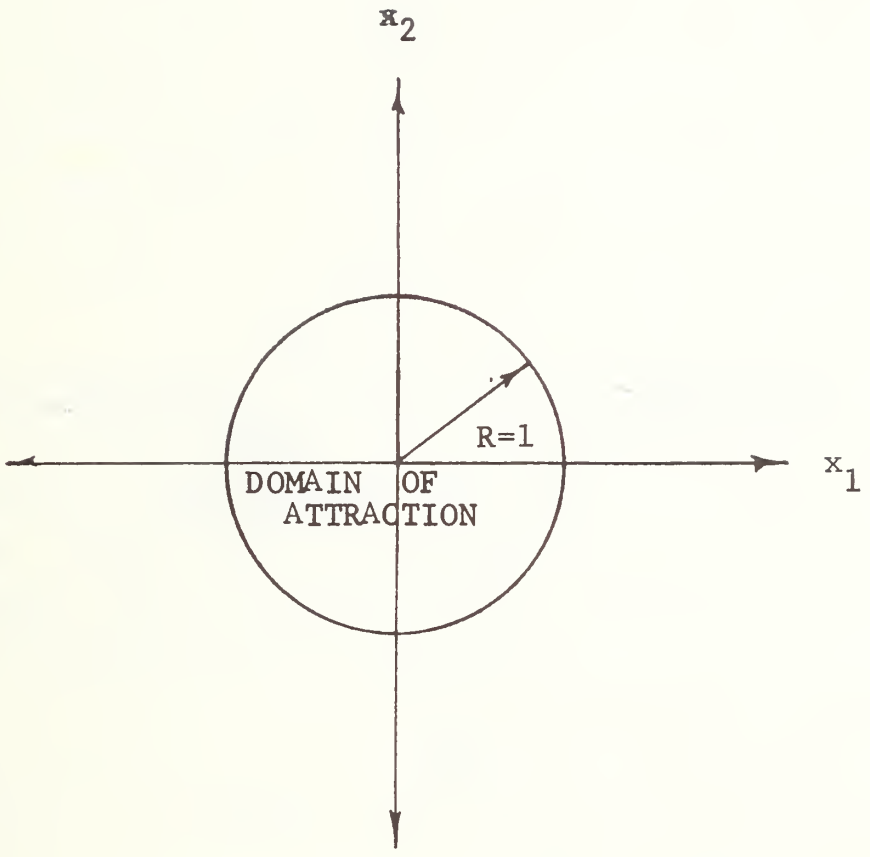


Figure C-5. Domain of attraction.

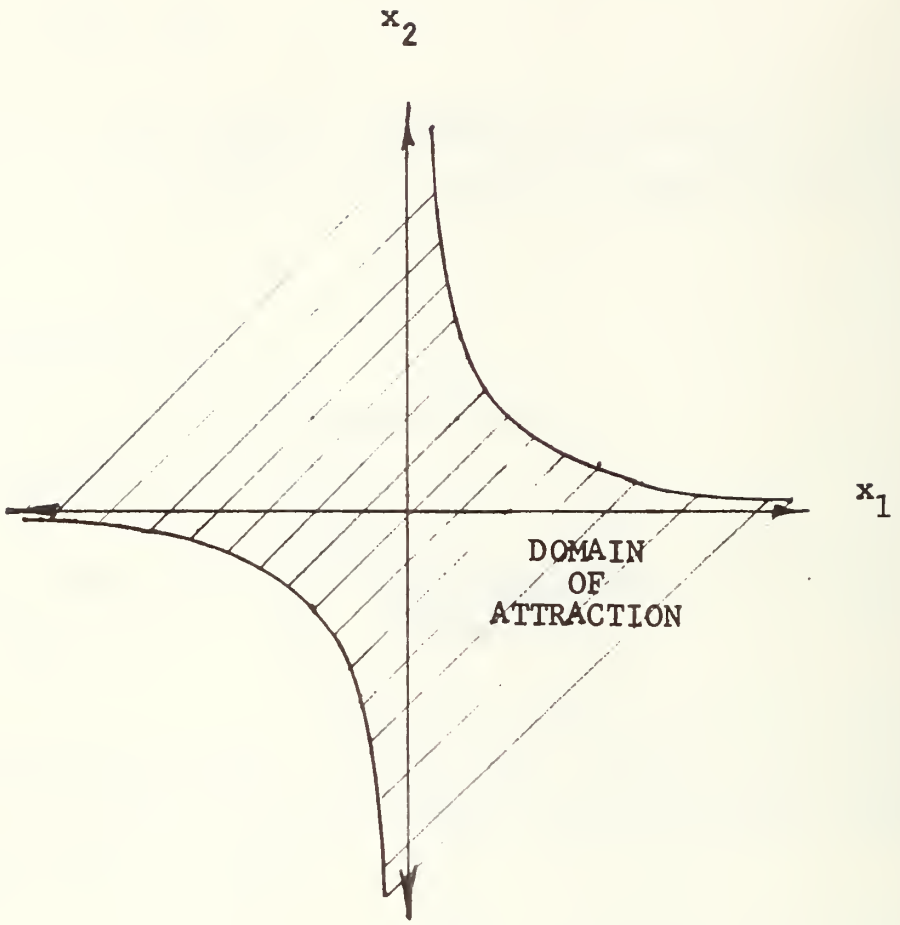


Figure C-6. Domain of attraction.

For articulated vehicles, Equation (C-85) or (C-87) cannot be solved for $V(\underline{x})$ or $W(\underline{x})$ exactly in closed form. The next best thing is to be able to approximate the exact solution. But since it is the domain of asymptotic stability which is of interest, the approximate solution should at the same time insure that the domain indicated is actually a part of the domain of asymptotic stability. Furthermore, it should permit the possibility of approaching as closely as desired to the exact stability domain. For this reason, $V(\underline{x})$ is expanded in a series form as

$$V(x_1),\ldots,x_n) = V_2(\underline{x}) + V_3(\underline{x}) + \ldots + V_m(\underline{x}) + \ldots \quad \text{(C-94)}$$

where $V_m(\underline{x})$ is a homogeneous form relative to $x_1,\ldots,x_n$ of the m^{th} power. Hence, $V_2(\underline{x})$ is a quadratic function of $x_1,\ldots,x_n$. To determine $V_m(\underline{x})$, Equation (C-94) is substituted into Equation (C-85), yielding the system of recurrence relations

$$\begin{aligned}
&\frac{\partial V_2}{\partial x_1}(a_{11}x_1 + \ldots + a_{1n}x_n) + \ldots + \frac{\partial V_2}{\partial x_n}(a_{n1}x_1 + \ldots + a_{nn}x_n) \\
&\qquad = -\varphi_2(x_1,\ldots,x_n) \\
&\frac{\partial V_m}{\partial x_1}(a_{11}x_1 + \ldots + a_{1n}x_n) + \ldots + \frac{\partial V_m}{\partial x_n}(a_{n1}x_1 + \ldots + a_{nn}x_n) \\
&\qquad = R_m(x_1,\ldots,x_n)
\end{aligned} \quad \text{(C-95)}$$

where $\varphi_2(x_1,\ldots,x_n)$ is a quadratic form of the variables $x_1,\ldots,x_n$ and $R_m(x_1,\ldots,x_n)$ is a function of the m^{th} degree in $x_1,\ldots,x_n$ which is known if each of the $V_2,V_3,\ldots,V_{m-1}$ have already been determined. The quadratic function V_2 can be chosen as $\underline{x}'P\underline{x}$ as in Equation (C-27) where P is obtained from the solution of $A'P + PA = -Q$ after choosing Q to be a positive definite matrix. We note that V_2 is positive definite since the origin is asymptotically stable in the small. The sequence in Equation (C-95) can then be found recursively. Let the series of Equation (C-94) be truncated by including terms up to the m^{th} order as

$$V^{(m)}(x_1,\ldots,x_n) = V_2(\underline{x}) + V_3(\underline{x}) + \ldots + V_m(\underline{x}) \qquad \text{(C-96)}$$

Let

$S^{(m)}(x_1,x_2,\ldots,x_n)$ = all points $(x_1,\ldots,x_n)$ other than the origin on which $\dot{V}^{(m)}(x_1,\ldots,x_n) = 0$

$C_{min}^{(m)} = \min [V^{(m)}(x_1,\ldots,x_n)$ on $S^{(m)}(x_1,\ldots,x_n)]$

$\Omega^{(m)}$ = all points $(x_1,\ldots,x_n)$ for which $V^{(m)}(x_1,\ldots,x_n) \leq C_{min}^{(m)}$

Zubov [28] has proved that the surface $V^{(m)}(x_1,\ldots,x_n) = C_{min}^{(m)}$ is wholly contained in the domain of attraction Ω. If Ω is bounded, all $V^{(m)}(x_1,\ldots,x_n) = C_{min}^{(m)}$ are bounded irrespective of the form of admissible $\varphi(x_1,\ldots,x_n)$. If any $V^{(m)}(x_1,\ldots,x_n) = C_{min}^{(m)}$ is unbounded for some admissible $\varphi(\underline{x})$, the stability domain is unbounded. Hence, $\Omega^{(m)}$ is a conservative estimate of the actual domain of attraction Ω.

The objective of the series solution for V is to be able to come as close to the stability boundary as desired. It might be expected that as more terms are included in the truncated

series expansion of V, the closer one would come to the actual stability boundary. But the experience of Margolis and Vogt, [30] Rodden [31], Julich [32], and Geiss et al. [29], indicates that it is not necessarily true that $\Omega^{(m)}$ is contained in $\Omega^{(m+1)}$. The method converges poorly and the quadratic estimate, i.e., retaining only V_2 in Equation (C-96), is often better than much higher order estimates.

It appears that Margolis and Vogt [30] were the first to employ a digital computation with the Zubov's method to integrate the recurrence relations of Equation (C-95), beginning with the quadratic terms. The series is truncated as in Equation (C-96). The set of points other than the origin is found for which $\dot{V}^{(m)} = 0$ and this set is designated by $S^{(m)}$. The minimum value of $V^{(m)}$ on $S^{(m)}$ is designated $C_{min}^{(m)}$ and $V^{(m)} = C_{min}^{(m)}$ is an approximation to the stability boundary. Their computational procedure is however limited only to second order systems. In their example, they consider the Van der Pol equation which in the state variable form is represented by

$$\begin{aligned} \dot{x}_1 &= x_2 - \varepsilon\left(x_1 - \frac{x_1^3}{3}\right) \\ \dot{x}_2 &= - x_1 \end{aligned} \qquad \text{(C-97)}$$

With the choice $\varphi(x_1,x_2) = x_1^2$ i.e, a positive semidefinite quadratic form, and $\varepsilon = 0.7$, their results are shown in Figure C-7. When V is approximate by the quadratic V_2, the stability boundary is a circle of radius $\sqrt{3}$ (not shown in Figure C-7). The n = 6 approximation is completely enclosed by n = 2 approximation.

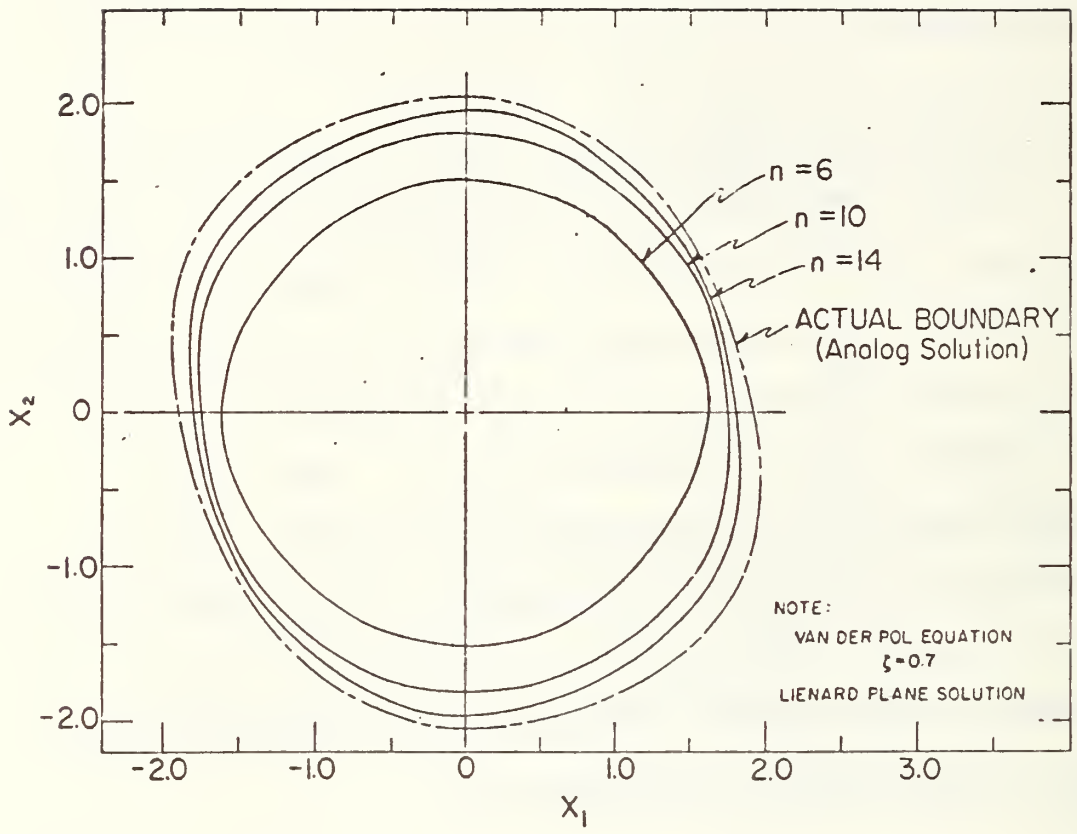


Figure C-7. Stability boundary as given by approximate solutions to Zubov equation.

On the other hand, n = 10 approximation completely enclosed both the n = 2 and n = 6 approximations and is, in turn, completely enclosed by n = 14 approximation. Thus, a surprisingly high order of approximation must be used in order to improve upon the quadratic results. However, it should be noted that their choice of the φ functions yields the maximum domain when V is approximated by quadratic function, V_2, and for higher order systems the optimal choice of the φ function is not a trivial problem.

The results of Margolis and Vogt, while limited to second order systems, have generated some interest in this problem and have served as basis for further study by others. The following discussion is useful to understand these methods. When the estimated region is finite, there is at least one surface in the state space on which $\dot{V} = 0$ and across which $\dot{V}$ changes sign. The largest constant c for which V = c is an estimated stability boundary, is the least upper bound on that surface. A subspace in which $\dot{V} = 0$ may be allowed in a stability region if it can be shown that no trajectory of the system equations lies wholly within it except at the origin. Figure C-8 shows a geometric portrait in the two dimensional case. The closed contours V = constant define stability regions when $\dot{V}$ is less than or equal to zero within them. The largest possible region is that contour which is just tangent to the $\dot{V} = 0$ surface. Global stability is the special case when $\dot{V} < 0$ and does not change sign throughout the state space. The following methods make use of this geometric interpretation.

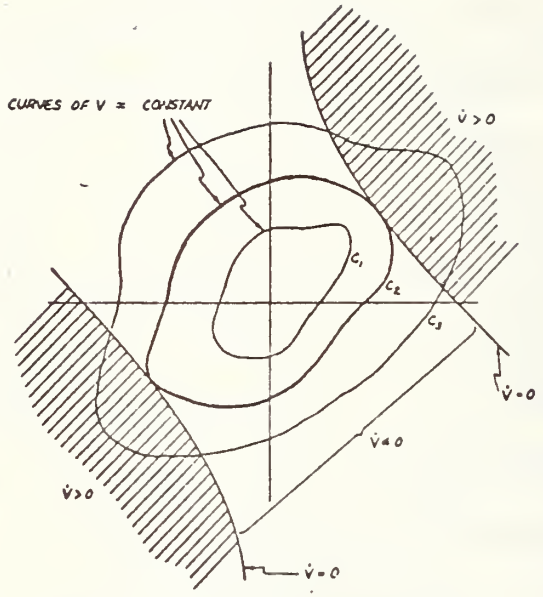


Figure C-8. A geometric portrait of the two-dimensional case.

Method 1.- Tangent Search. This method has been proposed by Rodden [31] and is a step-by-step method to seek the point of tangency of the V and $\dot{V} = 0$ surfaces. Rodden's computer algorithm makes use of the fact that at the point of tangency, the gradients of V and $\dot{V}$ are in the same direction, since at this point both the vectors V and $\dot{V}$ are increasing in the same direction. This point is clarified in Figure C-9 . In this figure, point P_1 is a tangency point whereas point P_2 is not. The disadvantage of this method is that it requires excessive computer time because it requires continual search steps to relocate the contour $\dot{V} = 0$.

Method 2 - Functional Minimization. This method has been proposed by Julich [32] and makes use of the fact that if the minimum of V subject to the constraint that $\dot{V} = 0$, the origin being excluded, is c, than $V < c$ is an estimate of the region of asymptotic stability. The constraint that $\dot{V} = 0$ is imbedded in the minimization problem by employing a penalty function. Hence, the function $V + r\dot{V}^2$ is minimized where r is a penalty function and $\dot{V}^2$ is the penalty for straying from $\dot{V} = 0$. It is known that the minimum of V converges to the minimum of $V + r\dot{V}^2$ as $r \rightarrow \infty$. A high penalty coefficient insures the accuracy of the minimum, but a high penalty coefficient during the initial steps, where $\dot{V}$ is not small, may lead to overflow in the digital computer. A minimization algorithm based on the method of Davidon or the method of Fletcher and Powell can be employed.

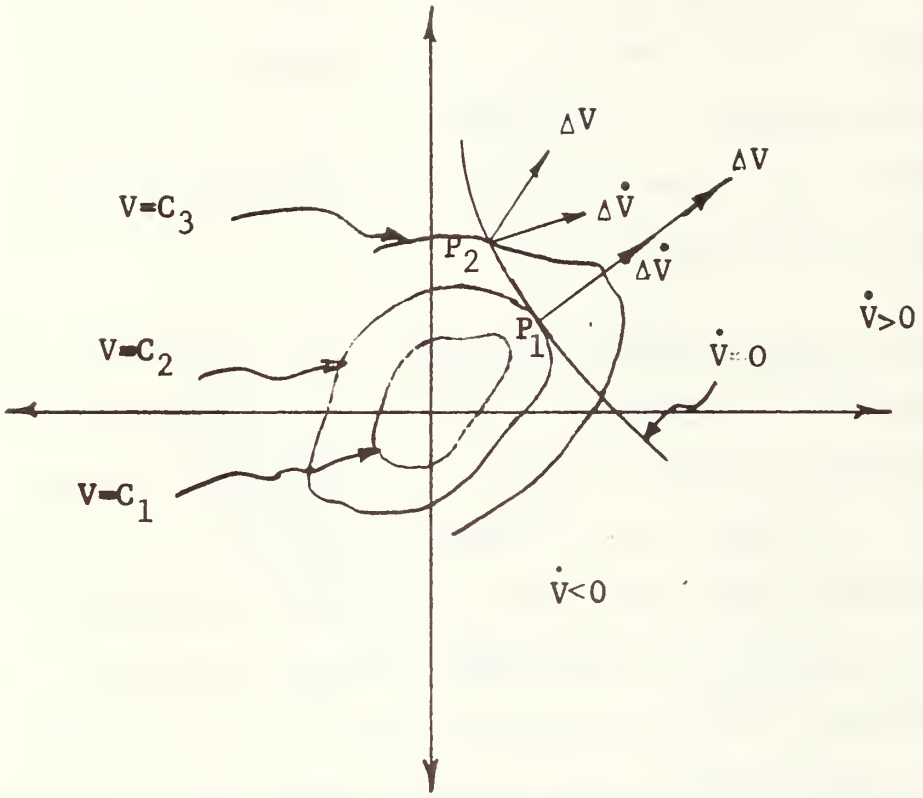


Figure C-9. Search for the tangent point.

The global minimum of V subject to the constraint that $\dot{V} = 0$ is zero which is to be excluded as trivial. Hence, the minimum sought is a local minimum nearest in value to the global minimum which is zero. If this minimum is c, then $V < c$ is an estimate of the domain of attraction. The disadvantage of this method is that there is no general search technique that can find the smallest local minimum from among the set of local minimums. It has also been suggested that the function $V + r_1\dot{V}^2 + r_2\rho(\underline{x})$ be minimized where $\dot{V}^2$ is the penalty for straying from $\dot{V} = 0$ and $\rho(\underline{x})$ is the penalty for nearing $\underline{x} = \underline{0}$. Here, r_1 and r_2 are to be chosen large enough to force the search to the constraint and small enough not to mask the minimization of V. Thus, a trade off is required and this delicate balance cannot be determined a priori.

Method 3 - Random Search. This method has been proposed by Geis et al., [29] and has been used by them for the solution of nontrivial problems. Its disadvantage is that no deterministic information is available as to how close one is to the solution. Thus, large numbers of random trials are required to develop confidence in the results, and the computer time is excessive. Geis et al., [29] analyzed a nine dimensional system by searching for a best quadratic Lyapunov function. Random search was employed for the minimization. They found that for one set of parameters, 4 hours of IBM 360/95 computer time was required.

Method 4. This method utilizes the fact that at the point of tangency of the V and $\dot{V} = 0$ surfaces, the gradients of V and $\dot{V}$ are in the same direction as shown in Figure C-9. Hence, we get

$$\nabla V = k \nabla \dot{V} \tag{C-98}$$

where k is an unknown constant. In addition, the equation $\dot{V} = 0$ has to be satisfied. For the tractor semitrailer with five state variables, the equations can be written in the form

$$\begin{aligned}
&\frac{\partial V}{\partial x_1} = k \frac{\partial \dot{V}}{\partial x_1} \\
&\frac{\partial V}{\partial x_2} = k \frac{\partial \dot{V}}{\partial x_2} \\
&\quad\vdots \qquad\quad \vdots \\
&\frac{\partial V}{\partial x_5} = k \frac{\partial \dot{V}}{\partial x_5} \\
&\dot{V} = 0
\end{aligned} \tag{C-99}$$

We note that when $V = \underline{x}' P \underline{x}$, we get $\nabla V = 2P\underline{x}$ and $\nabla \dot{V} = -2Q\underline{x} + 2P\underline{h}(\underline{x}) + 2(\frac{\partial \underline{g}}{\partial \underline{x}})' P\underline{x}$.

Equation (C-99) has six nonlinear algebraic equations in the six unknowns $x_1,\ldots,x_5,k$. A solution of this set of six nonlinear simultaneous algebraic equations can be obtained with a digital computer. If the minimum value of V evaluated at this solution point is c, then $V < c$ is an estimate of the domain of attraction.

4.2 Quadratic Estimation

It has been observed earlier that the series method of Zubov for generating a Lyapunov function converges poorly and the quadratic estimate it produces is often better than much higher order estimates. These problems with the Zubov method led Geis et al.[29], to the concept of developing an optimal quadratic estimate of the domain of attraction. A quadratic estimate yields a hyperellipsoid for the domain of attraction which can be visualized much more readily than the hypervolumes of the higher order estimates. This is a distinct advantage. A quadratic function is chosen as a Lyapunov function, i.e.,

$$V(\underline{x}) = \underline{x}'P\underline{x} \tag{C-100}$$

The time derivative of $V(\underline{x})$ along the trajectory of Equation (C-11) is given by

$$\dot{V}(\underline{x}) = \underline{x}'(A'P + PA)\underline{x} + 2\underline{x}'P\underline{h}(\underline{x}) \tag{C-101}$$

Let

$$A'P + PA = - Q \tag{C-102}$$

If Q is chosen as positive definite, then P is also positive definite since it is assumed that the origin of (C-11) is asymptotically stable in the small. Also $\dot{V}$ will be negative definite at least in a sufficiently small region around the origin since $\underline{h}(\underline{x})$ consists of higher order terms in $\underline{x}$. Since A is a stable matrix, every

positive definite Q will result in a positive definite P. Theorem 8 requires that $\dot{V}(\underline{x})$ be negative within Ω_k, exclusive of the origin. The selection of the largest such Ω_k is equivalent to finding the minimum value of $V(\underline{x})$ on the surface $\dot{V}(\underline{x}) = 0$, i.e., $c = \min V(\underline{x})$ subject to the constraint $\dot{V}(\underline{x}) = \underline{0}$, $\underline{x} = 0$.

Then, $\underline{x}'P\underline{x} = c$ is the hyperellipsoid which is an estimate of the domain of attraction Ω. This domain is a function of P or equivalently of Q and it is desired to find the optimal value of this domain. The optimal estimate is given by that choice of Q which maximizes the volume of Ω. Here, Q is chosen as the variable since P is then guaranteed to be positive definite whereas the reverse does not hold. The volume of Ω is proportional to

$$J(Q) = \left[\frac{c^n}{\prod_{i=1}^{n} \lambda_i(P)} \right]^{1/2} = \left[\frac{c^n}{\det P} \right]^{1/2} \qquad \text{(C-103)}$$

where $\lambda_i(P)$ are the eigenvalues of P, n is the dimension of $\underline{x}$, and the expression J(Q) is the product of the principal axes of the hyperellipsoid Ω. The optimal quadratic estimate of the domain of attraction is given by that choice of Q that maximizes the volume of Ω. The procedure is to pick $Q > 0$, calculate P from Equation (C-102), $V(\underline{x})$ from Equation (C-100), and $\dot{V}(\underline{x})$ from Equation (C-101). Then calculate c by employing one of the methods outlined in the last subsection (preferably method 4), calculate

J(Q) and repeat with a new Q > 0 until no further improvement is made in J(Q). The two subsidiary problems to be solved consist of (1) devising a technique for generating arbitrary elements of the entire collection of positive definite symmetric matrices and (2) devising an algorithm for the solution of Q°, the optimal Q. These subsidiary problems are discussed next and their solution is given as proposed by Geis, et. al [29].

4.2.1 Generation of Positive Definite Matrices

An inefficient method for the generation of the elements of the Q matrix is to randomly generate matrices and select only those that pass the test for positive definiteness. An efficient procedure suggested by Geis et al.[29], is based on the parameterization of unitary matrices. Matrix Q is computed by taking advantage of its orthogonal similarity to a positive diagonal matrix, i.e.,

$$Q = T'\Lambda T \text{ where } T'T = I \tag{C-104}$$

and

$$\Lambda = \text{diag}\left\{\lambda_1, \lambda_2, \ldots, \lambda_n\right\} \quad , \lambda_i > 0 \quad i = 1,2,\ldots,n$$

$$T = \prod_{k=1}^{n-1} T_{n-k} = t_{n-1}T_{n-2}\cdots T_1 \tag{C-105}$$

$$T_k = \left[\prod_{\ell=k+1}^{n-1} T_{k\ell}(\theta_\mu)\right] T_{kn}(\varphi_k)$$

$$\mu = \frac{(2n - k - 2)(k - 1)}{2} + n - \ell$$

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$-\pi \leq \varphi < \pi$$

(C-105 contd)

$$T_{k\ell}(\theta_\mu) = S_{ij}$$

In Equation (C-105), S_{ij} is defined as follows:

$$\begin{aligned} S_{ii} &= 1 \quad i = k, \ell \\ S_{kk} &= S_{\ell\ell} = \cos \theta_\mu \\ S_{ij} &= 0 \quad i \neq j;\ i, j \neq k, \ell \\ S_{k\ell} &= -\sin \theta_\mu \\ S_{\ell k} &= \sin \theta_\mu \end{aligned} \tag{C-106}$$

Thus, the T_k are products of (n x n) rotation matrices where the φ's are longitude angles and the θ's are latitude angles. In general, there are n lambdas, (n - 1) phis, (n - 1)(n - 2)/2 thetas in this parameterization or in total n(n + 1)/2 parameters which equal the number of arbitrary elements in a symmetric n x n matrix. In the case of the tractor-semitrailer where n = 5, we have 5 lambdas, 4 phis, 6 thetas which sum to 15, the order of the parameter space. The constraints are removed by defining $\lambda_i = e^{n_i}$ and $\theta_i = \frac{1}{2}(-\pi + \theta_i' \bmod 2\pi)$ where n_i and θ_i' are arbitrary real numbers. Since the φ_i only appear as arguments of trigonometric functions, they too can be arbitrary real numbers.

4.2.2 Maximization of J(Q)

Geis et al.[29], initially conducted a search on the η_i parameters of Q one at a time while the angles θ_μ, φ_k were fixed at zero. Beginning with η_1 free and $\eta_j = 0$ for $j \neq 1$, they calculated η_1^o, the optimal value; then with $\eta_1 = \eta_1^o$, η_2 free, and $\eta_j = 0$ for $j \neq 1, 2$, they repeated the experiment and continued in this fashion. They then employed Barron's "accelerated random search." The basic idea is to select a starting point and a random perturbation from that point to establish a direction and evaluate the performance J(Q) at each point.

In conclusion, it is observed that all methods of analytically obtaining a Lyapunov function are not useful for the articulated vehicle study. The only systematic numerical method available for the generation of Lyapunov functions has been discussed here in detail. The approach recommended for estimating the domain of asymptotic stability of autonomous maneuvers of articulated vehicles is the quadratic estimation method which involves the following steps: (1) generate arbitrary elements of the Q matrix; (2) solve the Lyapunov matrix Equation (C-102) for P as a function of Q and A; (3) find the minimum of V subject to the constraint that $\dot{V} = 0$ by solving the nonlinear algebraic Equation (C-99); and (4) optimize Q matrix so that the volume of the estimated domain of asymptotic stability is maximum.

APPENDIX C

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APPENDIX D

EXPERIMENTAL EQUIPMENT AND PROCEDURES

APPENDIX D

EXPERIMENTAL EQUIPMENT AND PROCEDURES

1. DESCRIPTION OF EXPERIMENTAL EQUIPMENT

The experimental verification tests were conducted at the Bendix Automotive Development Center (BADC) on their skid pad which had a dry and wet jennite surface. A Diamond Reo single-axle tractor and double-axle 40 ft trailer van, described physically in Table 2 of the main text and shown in Figure D-1, shows the Diamond Reo tandem axle tractor along with two 27 ft single-axle trailer vans and a dolly used in the second series of tests. Table 2 includes a physical description of this equipment also. The tape recorder, control consoles and stable platform (Figure D-3) were mounted in the tractor bunk area. The trajectory marker fluid storage tank was attached to the tractor frame. Pressure for operation was obtained from the tractor. Transducers were attached at appropriate locations. Instrumentation used during these tests is described here in detail.

Transducers, signal conditioning equipment and a magnetic tape recorder comprised the instrumentation package. Statham accelerometers mounted in a biaxial configuration on the tractor sensed axial and transverse accelerations. Duncan rotary potentiometers were used to measure relative angular displacement between the tractor and the trailer, dolly and trailer and the steering angle. The resistance elements of the potentiometers were wired in a voltage balanced bridge circuit. This

configuration provides plus and minus data signals about a predetermined reference axis.

The brake electrical circuit was monitored to produce an on-off indication of the vehicle brake system. The tractor and trailer brake signals were amplitude and polarity coded so that they could be recorded on a single recording channel. A tachometer, mounted on a trailing fifth wheel assembly, was used to monitor the tractor velocity along its own axis. A biaxial gyro was used as a basic sensor to measure vehicle yaw positions.

Signal conditioning equipment was used to convert the transducer signals to a form suitable for recording. The conditioning equipment used includes the following: A Consolidated Electrodynamics Corp. (C.E.C.) Model 8-108 bridge balance unit, Burr-Brown Model 1660B transducer amplifiers, and a signal monitor chassis.

The C.E.C. Model 8-108 bridge balance unit provides the coupling between the resistive type transducers and the transducer amplifiers. The unit contains calibration circuitry and a bridge balance network to compensate for small residual voltages in the Wheatstone bridge circuitry.

For vehicle roll, pitch, and yaw attitudes, an inertial reference system, Humphrey Inc. Model CF-18-0904, was used to measure acceleration vectors and three axes of angular displacement.

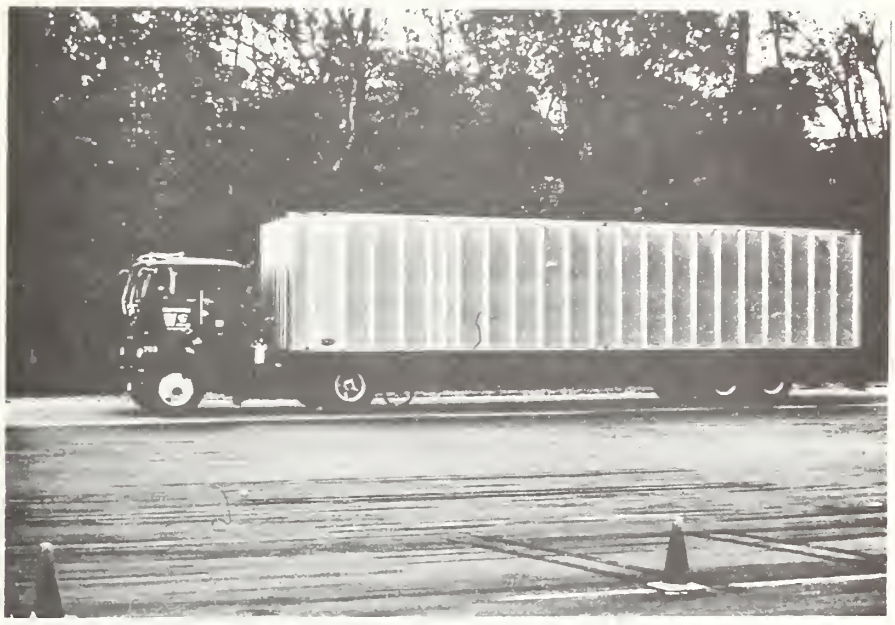

Figure D-1. Tractor single-semitrailer vehicle used in experimental verification tests.

Figure D-2. Tractor double-semitrailer vehicle used in experimental verification tests.

Figure D-3. Tractor cab instrumentation.

The inertial sensing system consists of a cageable vertical gyroscope with potentiometric pick offs in the pitch and roll axis. Slaved to the vertical gyroscope is a pitch and roll platform containing three potentiometric accelerometers mounted to sense accelerations in the longitudinal, lateral and vertical planes. The accelerometers have a range of $\pm$ 2.0 g. The resonant freguency is 20 Hz flat to 7 Hz, with a damping factor of 0.7 of critical. The accelerometers employ center tapped 5000 ohm potentiometers as their sensing elements. The threshold value is about 0.5 percent of full scale. The acceleration measurements can be made with respect to either the vehicle body axis or the stabilized vertical axis as required.

As previously mentioned, the system contains a cageable vertical gyroscope for measuring pitch and roll. Also within the system is a cageable directional gyroscope for measuring yaw position. The range of the vertical platform is $\pm$ 30 deg in the pitch and roll axis. The range of the directional gyroscope is $\pm$ 80 deg. Center tapped 5000 ohm potentiometers are used as sensing elements.

The vertical gyroscope measures with respect to the local vertical (within $\pm$ 0.25 deg of true vertical). The directional gyroscope measures with respect to its own orientation at the time of uncaging. The drift rate is less than 0.5 deg/min.

A pulsed trajectory marker (PTM) was used for measuring vehicle trajectory. This system was developed at IITRI for use on previous vehicle handling programs. On command, the unit deposits spots of water soluble dye on the pavement at 1.0 and 0.1 sec time intervals. The resulting locus of points marks the vehicle's center of gravity path throughout the test maneuver. An electrical pulse is recorded for each road mark, thus providing a means of correlating the trajectory and data signals.

A Laboratory Equipment Corp. (Labeco) fifth wheel Track-test unit was used to measure the vehicle longitudinal velocity. This unit consists of a DC tachometer mounted on a tracking fifth wheel assembly. The tachometer output voltage is linearly related to the vehicle speed.

The transducer amplifiers are DC to 10 kHz amplifiers with balanced differential input. The normal voltage gain is 1000, however, the gain can be reduced for special applications. These units have a single ended output, compatible with the input to the magnetic tape recorder.

The monitor chassis provides a convenient method for monitoring the condition of the data channels prior to recording. This unit is equipped with calibration circuitry and a time base unit. The monitor chassis contains an oscilloscope and a meter to provide a visual indication of signal levels.

An Ampex AR200 instrumentation magnetic tape recorder was used to record the data signals. The AR200 is designed to record the data signals under environmental conditions encountered in airborne and mobile applications.

The AR200 employs a closed loop drive system and an IRIG standard, 1 in. head configuration. The electronics meet IRIG intermediate band specifications. Fourteen channels are available for use on the test program. Twelve FM record amplifiers were used for data channels and two direct record channels were used for voice and time code recording. The data was recorded at a speed of 3-3/ips, providing a data bandpass from DC to 1250 Hz. One mil tape, stored on 10-1/2 in. reels, provided a recording interval of 3 hr 12 min.

The equipment described thus far was assembled to form a data acquisition system. The system was powered to an on-board power supply. Provisions were made to use external power for setting up and servicing the system.

Prior to conducting the field test, the instrumentation system was calibrated and evaluated in the instrumentation laboratory. The transducers were calibrated to determine the conversion factor from mechanical to electrical for each unit. This was accomplished over the anticipated range of application by using secondary calibration standards to apply the stimulus to the transducer and then recording the output signal voltage. For linear systems, the conversion factor is a constant equal to the signal voltage divided by the applied stimulus. The principal factors which were included in the system evaluation are:

- Signal-to-noise ratio
- Linearity
- Hysteresis
- Sensitivity and resolution
- Range (maximum span)
- Response to extraneous signals
- High and low frequency response

2. TEST PROCEDURES

Both experimental verification test series were planned on the basis of analytically obtained stability and handling criteria. Tests were planned to verify stability limits for prescribed lane change, evasive, and cornering maneuvers (those utilized in the analysis) under varied speeds and acceleration/braking sequences. Since it was anticipated that these tests would be run in the regime of tire nonlinear side forces, all precautions were taken to insure vehicle control. Therefore, the experimental test series was run on the AVDS3 computer program to examine the potential dynamic response of the vehicle. A twofold advantage was thereby gained: (1) an anticipated range of measured control variables was obtained; and (2) another check was made on the analytical model. Anticipated tire slip angles were noted and the proximity to the vehicle of the unstable operation regime was examined.

The center of gravity location and weights of the test vehicles were determined by Bendix while the equipment was in

the garage. The center of gravity of a vehicle was determined by Bendix through obtaining unit forces when lifting one axle of the vehicle.

The fully instrumented articulated vehicle was run on the test track to check out the instrumentation system under actual operating conditions including a typical lane change maneuver. Instrumentation modifications and/or calibrations were made after this checkout. Prior to actual testing, the driver practiced the test maneuvers to become familiar with the test equipment, the instrumentation and the road maneuvers.

For each experimental series, the vehicle maneuver was designated and the approximate bounds of the maneuver were marked on the test track. The driver performed the maneuver at the highest safe speed using the IITRI pulse marker system (1/10 sec mark intervals) to mark the tractor trajectory and therefore establish compliance or noncompliance of the test to the specifications. Where test trajectory data were desired, a steel tape was used to measure the distance between the reference line and the geometrical center of each marker in the x and y direction. The data recorded on the AR200 magnetic tape recorder was reproduced on an Ampex CP100 magnetic tape recorder and oscillograph.

The actual test procedure during the articulated vehicle experimental verification tests is given in concise form:

- Designate maneuver (lane change, evasive or corner).
- Designate conditions of maneuver including vehicle speed, and braking, or acceleration.
- Mark maneuver distance on the track with pylons.
- Calibrate transducers.
- Perform test maneuver.
- Place time marks on trajectory.
- Place reference lines for measurement references.
- Record distance of each mark from reference.(Figures D-4 through D-6 show details of trajectory marker measurements for cornering, lane change and evasive maneuvers respectively.)
- Record track surface condition.*

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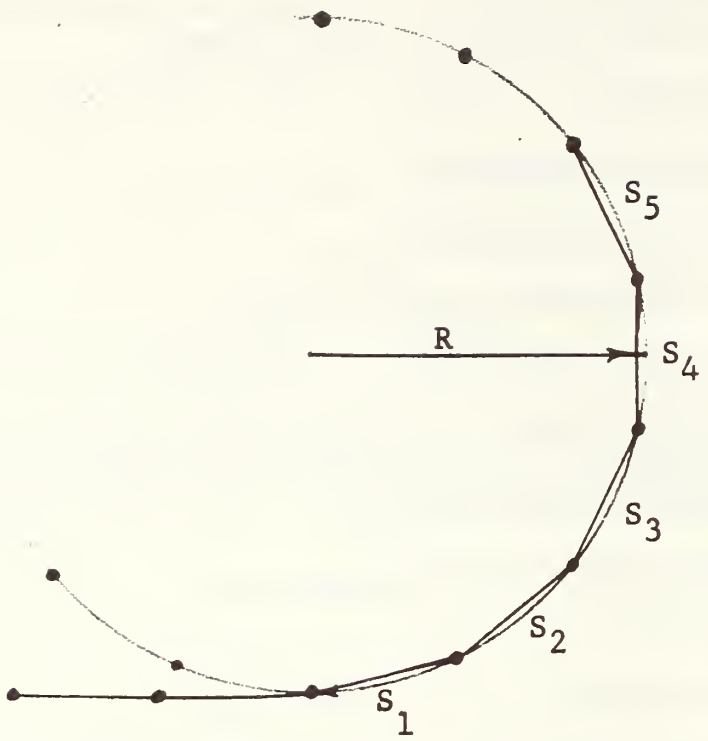
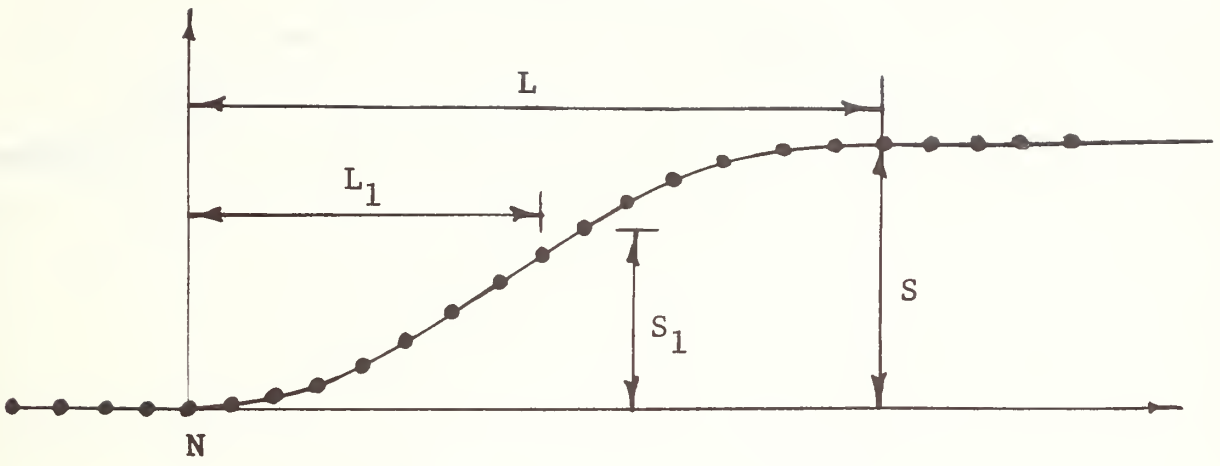
* ASTM skid number measurements were conducted on track locations utilized after each series of tests. These data were used to quantify tire side force curves.



Cornering (Circle or Arc) Constant Speed-Braking

1. Measure nominal radius, R.
2. Identify first mark.
3. Count evenly spaced marks, S.
4. Measure distance between marks during braking.

Figure D-4. Trajectory marker measurents - cornering.



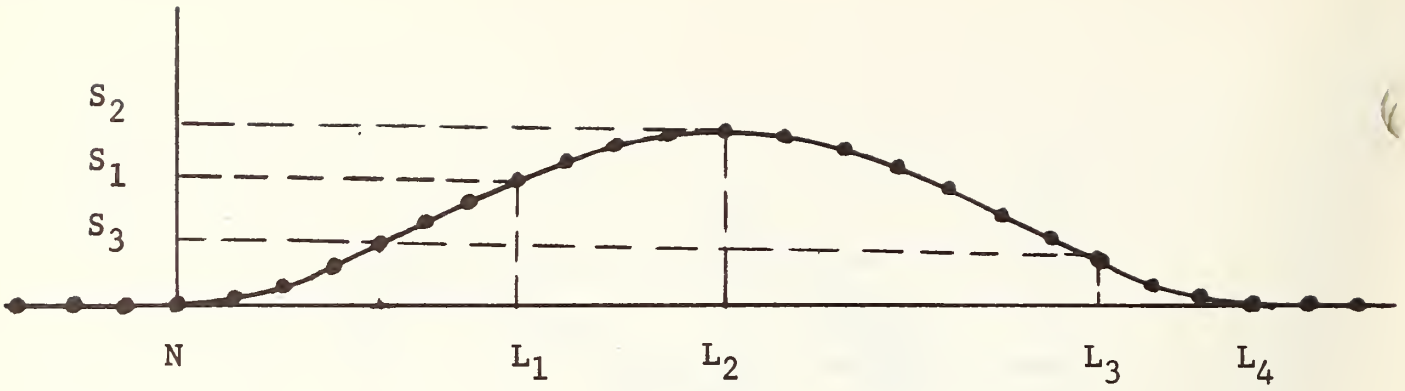
Lane Change, Constant Velocity

1. Count marks to beginning of maneuver.
2. Measure L,S.
3. Measure L_1, S_1.

Lane Change, Braking

1. Count marks to start.
2. Measure all marks start to finish.

Figure D-5. Trajectory marker measurements - lane change.



Evasive, Constant Velocity

1. Count N.
2. Measure L_1, L_2, L_3, L_4 and S_1, S_2, S_3.
3. Record position of (S_1, L_1), (S_2 L_2), (S_3 L_3), L_4 (number of trajectory marks).

Evasive, Braking

1. Count marks to start.
2. Measure every other mark from start to finish except in critical areas.

Figure D-6. Trajectory marker measurements - evasive maneuver.

APPENDIX E

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August 1973

Final Report

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15. Supplementary Notes

16. Abstract

An analytical and experimental investigation on the stability and handling of articulated vehicles was conducted to develop stability criteria for tractor single-, double-, and triple-semitrailer vehicles during commonly encountered road and speed maneuvers subject to environmental influences. To this end, an experimentally validated digital simulation model, AVDS3, was further developed and validated. It was utilized to generate physical stability limits for articulated vehicles. During this investigation, the direct method of Lyapunov was employed to generate mathematical stability limits of cornering articulated vehicles. In addition, an effort was made to apply the concept of finite time stability to this problem.

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STABILITY AND HANDLING CRITERIA OF ARTICULATED VEHICLES

1. INTRODUCTION

The accomplishments, results, conclusions and recommendations derived from a research program entitled "Stability and Handling Criteria of Articulated Vehicles" conducted by IIT Research Institute (IITRI) and sponsored by the National Highway Traffic Safety Administration (NHTSA) are summarized in this report. An analytical and experimental investigation was conducted to develop stability limits for articulated vehicles based on mathematical and physical stability criteria. The facility for prediction of the dynamic behavior of triple-trailer articulated vehicles during commonly encountered road and speed maneuvers subject to environmental influences has been developed.

In this program, stability and handling criteria were developed for tractor single-, double-, and triple-semitrailer vehicles supporting varied loads with a practical range of physical parameters; e.g., fifth wheel location, center of gravity location, trailer length, etc. Mathematical models were used to investigate stability limits featuring the use of the Articulated Vehicle Dynamic Simulation (AVDS3) computer program, Lyapunov's direct method, and the finite time stability concept. Stability limits were verified experimentally for tractor single- and double-semitrailer vehicles. Special emphasis was placed on the assessment of vehicle maneuverability and stability in its normal road environment. The AVDS3 model simulates the dynamic response and evaluates the stability of single-, double-, and triple-trailer articulated vehicles of varied geometry and load distribution subject to:

- Tractive and braking forces which can be distributed as desired.
- Nonlinear cornering forces described by an empirical model developed by IITRI which is constrained by a tire cornering-tractive force relationship as indicated by the "friction ellipse" concept.
- Steady and transient aerodynamic forces.
- Fifth wheel friction and damping.
- Prescribed road maneuvers including grades and banks.

The scope of this program includes a literature review of recent developments in articulated vehicle technology and of the application of the Lyapunov direct method to the determination of stability of dynamic systems. Mathematical models were reviewed for their applicability to stability analyses and the ability to calculate vehicle nominal motions while performing a prescribed trajectory using the AVDS3 model was developed. From the mathematical models, system perturbation equations were developed which describe the response of the system to perturbed variables. Mathematical stability analyses using Lyapunov's direct method, which considers stability during infinite time, and finite time stability concepts were investigated for their applicability to the problem. Physical stability criteria considering the response of the vehicle were developed. An experimental verification program involving tractor single- and double-semitrailer systems was conducted to validate mathematically and physically inferred stabilit limits. Some parameter variations were conducted to determine the effect of road surface condition on vehicle stability for cornering, lane changing and evasive maneuvers.

The trajectory approach, in which the articulated vehicle is required to maintain stability while traveling through a prescribed maneuver involving braking and/or acceleration, was selected because it yields results directly applicable to the needs of DOT/NHTSA, i.e., it relates vehicle performance to road environment. Figure 1 shows the IITRI approach to vehicle handling and stability problems.

A prescribed maneuver commonly needed on the highway, such as lane changing or cornering, serves as input data. The mathematical model reflects physical characteristics of the vehicle, the road and the environment. The mathematical model yields the driver control requirements, information on nonprescribed vehicle response, and information on its stability for the prescribed maneuver. The AVDS3 computer program (developed by IITRI for NHTSA) was used to determine physical stability (based on available cornering and tractive/braking forces, tire and yaw angles) and to provide nominal motions for mathematical stability analyses. Techniques for the mathematical determination of articulated vehicle stability were investigated and developed for limited applications using the second or direct method of Lyapunov.

2. PROGRAM TASKS

The objectives of this program were fulfilled in accordance with the specified scope through the completion of nine major program tasks shown schematically in relationship to each other in Figure 2.

2.1 Literature Review

A literature survey and review were conducted on technical material relevant to the development of stability and handling criteria for articulated vehicles. Specifically, the

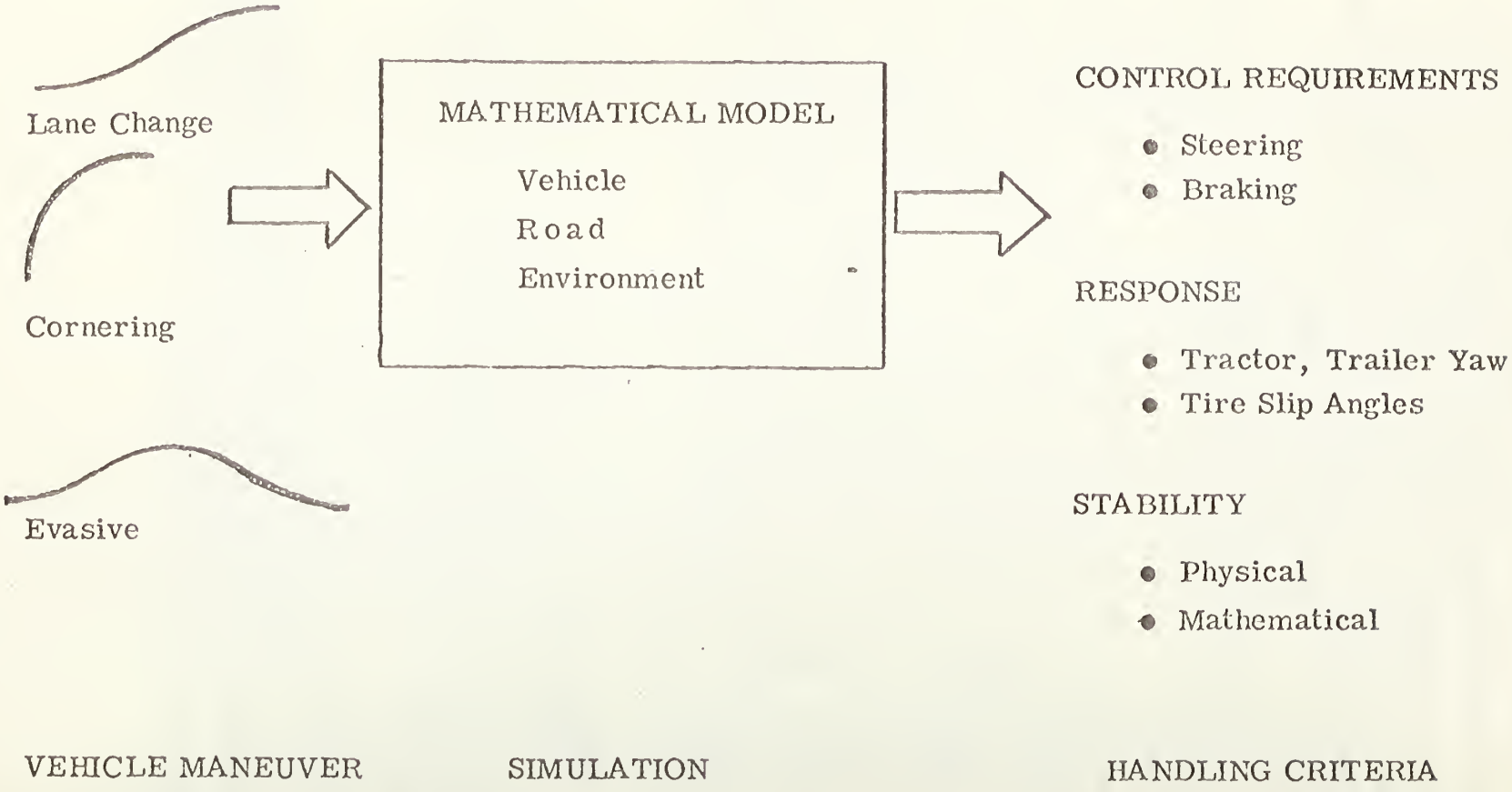


Figure 1. Articulated vehicle handling methodology.

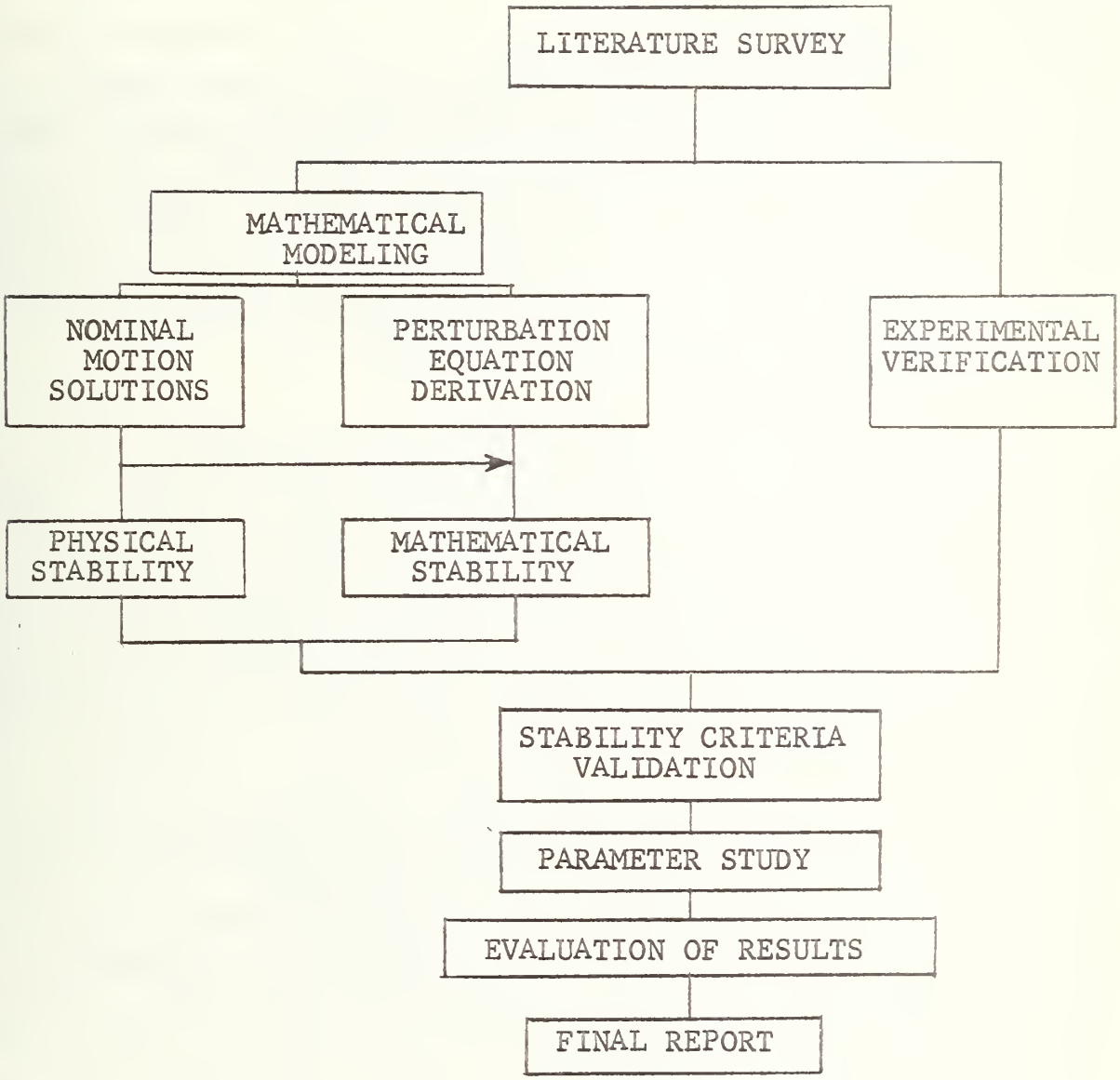


Figure 2. Work plan for stability and handling of articulated vehicles.

bibliography on articulated vehicle handling was updated and literature involving the application of the Lyapunov direct method and the method of finite time stability to the generation of mathematical stability limits was reviewed.

2.2 Mathematical Modeling

The task on mathematical modeling was conducted to obtain the best coordinate system and mathematical description of the problem to effect the least complicated, solvable problem formulation. Coordinate systems were examined and tractor body fixed coordinates were selected for mathematical stability analyses. Space fixed Cartesian coordinates are used in the AVDS3 simulation model.

The equations of motion and constraint conditions derived for triple-trailer articulated vehicles were examined for means of obtaining the least complicated meaningful mathematical model. The development of the vehicle motion perturbation equations required much algebraic manipulation; therefore, the results of the first series of articulated vehicle stability tests were used to justify simplifications in the model. The use of semistatic load transfer to model the effects of roll and pitch motions was examined. The nonlinearities involved in vehicle yaw motions and the tires were retained because they are fundamental to a realistic model. Assumptions on the constraint conditions, e.g., the dolly hitch to the semitrailer where only lateral and forward forces are transmitted, were made prior to the elimination of the excess variables.

Tire side forces and braking/tractive forces are introduced to the differential equations through the suspension system. In order to permit more refined tire modeling, multi-suspension attachment points were introduced in the tractor and trailer models. This feature allows modeling of single

and tandem axles. In addition, the mathematical model was expanded to include individual tire characterization. Tractive/braking forces are related to side forces using the friction ellipse concept.

2.3 Nominal Motion Solutions

The motions involved in articulated vehicles performing a prescribed nominal maneuver were obtained for two classes of problems. The first is a stationary maneuver where the nominal motion is steady state and is not a function of time. Specifically, this nominal motion is a circular trajectory at a constant forward velocity and the state variables, yaw rates and velocities, and the control inputs, braking and steering, are constants.

The second class of problem was described by a nonstationary maneuver where the nominal motion is unsteady state and hence is time varying. During this nominal motion, the state variables and the control inputs varied with time. The AVDS3 digital simulation model using numerical integration was modified to obtain these nominal motions for tractor single-, double-, and triple-semitrailer vehicles.

2.4 Perturbation Equation Derivation

This task involved the derivation of a set of equations describing the perturbed motion of the articulated vehicles. For steady state nominal motions, the equations are autonomous nonlinear differential equations. For unsteady state nominal motions, the equations are nonautonomous nonlinear differential equations. This process and the succeeding stability analyses are described in detail in the technical report.

2.5 Mathematical Stability Analysis

Investigation of the linearized differential equations of perturbed motion yields stability in the small without a priori knowledge of the size of the region of stability. Asymptotic stability implies that the perturbed motion decays in time to the nominal motion. Hence, the nominal motion is asymptotically stable. For steady state nominal motion, Routh criterion was employed on the characteristic polynomial of the perturbation equations to obtain information on stability in the small. Mathematical stability analyses of articulated vehicles were successfully conducted for autonomous (constant speed cornering) motions using Lyapunov's direct method. It was found that the concept of Lyapunov stability is too restrictive for nonautonomous motions (time-varying response). Whereas the Lyapunov method implies stability ove infinite time, the articulated vehicle can be instantaneously unstable in the sense of mathematics and yet recover at a new nominal motion. It was found that a modified Lyapunov stability concept called "finite time stability" is physical more applicable to articulated vehicle stability analysis than the Lyapunov direct method. Several finite time stability techniques were unsuccessfully applied to this problem. In each case, results were very conservative; i.e., stable motions were judged to be unstable.

2.6 Physical Stability Analysis

Physical stability criteria were developed through the examination of articulated vehicle response. The AVDS3 simulation model was modified to obtain stability limits based on bounded articulated vehicle response. The limits thus developed imply an ideal driver and vehicle performance. The criteria for vehicle stability is based on available vehicle control (braking and side) forces. This stability concept is implemented through the use of the friction ellipse concept at the individual tires at each instant during the maneuver.

2.7 Experimentation

Full-scale experimental work was conducted to assess the validity of mathematical modeling assumptions, to validate the AVDS3 model for computation of the dynamic response of tractor double-semitrailers, and to develop stability and handling limits for articulated vehicles. Two series of tests were conducted involving three vehicle configurations and two drivers. The trajectory method of testing was used in which the driver performs a prescribed maneuver as fast as safely possible. In the initial test series a tractor 45 ft semitrailer van vehicle was used to conduct modeling investigations and to further develop the AVDS3 model. In the second test series a tractor 27 ft semitrailer van vehicle and a tractor double 27 ft semitrailer van vehicle were used to verify AVDS3 for double articulated vehicles and to verify mathematical and physical stability limits.

2.8 Stability Criteria Validation

Fifteen articulated vehicle tests conducted at limit handling performance were chosen to verify the AVDS3 model in an analog manner. Complete time histories of the experimental and analytical vehicle response were compared to judge the validity of the model in simulation of limit maneuvers. In addition, specific test points were plotted on the stability limit curves derived through parameter variation to show point verification of parameter studies.

2.9 Parameter Studies

A parameter study was conducted on the vehicles tested in this program to determine their stability limits using the stability criteria developed in this program. Stability limits were developed for varied maneuvers, braking, road-tire friction interface, loading and speed. Since road-tire friction is the most important factor in articulated vehicle stability, it was utilized as the independent variable in the stability limit diagrams.

3. RESULTS

The results of the investigation on the development of stability criteria for articulated vehicles are summarized herein. For detailed analyses and engineering evaluations refer to the technical report - Part 1.

3.1 Experimental Verification

The results of the experimentation conducted at the Bendix Automotive Development Center (BADC), New Carlisle, Indiana, are shown in Tables 1 and 2. Peak values of the vehicle response variables are listed for cornering, lane changing and evasive maneuvers.

The test results shown in Table 1 are grouped according to maneuver.

- 75 ft radius cornering
- 200 ft radius cornering
- Lane changing at speeds varied between 32 and 45 mph.

Within these groups, the individual tests reflect varied load, road surface, and axle combinations. In the cornering tests, the tractor yaw angles are not given because they were measured with respect to an initial reference axis and show yawing due to the maneuver geometry plus the actual vehicle yaw. The tractor-trailer relative yaw angle provides a more significant measure of the vehicle's yawing motion. In all cases, peak forward accelerations are given when the vehicle employs braking during the maneuver.

In the 75 ft radius cornering maneuvers of test series No. 1, a peak of 0.46 g lateral acceleration was obtained for an unloaded tandem axle configuration. Roll and steering angles had peak values of 4.9 and 12.5 deg respectively. These values agree well with the stability limits. In general the cornering limits decreased only slightly for wet surfaces.

Table 1

SUMMARY OF EXPERIMENTAL RESULTS FROM ARTICULATED VEHICLE LIMIT MANEUVERS (peak values)

Maneuver Description		Axle/Load Conf.*	Road Cond.	Tractor							Tractor-Trailer Yaw Angle (deg)	Experiment No.
				Nominal Speed (mph)	Lat. Acc. (g)	For. Acc. (g)	Steer. Angle (deg)	Yaw Angle (deg)	Roll Angle (deg)	Pitch Angle (deg)		
Corner 75 ft Radius (Nominal)		T/U	Dry	22.2	0.46	---	9.1	---	4.9	---	20.8	14
		S/U	Dry	20.0	0.35	---	11.0 (±1.5)	---	2.9	---	24.0	15
		S/U	Wet	17.5	0.28	---	10.3	---	2.6	---	26.2	21
		T/20	Wet	21.5	0.37	---	12.4 (+2.0)	---	3.3	---	27.9	23
		T/10	Wet	20.0	0.36	---	11.5 (±1.5)	---	3.8	---	25.9	37
Corner 200 ft Radius (Nominal)		S/U	Dry	36.2	0.47	---	4.1	---	3.2	---	9.0	16
	B**	S/U	Dry	36.2	0.43	0.53	3.4	---	2.5	1.2	9.5	17
		T/20	Dry	32.4	0.35	---	5.6	---	2.8	---	8.1	24
	B	T/20	Dry	33.0	0.38	0.41	5.3	---	2.5	---	10.3	25
	B	T/10	Dry/Wet	37.5	0.42	0.23	5.6	---	3.7	0.7	9.3	35
	B	T/10	Dry/Wet	37.5	0.36	0.40	4.0	---	1.8	1.1	8.9	36
14'x145'		T/U	Dry	37.7	0.34	---	3.9	10.8	1.7	---	5.2	9
12'x208'		T/U	Dry	44.2	0.32	---	3.4	7.9	1.6	---	4.1	10
14'x221'	B	T/U	Dry	45.2	0.28	0.51	2.5	9.5	1.5	1.0	4.7	12
12'x	B	T/U	Dry	45.0	0.29	0.49	2.9	9.9	1.6	1.1	4.8	13
12'x120'		S/U	Wet	37.1	0.28	---	2.9	9.2	1.6	---	4.0	19
12'x133'		T/20	Dry	39.4	0.25	---	3.9	9.8	1.9	---	3.6	28
12'x160'	B	T/20	Dry	38.1	0.23	0.4	4.0	9.6	1.5	1.1	5.6	30

* S = single axle
T = tandem axle
U = unloaded trailer
10 = 10,000 lb load
20 = 20,000 lb load

** B = braking

Table 2

EXPERIMENTAL RESULTS FOR TEST SERIES NO. 2

Measured Variable	Channel	Units	1	2	3	4	5	6	7	8	9	10	11	12	13
Tractor Forward Acceleration	1	ft/sec^2			-0.353	-0.522		+0.099 -0.323		-0.420		-0.40		-0.32	
Tractor Lateral Acceleration	2	ft/sec^2	+0.191 -0.254	+0.242 -0.235	-0.026 +0.026	+0.078 -0.076	+0.202 -0.268	+0.100 -0.184	+0.265 -0.308	+0.205 -0.245	+0.378		+0.29 -0.30	+0.25 -0.18	+0.31 -0.29
Tractor Yaw	3	deg	-9.34	-10.10 +9.38		-3.09	-10.0	-10.21	-9.28 +8.66	-8.97 +10.93	-19.9		-10.14 +1.99	-8.76	-11.42
Tractor Roll	4	deg	+1.43 -1.53	+1.80 -1.35		+0.313	+2.23 -1.57	+1.88 -1.15	+2.12 -2.16	+2.16 -1.50	+3.23		+2.48 -2.48	+1.72 -1.52	+2.84 -2.16
Dolly Yaw	5	deg	-8.08	-8.86 +8.06	-0.805	+1.03 -1.98	-8.95	-8.47	-8.32 +7.44	-8.00 +9.03	-19.96		-9.74 +3.09	-8.08	-11.64 +1.20
Steering	6	deg	-4.71 +4.86	-6.43 +5.43	+1.14 -3.10	+2.23 +0.216	-5.69 +4.68	-3.88 +4.32	-5.23 +5.08	-4.90 +4.43	-15.41		-5.73 +5.84	-4.90 +3.99	-6.15 +5.62
Tractor-Trailer Yaw	7	deg	-2.66 +3.22	-3.63 +4.19		-1.31 +0.51	-3.12 +2.64	-4.22 +2.61	-2.26 +2.98	-2.70 +3.66	-12.32		-2.51 +3.56	-3.08 +3.08	-2.72 +3.59
Dolly-Trailer Yaw*	8	deg	-2.87 +1.91	-3.56 +3.07		-0.914 +1.60	-2.96 +1.60	+0.614 -2.88	+0.50 -1.34	-3.29 +2.63	-11.56	-0.44	-3.48 +3.62	-3.04 +1.10	-4.06 +3.66
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.215	+0.204			+0.273	+0.246	+0.286	+0.275	+0.389		+0.30	+0.26	+0.35
Trailer Lateral Acceleration, 2**	10	ft/sec^2	+0.230 -0.199	+0.207 -0.246			+0.311 -0.287	+0.297 -0.154	+0.284 -0.377	-0.237	+0.381		+0.35 -0.41	+0.26 -0.11	+0.42 -0.41
Speed	11	mph	33.10	42.86	45.71	40.91	39.09	38.18	39.09	39.09	19.54	36.9	36.4	37.3	38.7
Load E=empty F=full H=half		---	E	E	E	E	E	E	E	E	E	F	F	F	F
Braking		---	No	No	Yes	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
Nominal Speed		mph	40	40	40	40	40	40	40	40	-	40	40	40	40
Surface W=wet D=dry		---	W	W	W	D	D	D	D	D	D	D	D	D	D
Maneuver***		---	LC	E	SLB	SLB	LC	LC	E	E	C	SLB	LC	LC	E

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 2 (contd)

Measured Variable	Channel	Units	14	15	16	17	18	19	20	21	22	23	24	25	26
Tractor Forward Acceleration	1	ft/sec^2		-0.33				-0.375		-0.450	-0.424			-0.350	
Tractor Lateral Acceleration	2	ft/sec^2	+0.33 -0.40	+0.28 -0.23	+0.40	+0.262	+0.357		+0.312 -0.284	+0.215 +0.096	+0.282	+0.293 -0.370			+0.299 -0.359
Tractor Yaw	3	deg	-10.28 9.34	-10.21 +10.41	-19.07	-16.29	-19.87	-1.55	-10.83	-9.28	-9.18	-9.79 +8.14			-11.83 +1.76
Tractor Roll	4	deg	+2.57 -3.59	+1.98 -1.87	+8.40 -5.68	+2.88	+4.16		+2.52 -2.00	+1.56 -4.00	+1.52	+2.20 -3.52			+2.24 -3.68
Dolly Yaw	5	deg	-10.30 +10.77	-9.42 +8.55	-17.34	-14.81	-18.71	+0.55 -0.94	-11.71 1.65	-7.70	-7.15	-9.98 +8.65		-0.634	-11.72 +4.12
Steering	6	deg	-6.33 6.44	-4.75 +5.09	-14.84	-13.85	-14.7	+1.80	-5.41 +5.67	-3.64 +2.54	-3.97 +3.64	-5.45 +5.70		+4.79	-5.99 +7.26
Tractor-Trailer Yaw	7	deg	-2.46 +4.31	-3.66 +4.27	-16.42	-15.32	-12.85	-0.697	-2.46 +3.71	-3.71 +2.68	-2.92 +4.48	-2.18 +4.11		-3.86	-3.68 +5.03
Dolly-Trailer Yaw*	8	deg	-3.66 +4.85	-3.47 +2.85	-14.87	-15.50	-15.38	-0.645	-4.00 5.42	-2.58 +1.68	-2.71 +1.49	-3.67 +5.07		+1.65 -0.506	-3.88 +7.13
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.31 -0.44	+0.30 -0.20	+0.27	+0.238	+0.316	+0.047	+0.276 -0.331	+0.211 -0.076	+0.165	+0.271 -0.373			+0.287 -0.327
Trailer Lateral Acceleration, 2 **	10	ft/sec^2	+0.35 -0.49	+0.31 -0.16	+0.30	+0.239	+0.342	+0.051	+0.393 -0.471	+0.203 -0.032	+0.194	+0.330 -0.439			+0.320 -0.367
Speed	11	mph	39.5	36.9	20.7	16.82	19.77	38.63	23.18	37.27	37.27	39.54		36.40	34.15
Load E = empty F = full H = half		---	F	F	F	F	F	F	F	F	F	F	F	H	H
Braking		---	No	Yes	No	No	No	Yes	No	Yes	Yes	No	Yes	Yes	No
Nominal Speed		mph	40	40	-	-	-	40	40	40	40	40	40	40	40
Surface W = wet D = dry		---	D	D	D	W	D	D	D	D	D	D	D	W	W
Maneuver***		---	E	E	C	C	C	SLB	LC	LC	LC	E	E	SLB	LC

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 2 (contd)

Measured Variable	Channel	Units	27	28	29	30	31	32	33	34	35	36	37	38	39
Tractor Forward Acceleration	1	ft/sec^2	-0.363		-0.502						-0.381		-0.452	-0.438	
Tractor Lateral Acceleration	2	ft/sec^2	+0.275 -0.136	+0.402 -0.355	+0.355 -0.202	+0.241	+0.417	+0.372 -0.366		+0.280 -0.368	+0.323 -0.244	+0.372 -0.422	+0.310 -0.297	+0.064	+0.387 -0.484
Tractor Yaw	3	deg	-0.882 +6.95	-13.18 +9.45	-12.87 +15.67	-15.88	-22.06	-12.06 +2.37		-11.78 +1.33	-9.63	-11.67 +10.96	-10.83	-7.94	-13.41 +1.74
Tractor Roll	4	deg	+1.72 -0.80	+2.88 -3.32	+2.68 -2.60	+2.94	+4.08	+2.80 -3.16		+3.20 -2.84	+1.91 -1.27	+2.68 -3.72	+2.44 -2.88		+2.70 -3.45
Dolly Yaw	5	deg	-16.0 +1.11	-13.23 +9.98	-13.70	-14.89	-19.87	-12.81 +4.48	-2.97	-13.13 +3.93	-9.44 +5.38	-12.73 +12.73	-12.28	-3.48	-14.23 +4.24
Steering	6	deg	-4.97 +3.74	-7.88 +7.88	-7.62 +6.79	-17.15	-17.36	-7.77 +7.95	+1.45 -0.327	-8.24 +7.26	-6.17 +4.64	-8.07 +8.21	-7.22 +6.61	+5.54	-7.41 +8.97
Tractor-Trailer Yaw	7	deg	-3.14 +6.43	-4.39 +5.50	-4.13 +5.92	-15.43	-15.43	-4.02 +4.72	-0.975 -0.663	-3.82 +5.03	-3.04 +3.63	-3.82 +5.54	-4.37 +5.23	-5.93	-6.55 +6.63
Dolly-Trailer Yaw*	8	deg	-10.22 +10.61	-5.28 +6.71	-6.46 +10.55	-15.71	-15.39	-6.21 +7.43	-2.58 +2.63	-6.80 +8.56	-5.21	-6.75 +7.88	-6.48 +8.43	-2.54	-6.34 +9.29
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.222 -0.156	+0.320 -0.356	+0.340 -0.201	+0.252	+0.404	+0.348 -0.465		+0.342 -0.412	+0.294	+0.339 -0.570	+0.318 -0.315		+0.234 -0.443
Trailer Lateral Acceleration, 2 **	10	ft/sec^2	+0.141 -0.239	+0.342 -0.410	+0.335 -0.220	+0.246	+0.420	+0.427 -0.517		+0.483 -0.531		+0.414 -0.552	+0.393 -0.299		+0.406 -0.525
Speed	11	mph	35.05	34.60	35.50	18.87	21.78	37.75	40.00	36.89	37.72	36.81	35.90	39.55	37.30
Load E = empty F = full H = half		---	H	H	H	H	H	H	H	H	H	H	H	E	E
Braking		---	Yes	No	Yes	No	No	No	Yes	No	Yes	No	Yes	Yes	No
Nominal Speed		mph	40	40	40	-	-	40	40	40	40	40	40	40	40
Surface W = wet D = dry		---	W	W	W	W	D	D	D	D	D	D	D	D	D
Maneuver***		---	LC	E	E	C	C	LC	SLB	LC	LC	E	E	SLB	LC

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 2 (contd)

Measured Variable	Channel	Units	40	41	42	43	44	45	46	47	48	49	50	51	52
Tractor Forward Acceleration	1	ft/sec^2	-0.478		-0.429 -0.548		-0.421		-0.394		-0.396		-0.487		-0.445
Tractor Lateral Acceleration	2	ft/sec^2	+0.314 -0.148	+0.400 -0.441	+0.347 -0.293	+0.247		+0.368 -0.268	+0.274 -0.118	+0.396 -0.385	+0.338 -0.278	+0.278		+0.333 -0.323	+0.310 -0.305
Tractor Yaw	3	deg	-9.48 +0.93	-13.62 +9.22	-10.04 +8.70	-18.15	-0.722 +0.619	-10.31	-14.43 +6.80	-11.86 +9.18	-10.21 +16.08	-20.62		-11.0	-7.47 +2.80
Tractor Roll	4	deg	+1.95 -1.27	+3.00 -3.92	+2.28 -2.36	+2.52		+2.36 -1.72	+1.48 -0.64	+2.24 -2.80	+2.04 -2.32	+3.40		+2.68 -2.44	+1.84 -0.40
Dolly Yaw	5	deg	-9.51	-14.93 +8.80	-10.22 +12.67	-15.84	+0.396 -0.317	-10.53	-24.79	-12.67 +7.76	-11.01 +17.11	-16.27		-9.43	-6.60
Steering	6	deg	-5.66 +3.32	-9.04 +7.41	-6.72 +5.06	-14.83	+0.731 -0.292	-6.05 +4.87	-4.48 +10.58	-6.84 +5.02	-5.76 +5.41	-15.47		-6.99 +6.35	-5.12 +2.61
Tractor-Trailer Yaw	7	deg	-5.57 +6.26	-7.18 +6.07	-5.77 +4.30	-14.31	-1.40 +0.53	-5.55 +2.46	-7.58 +8.69	-5.94 +4.06	-4.83 +8.69	-14.75	-1.07	-4.10 +5.12	-3.42 +7.12
Dolly-Trailer Yaw*	8	deg	-4.81 +7.30	-7.57 +8.33	-5.25 +7.02	-14.90	+0.711 -0.610	-4.72 +6.40	-18.29 +9.65	-6.05 +5.84	-5.44 +13.21	-14.40		+1.08 -0.538	+0.806
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.173 -0.322	+0.399 -0.476	+0.313	+0.220		+0.323 -0.272	+0.203	+0.334 -0.323	+0.312 -0.385	+0.287		+0.312 -0.294	+0.276
Trailer Lateral Acceleration, 2**	10	ft/sec^2		+0.487 -0.548	+0.359 -0.188	+0.250		+0.333 -0.299	+0.260 -0.192	+0.307 -0.371	+0.333 -0.414	+0.265	-0.534	-0.192	-0.495
Speed	11	mph	38.20	38.66	36.85	17.08	39.55	38.22	37.75	36.85	37.30	19.77	40.91	37.72	39.54
Load E = empty F = full H = half		---	E	E	E	E	E	E	E	E	E	E	E	E	E
Braking		---	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Nominal Speed		mph	40	40	40	-	40	40	40	40	40	-	40	40	40
Surface W = wet D = dry		---	D	D	D	W	W	W	W	W	W	W	D	D	D
Maneuver***		---	LC	E	E	C	SLB	LC	LC	E	E	C	SLB	LC	LC

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

Table 2 (concl)

Measured Variable	Channel	Units	53	54	55	56	57	58	59
Tractor Forward Acceleration	1	ft/sec^2		-0.452		-0.276		-0.304	
Tractor Lateral Acceleration	2	ft/sec^2	+0.398 -0.415	+0.430 -0.368	+0.396 -0.387	+0.303 -0.178	+0.409 -0.378	+0.417 -0.327	+0.381
Tractor Yaw	3	deg	-12.77 +11.21	-11.63 +10.90	-11.96	-8.82 +9.45	-12.37 +7.73	-10.21 +15.88	-23.77
Tractor Roll	4	deg	+2.96 -3.16	+3.20 -2.92	+2.68 -2.20	+2.44 -0.76	+2.76 +2.80	+2.76 -2.00	+4.04
Dolly Yaw	5	deg	-10.45 +9.35	-9.27 +7.86	-10.06	-8.02 +1.73	-10.30 +6.60	-8.10 +7.62	-16.03
Steering	6	deg	-8.27 +7.04	-8.71 +6.74	-6.70 +6.53	-6.75 +2.48	-7.80 +6.70	-7.28 +5.09	-17.71
Tractor-Trailer Yaw	7	deg	-5.27 +5.71	-5.56 +6.10	-4.63 +5.42	-3.94 +10.28	-4.68 +5.77	-3.94 +10.85	-15.83
Dolly-Trailer Yaw*	8	deg	+1.25 -1.12	+1.03 -0.672	+0.844 -0.444	+0.933 -0.835	+1.07 -0.710	+0.835 -0.800	+1.97 -5.82
Trailer Lateral Acceleration, 1	9	ft/sec^2	+0.348 -0.407	+0.345 -0.330	+0.316 -0.276	+0.265 -0.290	+0.309 -0.276	+0.280 -0.454	+0.334
Trailer Lateral Acceleration, 2**	10	ft/sec^2	-0.367	-0.474	-0.192	-0.286	-0.192	+0.256 -0.265	
Speed	11	mph	37.27	40.91	40.45	39.09	39.54	41.36	32.72
Load E = empty F = full H = half		---	E	E	E	E	E	E	E
Braking		---	No	Yes	No	Yes	No	Yes	No
Nominal Speed			40	40	40	40	40	40	-
Surface W = wet D = dry		---	D	D	W	W	W	W	W
Maneuver***		---	E	E	LC	LC	E	E	C

*For tests 50-59, this column becomes trailer roll.
**For tests 50-59, this column becomes tractor forward acceleration.
***Maneuvers: C=cornering, E=evasive, LC=lane change, SLB=straight line braking

The 200 ft radius cornering maneuvers show high lateral and forward acceleration. Test No. 17 shows a nominal speed of 36.2 mph with a lateral acceleration of 0.43 g and a forward deceleration of 0.53 g due to braking. Steering angles show values around 5 deg which indicate a probable reserve in steering. In these cornering experiments on wet and dry surfaces, roll angles peak at about 3.5 deg and pitch angles peak at about 1.0 deg. These values are similar in magnitude to those predicted by Mikulcik[1] in his analytical work.

The lane change maneuvers were performed at speeds ranging from 38 to 45 mph with and without braking. Lane changing peak lateral and forward accelerations of 0.34 to 0.51 were recorded. Steering angles of 4.0 deg indicate that steering was not critical in these maneuvers. Yaw, roll, and pitch angle peaks were 11.8, 1.9 and 1.1 deg respectively in this series of experiments and selected response records are discussed in the technical report where analytical-experimental validation is presented.

The second series of experimental validation tests are summarized in Table 2 and are grouped according to testing order while performing the following maneuvers.

- 82 ft radius cornering.
- Lane changing at 40 mph with and without braking.
- Evasive maneuvers at 40 mph with and without braking.

Forty-nine experiments were conducted on a tractor double-semitrailer unit and 10 experiments were conducted on a tractor semitrailer. In the test matrix, the following parameters were varied.

- Tires (mixed Firestone and Goodyear and all Goodyear were used).
- Load (empty, half, and full).
- Surface (dry and wet).
- Maneuvers (lane change (LLC), evasive (EM), and corner (CCW)).
- Braking (straight line (SLB), and braking with maneuvers).

Results indicate that handling of tractor double-semitrailer vehicles is tedious on wet surfaces. Movies of the experiments show that control of the vehicle is lost at the dolly during severe braking resulting in jackknifing. In violent maneuvers with empty trailers it was found that large unallowable (in the sense of road operation) trailer swing occurred.

3.2 Stability Model Validation

The 15 tests listed in Table 3 represent a series of limit maneuvers conducted with the following varied parameters

- Maneuver (lane change, corner and evasive)
- Vehicle configuration (single 27, single 45 and double 27)
- Load configuration (empty, loaded, one trailer empty and the other loaded
- Surface (wet and dry)
- Tires (mixed, Goodyear, mixed Firestone and Goodyear)
- Braking

The simulations of limit maneuvers are compared on a time history basis throughout the maneuver to provide credibility of the AVDS3 simulation model to develop stability limits. Since the cornering maneuvers were performed at steady state, peak response values are compared for experiment and AVDS3 simulation. A summary of the peak values of analytical-experimental responses is shown in Table 3.

Table 3

SUMMARY OF ANALYTICAL EXPERIMENTAL RESPONSE FOR ARTICULATED VEHICLES PERFORMING LIMIT MANEUVERS

Simulation Number	Test Number	Vehicle Configuration	Surface	Maneuver	Tractor Accel. (ft/sec^2) Forward	Tractor Accel. (ft/sec^2) Lateral Exper.	Tractor Accel. (ft/sec^2) Lateral Anal.	Steering (deg) Exper.	Steering (deg) Anal.	Tractor Yaw (deg) Exper.	Tractor Yaw (deg) Anal.	Tractor-1st Trailer Yaw (deg) Exper.	Tractor-1st Trailer Yaw (deg) Anal.	Dolly Yaw (deg) Exper.	Dolly Yaw (deg) Anal.	Dolly-2nd Trailer Yaw (deg) Exper.	Dolly-2nd Trailer Yaw (deg) Anal.	Trailer Yaw (deg) Exper.	Trailer Yaw (deg) Anal.	Maximum Tire Slip (deg)
1	2- 6	Double 27' Trailer (empty)	D	Lane Change with braking	-12.10	+ 8.68 - 6.51	------	+ 5.23 - 5.08	+ 4.95 - 3.90	+10.21	+11.18	+ 4.22 - 2.61	+ 4.19 - 3.21	+ 8.47	+10.86	+ 2.88 - 1.69	+ 3.18 - 2.74	------	------	- 3.61
2	2-14	Double 27' Trailer (full)	D	Evasive	------	+11.08 -14.57 + 7.67	------	+ 6.33 - 6.44 + 3.33	+ 4.93 - 6.14 + 3.01	+10.28 - 9.34	+ 9.82 -12.01	+ 2.46 - 4.31 + 4.30	+ 4.01 - 5.67 + 2.65	+10.30 -10.77	+10.60 -13.67	+ 3.66 - 4.85 + 3.80	+ 4.39 - 6.70 + 2.46	------	------	+ 5.22
3	2-15	Double 27' Trailer (full)	D	Evasive with braking	- 9.45	+ 9.33 - 7.50 + 3.82	------	+ 4.75 - 5.09 + 3.73	+ 3.87 - 4.36 + 2.51	+10.21 -10.41	+ 9.73 -12.70	+ 3.66 - 4.27 + 3.87	+ 3.43 - 4.10 + 3.57	+ 9.42 - 8.55	+ 9.86 -13.59	+ 3.47 - 2.85 + 1.35	+ 3.53 - 4.77 + 5.60	------	------	- 5.10
4	2-39	Double 27' Trailer (empty)	D	Lane Change	------	+14.94 -15.99	------	+ 7.41 - 8.97	+ 8.43 - 8.13	+13.41 - 1.74	+15.38 - 1.37	+ 6.55 - 6.63	+ 6.85 - 8.08	+14.23 - 4.24	+16.53 - 3.70	+ 6.34 - 9.29 + 4.08	+ 7.21 - 9.13 + 4.65	------	------	+ 6.81
5	2-41	Double 27' Trailer (empty)	D	Evasive	------	+14.64 -13.56 + 4.08	------	+ 9.04 - 7.41 + 2.25	+ 7.94 - 5.80 + 1.66	+13.62 - 9.22	+13.61 - 6.93	+ 7.18 - 6.07 + 2.88	+ 6.66 - 5.74 + 1.59	+14.93 - 8.80	+14.76 - 7.33	+ 7.57 - 8.35 + 2.31	+ 6.43 - 6.38 + 1.31	------	------	- 6.35
6	2-51	Single 27' Trailer (empty)	D	Lane Change	------	+12.71 -11.24	------	+ 6.99 - 6.35	+ 6.34 - 5.17	+11.00	+11.81	+ 4.10 - 5.12	+ 5.42 - 5.08	------	------	------	------	+ 9.43	+11.20	- 4.74
7	2-53	Single 27' Trailer (empty)	D	Evasive	------	+13.94 -13.61 +12.29	------	+ 8.27 - 7.04 + 7.33	+ 7.49 - 5.75 + 5.35	+ 2.77 -11.21	+13.24 -11.25	+ 5.27 - 5.71 + 3.66	+ 6.23 - 6.00 + 4.91	------	------	------	------	+10.45 - 9.35	+12.47 -11.07	- 5.69
8	1- 9	Single 45' Trailer (empty)	D	Lane Change	------	+ 9.13 - 9.13	------	+ 3.27 - 3.90	+ 3.02 - 2.99	+10.80	+ 9.90	+ 4.51 - 5.18	+ 5.43 - 5.15	------	------	------	------	+ 8.79	+ 8.21	- 2.50
9	1-19	Single 45' Trailer (empty)	W	Lane Change	------	+ 7.59 - 7.70	------	+ 2.92 - 2.29	+ 3.84 - 3.37	+ 9.21	+ 7.94	+ 4.04 - 4.00	+ 4.39 - 4.45	------	------	------	------	+ 7.72	+ 6.52	- 3.06
10	2-26	Double 27' Trailer (1st full, 2nd empty)	W	Lane Change	------	+ 9.31 -10.43	------	+ 5.99 - 7.26 + 2.50	+ 5.69 - 7.88 + 7.13	+11.83 - 1.76	+11.83 - 3.60	+ 3.68 - 5.03 + 2.03	+ 4.46 - 5.90 + 3.69	+11.72 - 4.12 + 1.58	+15.25 - 8.22 + 3.18	+ 3.88 - 7.13 + 3.04	+ 6.29 -12.02 +14.14	------	------	+ 6.82
11	2-37	Double 27' Trailer (1st full, 2nd empty)	D	Evasive with Braking	-13.50	+11.28 -11.60	------	+ 7.22 - 6.61	+ 5.82 - 5.21	+10.83	+12.15	+ 4.37 - 5.23	+ 4.62 - 4.51	+12.28	+13.24	+ 6.48 - 8.43	+ 5.69 - 4.59	------	------	- 4.42
12	1-12	Single 45' Trailer (empty)	D	Lane Change with braking	-15.68	+ 9.92 - 6.39	------	+ 1.98 - 2.52	+ 3.09 - 3.85	+ 9.45	+10.93	+ 4.72 - 3.70	+ 6.39 - 5.55	------	------	------	------	------	------	-11.85
13	1-15	Single 45' Trailer (empty)	D	Cornering 72.5' radius	------	+10.95	+16.50	+11.00	+12.85	------	------	+24.00	+24.71	------	------	------	------	------	------	- 7.16
14	2- 9	Double 27' Trailer (empty)	D	Cornering 82.0' radius	------	+12.18	+14.92	+19.90	+12.48	------	------	+12.32	+13.63	------	------	+11.56	+14.03	------	------	- 6.14
15	2-59	Single 27' Trailer (empty)	W	Cornering 74.17' radius	------	+12.25	+12.15	+17.71	+14.53	------	------	+15.83	+15.46	------	------	------	------	------	------	- 7.65

Simulation No. 1 (test No. 2-6) shows the empty double-articulated vehicle response in a lane change maneuver at 40 mph with braking on a dry surface. A combination of Firestone and Goodyear tires was used on this and the two succeeding tests discussed herein. This vehicle carried a full load during the test. The results show high peak forward and lateral accelerations (0.39 and 0.28 g respectively) with a tractor yaw of about 10 deg. Peak values throughout the simulation were within 10 percent and phase differences were minimal. In this experiment, the tire model works well in simulating system response even though the friction ellipse concept is utilized.

Test results of an evasive maneuver performed on a dry surface with a fully loaded double-articulated vehicle are shown as simulation No. 2. No braking was performed during this violent maneuver (0.47 g lateral acceleration). Another fully loaded double-articulated correlation record is shown in simulation No. 3. Again an evasive maneuver was performed; however, the application of braking limited the lateral acceleration in the maneuver requiring an extra 2 sec for completion of the maneuver. This record shows remarkably good correlation between analysis and experiment.

The response of a double articulated vehicle in a change maneuver without braking is shown as simulation No. 4 where all Goodyear Super Hi-Miler tires were used in the test configuration (Figure 3). The analytical results shown agree well with those of the experiment which is better than a 0.5 g maneuver. It is interesting to point out that the driver steering response followed that of AVDS3 closely and simulated a decaying sine wave characteristic of a violent maneuver.

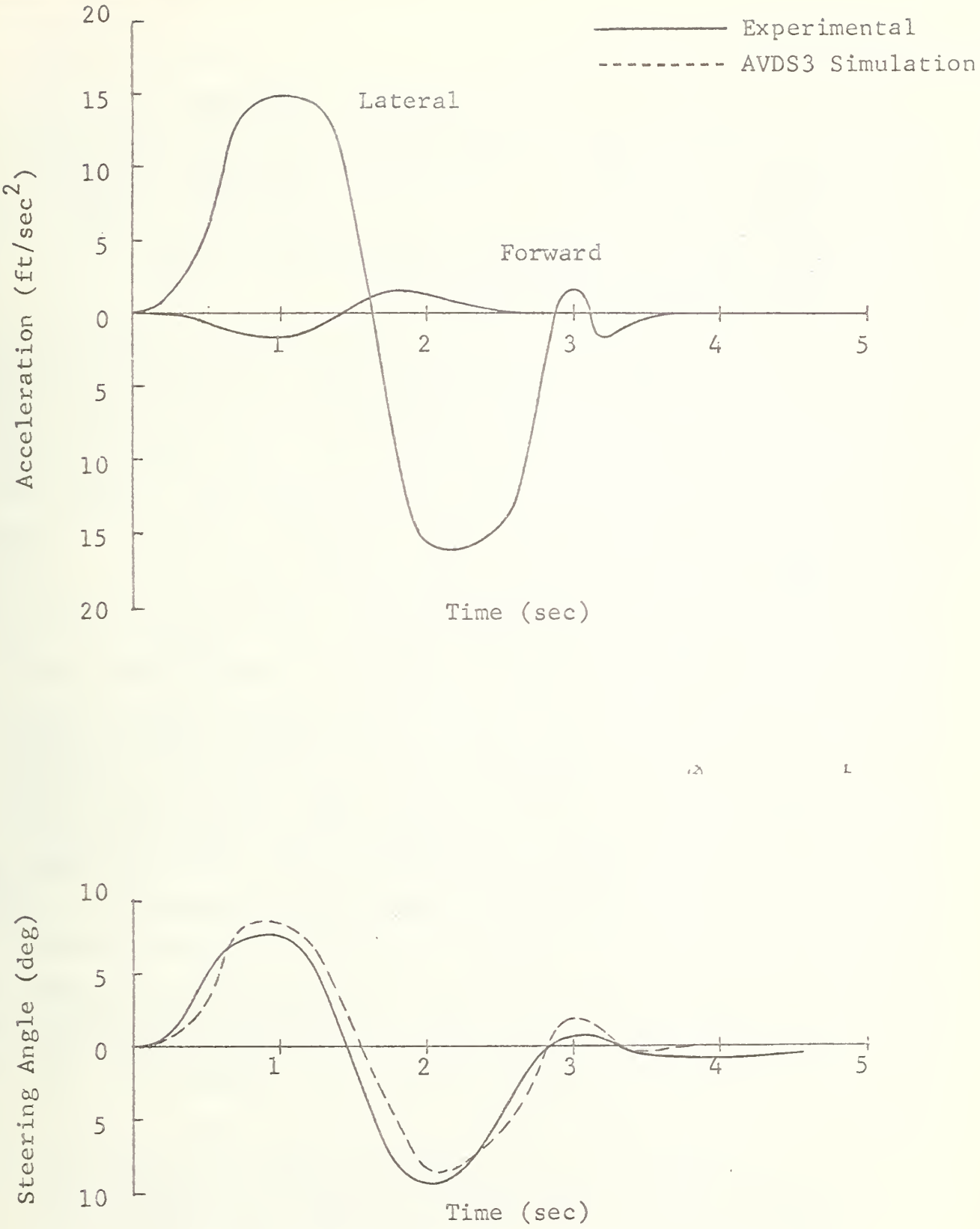


Figure 3. Analytical-experimental comparison of tractor double unloaded semitrailers (27 ft) response for experiment No. 2-39; lane change (14.4 ft x 191 ft) at 40 mph without braking.

Results of a similar test configuration in an evasive maneuver are shown in simulation No. 5. The peak lateral acceleration is about 0.5 g. In this experiment, tractor-trailer yaw and dolly-trailer yaw angles are both about 7 deg. Taken with large dolly and tractor yaw angles (15 deg) a large trailer swing was observed.

Simulations No. 6 and 7 show analytical-experimental results of an articulated vehicle with a 27 ft trailer performing lane changing and evasive maneuvers on a dry surface. It is noted that the lateral accelerations (o.43 g peak) are less for the single articulated vehicle than those obtained for the double trailer unit. The simulation follows the experimental results closely. The steering record in the evasive maneuvers indicates that the driver was extended on this maneuver. These records show about 3 sec to complete a lane change and about 5 sec to complete an evasive maneuver.

Results of tests with an articulated vehicle with a single 45 ft trailer on dry and wet surfaces are shown in simulations No. 8 and 9. Mixed tires were mounted on this vehicle. Peak lateral accelerations are lower for this trailer indicating greater demand on tires by the larger vehicle. On the dry surface a peak lateral acceleration of 0.28 g was obtained; this dropped to 0.23 g on the wet surface.

Despite mixed tires, good correlation was obtained. This indicates that the functional form of the tire cornering force curve obtained from experimental data simulates well the varied tires on this vehicle.

In the lane change maneuver with violent braking the simulation (No. 12) was barely acceptable. It appears that the friction ellipse concept as presently formulated did not properly apportion tire cornering and braking forces. In the simulation, tractor and trailer yaw angles were about 25 percent

higher than those obtained in the experiments. This indicates a lack of sufficient cornering force available in the simulation model. The friction ellipse concept constrained the available cornering force more than the presence of braking would have, in reality.

Simulations 13 through 15 were performed on cornering articulated vehicles yielding large steering requirements (up to 20 deg). This fact bears out the driver's observation that front wheel skid is the limiting factor in cornering. Tractor-trailer and dolly-trailer yaw angles up to 14 deg and 24 deg were obtained for 27 ft and 45 ft trailers respectively. Lateral accelerations up to one-half g were observed analytically and experimentally.

In general it can be observed that the analytical-experimental results obtained in this project correlate well. No experimental data is available on tire slip angles; however, simulation is available. Even though the tire slip angle results appear to be low in magnitude, it must be recognized that most tires are utilized in the low load regime owing to multiple tires per axle where the slip angle at which the maximum side force occurs is low. Therefore, these values are near the tire force saturation slip angle.

In the experimental-analytical records it must be noted that the experimental response values lag the analytical calculations (Figure 4). This phenomenon was first observed in these limit maneuvers (at lower values of lateral acceleration this phase discrepancy was not noted). In addition because of the fact that the basic vehicle control is exerted at the tractor, errors in response (either phase or amplitude) are propagated through the vehicle. Therefore, the tractor double-semitrailer system accentuates both phase and amplitude errors. The phase lag observed in the experimental results is explained by the system lateral flexibility and

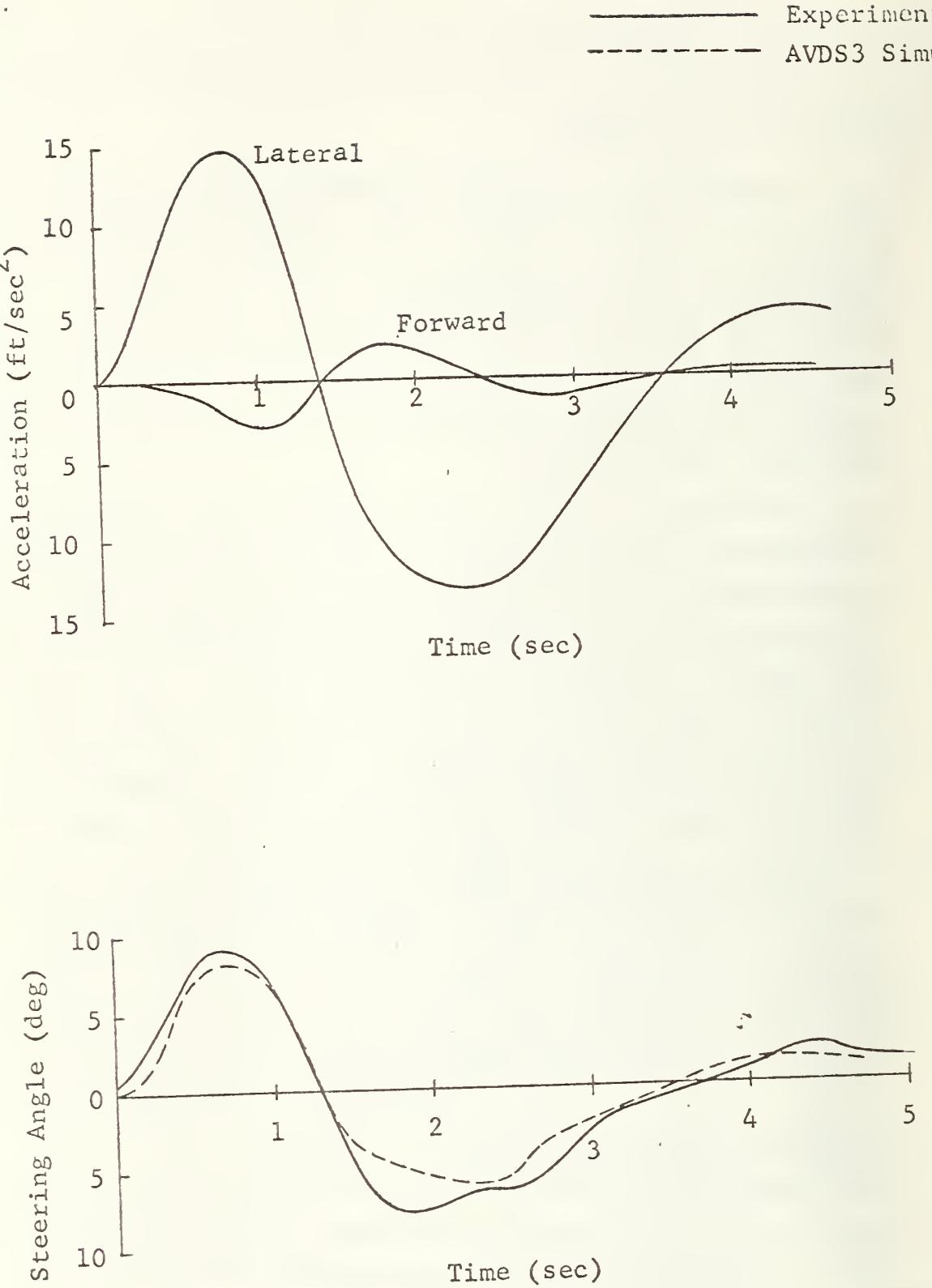
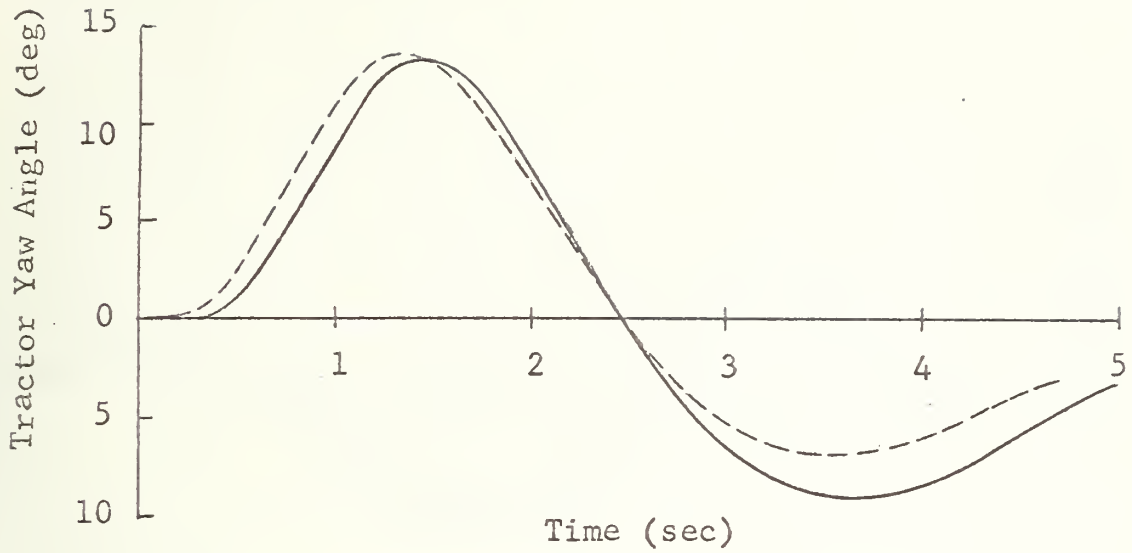


Figure 4. Analytical-experimental comparison of tractor double unloaded semitrailers (27 ft) response for experiment No.2-41; evasive (14.3 ft x 233 ft) at 40 mph without braking.



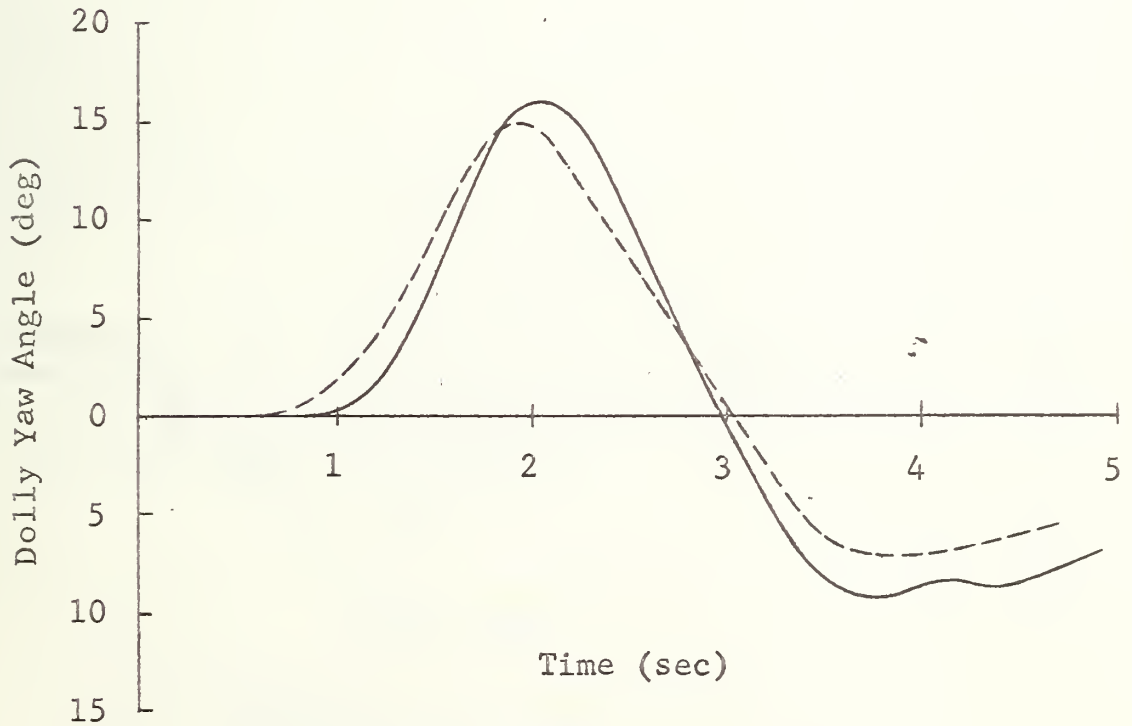
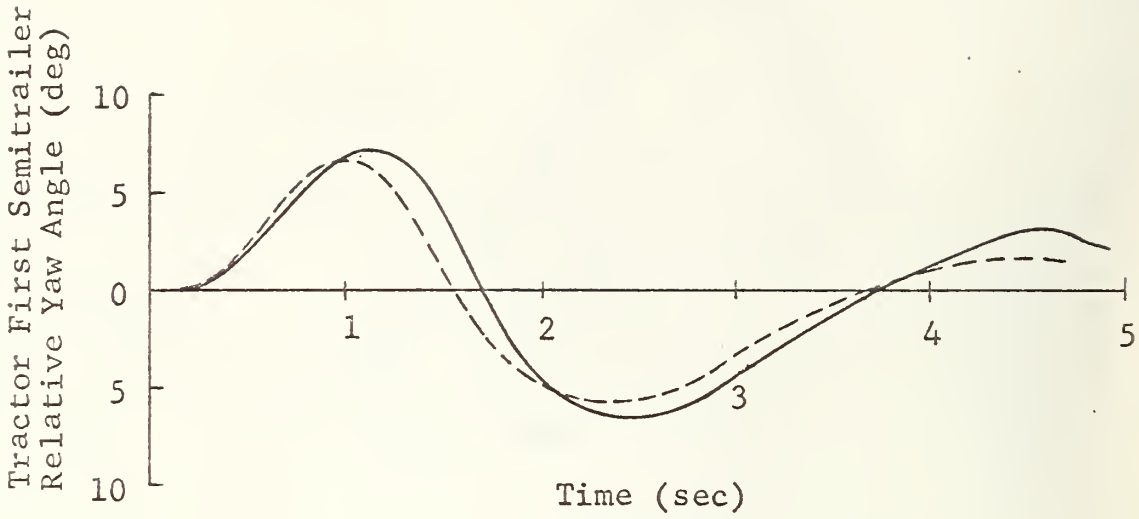


Figure 4. (contd)



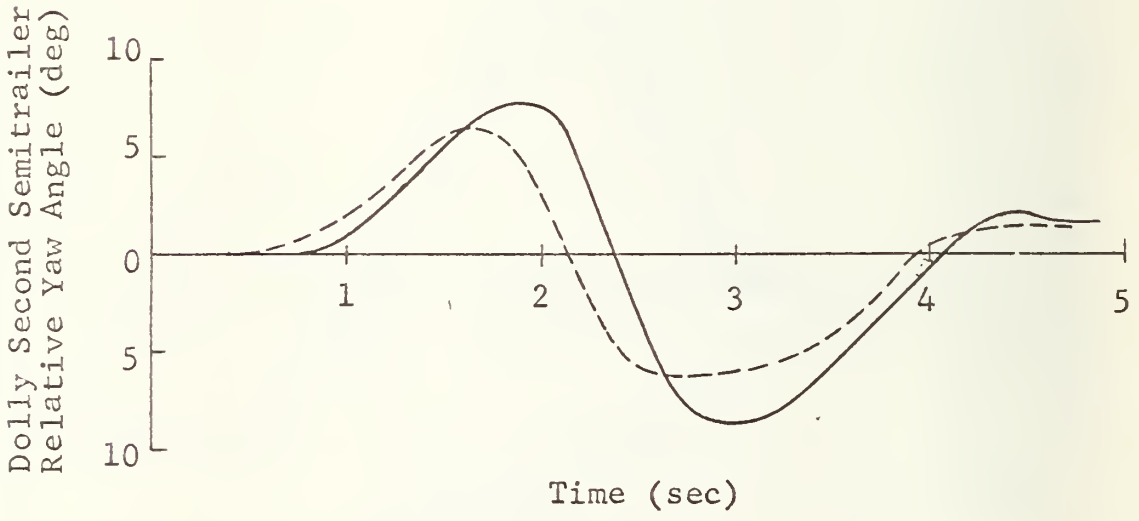


Figure 4. (concl)

damping not being modeled in AVDS3. In effect, the AVDS3, is a rigid body model forced directly by tire side and forward forces; whereas the experimental model (full-scale vehicle) is a rigid body mounted on flexible-damped springs and tires, with tire side and forward forces transmitted through the flexible-damped members. In somewhat analogous simple, single degree-of-freedom systems, the response at the mass lags the forcing function due to system flexibility and damping. Since AVDS3 does not model lateral flexibility and damping, its results do not reflect the lag in response between the tire forces and the rigid body mass. It should also be noted that this phenomenon has little effect on the response magnitudes.

3.3 Parameter Study

A parameter study was conducted on the test vehicles of this program (described in the technical report) to establish stability limits for varied maneuvers, braking, road-tire interface, loading and speed. The test stability criteria utilized to discern these stability limits are described in detail in Section 4 of the technical report. Basically, loss of side force on the tire at the road interface constitutes instability. Stability limits are given in the technical report in terms of tire-road friction.

An experimental-analytical comparison of specific stability results generated for single and double articulated vehicles in this program is shown in Table 4. In general the stability limits generated by Lyapunov and AVDS3 are higher than the experimental results.

EXPERIMENTAL-ANALYTICAL COMPARISON OF ARTICULATED VEHICLE STABILITY LIMITS

Vehicle Config.	Maneuver	Surface	Load Config.*	Test No. Series	Limit Velocity Exp. (ft/sec)	Anal. (ft/sec)
27 ft single	C	Wet	U	59-2	30.18	31.75
27 ft double	C	Dry	U	9-2	31.50	36.50
		Wet	U	43-2	25.32	31.60
		Dry	H	31-2	32.40	36.50
		Wet	H	30-2	25.50	31.60
		Dry	F	16-2	32.80	36.50
		Wet	F	17-2	26.25	31.60
45 ft single	C	Dry	U	14-1	33.80	37.50
		Wet	U	21-1	25.90	31.75
		Wet	L	23-1	29.25	31.75
					Limit Lateral Acceleration (g, ft/sec^2)	
27 ft single	E	Dry	U	53-2	0.415	0.580
		Wet	U	57-2	0.409	0.400
	L.C.	Wet	U	55-2	0.396	0.400
27 ft double	E	Dry	U	41-2	0.441	0.600
		Wet	U	47-2	0.396	0.400
		Dry	H	36-2	0.422	0.640
		Wet	H	28-2	0.402	0.335
		Dry	F	14-2	0.400	0.625
27 ft double	L.C.	Dry	U	39-2	0.484	0.600
		Wet	U	45-2	0.368	0.400
		Wet	H	26-2	0.359	0.335
45 ft single	L.C.	Dry	U	9-1	0.340	0.580
		Wet	U	18-1	0.258	0.400

*U = unloaded
H = second trailer unloaded
F = fully loaded
L = loaded

C = circle
E = evasive
L.C = lane change

The first series of analytical and experimental results are for cornering of tractor semitrailer systems performed without braking. Limit velocities obtained through simulation are about 10 percent higher than the experimentally obtained limits. This indicates that the stability criterion of requiring the total front axle to skid at the inception of instability yields limits which are unattainable. This is realistic since the AVDS3 simulation yields physical stability limits for the perfect driver, vehicle and road condition which should be unattainable in practice. The same conclusion can be drawn for lane changing and evasive maneuvers on a dry surface. For lane changing on a wet surface, test results (tests 28-2, 57-2 and 26-2) indicate that the driver exceeded the AVDS3 generated limits. This result can be attributed to either an incorrect assessment of the road surface/tire friction interface or the fact that the driver can tolerate some tire skid during the maneuver. Specific results of the parameter study are discussed individually in the technical report.

Figure 5 shows a typical experimentally verified analytically generated stability chart. The effect of load on the stability of a tractor double 27 ft semitrailer system performing an 82 ft radius cornering maneuver without braking is shown. The results obtained from the simulation model AVDS3 showed indistinguishable limit velocites for the three cases. The test results shown in the figure lead to a similar conclusion; however, there is a fine difference in limit velocity consistently showing highest limit for a loaded vehicle and lowest for an unloaded vehicle both on wet and dry surfaces. The difference in limit velocity is only 1 ft/sec.

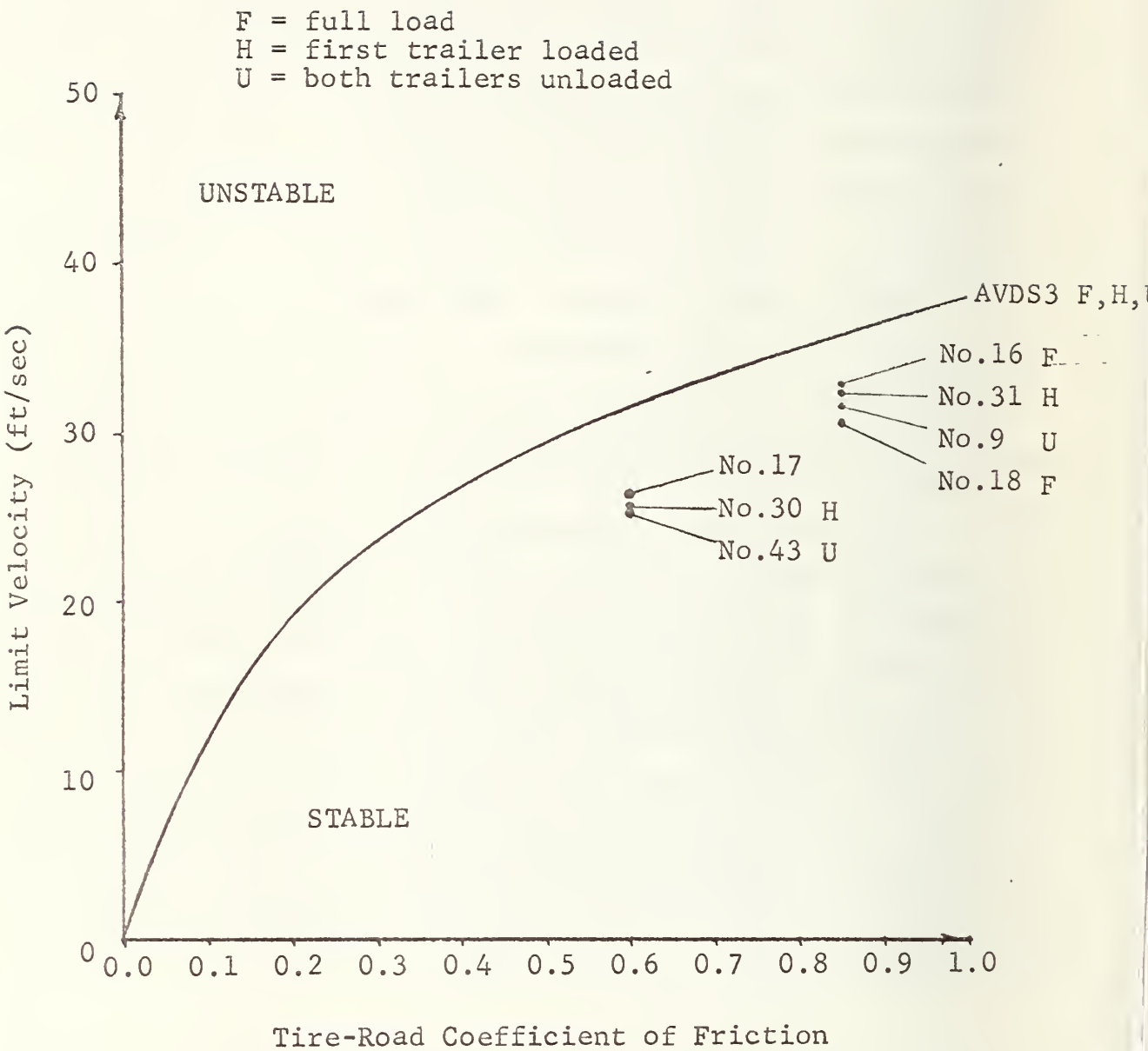


Figure 5. The effect of load on the stability of a tractor double 27 ft semitrailer system performing an 82 ft radius cornering maneuver without braking (all tests from test series No. 2).

4. CONCLUSIONS

Because of the complexity of meaningful articulated vehicle models which must retain nonlinearities in their formulations, it can be concluded on the basis of work conducted in this study that methods of analytically obtaining Lyapunov functions are difficult to implement. Systematic numerical methods appear to be useful in the study of the stability of articulated vehicles in steady state (autonomous) cornering maneuvers. For nonautonomous systems, it appears that the Lyapunov concept of stability is too restrictive because it requires stability over infinite time and no momentary movement to another stable trajectory is allowed. A modified Lyapunov concept called finite time stability is applicable to the articulated vehicle problem because it considers the stability of the vehicle over a finite time interval. In this study the techniques of mathematical stability were studied and exhaustive attempts were made to apply these techniques to the generation of articulated vehicle limits. The limited success of this program leads to the conclusion that the universal generation of stability limits utilizing the concepts of mathematical stability appears to be at present beyond the state of the art. The major difficulty encountered is that the practical articulated vehicle problem is nonlinear and nonautonomous. Stability techniques available in the literature are oriented toward linear, autonomous systems with the more forward techniques being able to solve specific nonlinear autonomous problems; however, it was found that most of these are contrived problems not of practical value.

Looking on the more positive aspects of this investigation, we found that stability limits based on physical stability criteria can be generated for articulated vehicles performing road maneuvers in varied environments. The results of this study indicate that:

- Stability limits are insensitive to vehicle load and trailor length.
- Stability limits are highly dependent on road-tire friction coefficient.
- Stability limits are relatively insensitive to added trailers. This result was reported in the final report on Articulated Vehicle Handling[2].
- Stability limits are sensitive to braking with decreased allowable maneuver velocity.
- Stability limits for evasive and lane change maneuvers are similar.

The stability criteria used to generate the stability limits through the digital computer program AVDS3 were verified by full-scale testing. It was shown that the computer simulation can perform successfully on multitrailer vehicles. The modeling of the hitch point between a trailer and dolly was verified. Experimental modeling studies on roll and pitch angle responses showed them to be small compared to yaw angles and therefore gives credibility to the use of load transfer assumptions. Finally discrepancies in analytical-experimental response phase indicate that vehicle lateral damping and flexibility may be important and should be included in the simulation model.

From the results of this study,the conclusion of the previous NHTSA study[2] on articulated vehicle handling concerning the importance of tire modeling is reinforced. Although much progress was made on this study in tire modeling, full-scale on-site tire parameter measurements which consider both cornering and tractive/braking forces are required. The friction ellipse concept needs experimental development.

From the experimental studies it can be concluded that articulated vehicles can safely perform violent high g maneuvers if maneuver space is available and professional drivers are used. The weakest link in the multitrailer systems appears to be the dolly. In violent maneuvers, especially involving braking, dolly jackknifing was frequently encountered. It was found that loading of multitrailer articulated vehicles is important for control in road maneuvering situations. Finally the analytical-experimental records indicate that professional drivers follow the most efficient driving modes acting with the proficiency of a good automatic control system.

It can be concluded that further study on the factors that affect articulated vehicle stability and handling is required; particularly on the tires, which were shown to be most important factors, in order to establish meaningful safety standards.

5. RECOMMENDATIONS

Recommendations for further work on articulated vehicle modeling, simulation, experimentation and stability criteria result from the findings of this program. It is recommended that in-depth, stability and response studies in articulated vehicle dynamics be continued to quantify the stability and maneuverability of articulated vehicles in normal traffic flow. The effect of parameters of road design including surfaces, curves, grades and banks should be evaluated thoroughly in a parameter study. Stability criteria involving trailer swing -- particularly with double- and triple-trailer units -- should be developed. In addition, an experimental-analytical program should be conducted to develop standard analytical-experimental procedures of articulated vehicles through stability and response criteria, the driver skill involved in safe vehicle operation, and the road design and surface condition requirements.

Although the models of articulated vehicles are sufficiently comprehensive for the prediction of gross vehicle response, effort should be expanded to determine the sensitivity of the response to modeling assumptions. This would involve formulating the problem in terms of more degrees of freedom and determining the effect of governing parameters on the response. Areas most likely to show fruitful investigation are the suspension system, wheels, tandem axles, and the tires. The vertical suspension system properties involve the forward or lateral load transfer from wheel to wheel caused by maneuvering, aerodynamic forces and braking forces. The lateral suspension system properties include lateral, forward and yawing motions of the vehicle, particularly in softly supported vehicles such as moving vans. In particular the problem of how lateral stiffness and damping affect computer simulated response should be thoroughly examined.

The assumptions made in fifth wheel connection and hitch point connections, particularly as they affect the transmission of forces and moments to other parts of the vehicle, should be critically examined. Wheel rotation degrees of freedom are involved in direct study of the dynamic implementation of braking systems, particularly antilock systems. Phenomena such as brake fade and how it affects articulated vehicle handling should be studied. Without a doubt, tires show the most pressing need for effort on modeling. A longitudinal/lateral force model that predicts the tractive acceleration/cornering force interaction for tractor semitrailer tires should be developed. This will in all probability involve curve fitting of empirical data. The model would be characterized for tires of typical size and construction with varied road surface, temperature and velocity effects included. Curve fitting or computer identification schemes may have to be utilized for individual tires under specified conditions. This work is necessary to evaluate the effect of tire properties on articulated vehicle handling and stability.

In view of the control and maneuverability problems confronting the driver of articulated vehicles, studies should be undertaken on the demands on driver skill, measured in terms of potential loss of control and in the context of the vehicle operating over its full range of environments and design configurations.

Experimental work should be utilized to verify the overall dynamic response and stability of single-, double- and triple-articulated vehicles. The experimental results of this program should be used with actual tire data to verify the Mikulcik[1] model for both trucks (as a special case of the tractor semitrailers) and tractor semitrailers. In addition this response model should be extended to triples. In this manner, both direct response and semi-inverse (AVDS) models would be available for vehicle evaluation. The experimental generation of tire characteristics should be pursued whether

it be accomplished indirectly with a special test fixture, directly from an articulated vehicle, or from analytical/experimental parameter identification studies. Actually, the test data obtained in the present program could be used with the AVDS3 or Mikulcik model to determine tire characteristics for the particular tires and operating conditions of this program.

Further work is required on the characterization of aerodynamic effects including the effects of steady wind and gusts on articulated vehicle response, and the interaction of these vehicles with other road vehicles. Aerodynamic constants associated with lateral and forward rectilinear motions and roll, pitch and yaw angular motions require further study. In addition the interaction effects of articulated vehicle units or the total characterization of the vehicle is required.

Roads should be characterized to facilitate study of the potential maneuverability conditions to be encountered by drivers of articulated vehicles. This characterization should include tire/road interaction for specific road surfaces, road conditions (oil, ice, water, dust) and geometry (hills, banks and curves).

Finally if the present work on the determination of mathematical stability limits is pursued further, it is recommended that the finite time stability concept be developed on a problem oriented basis. Techniques directly applicable to articulated vehicles will have to be developed.

6. REFERENCES

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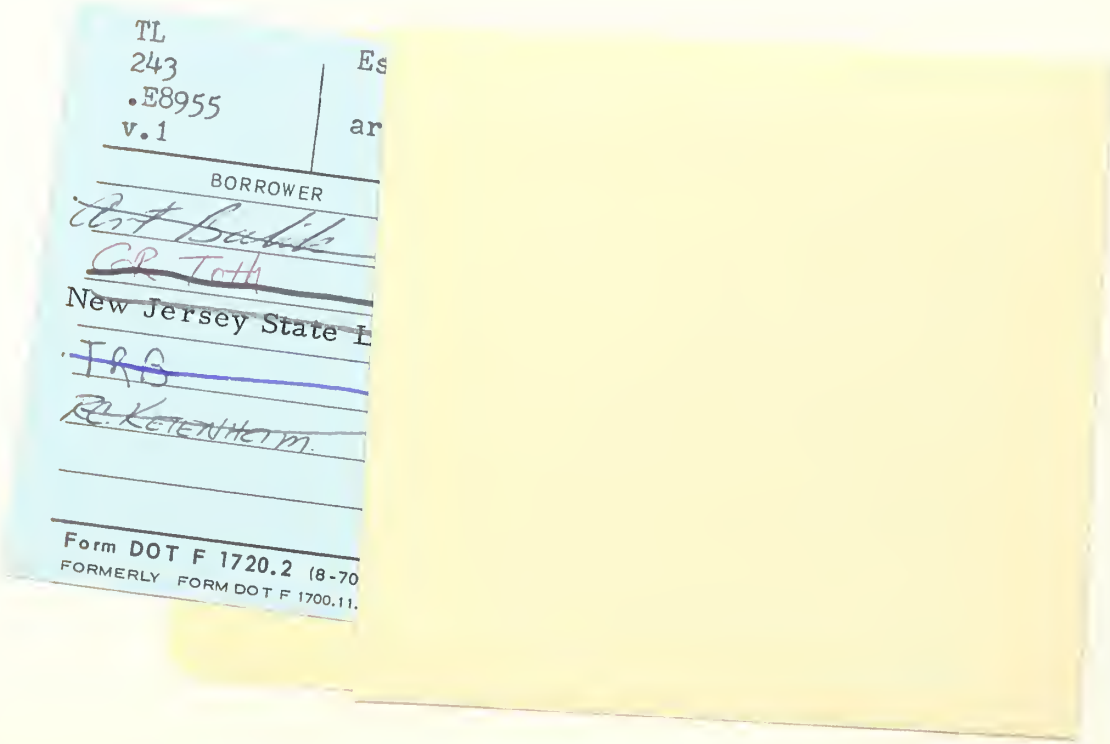
2. Eshleman, R. L. and Desai, S., "Articulated Vehicle Handling," Final Report, DOT Contract DOT-HS-105-1-151, IITRI Project J6255, HS 800 674, April 1972.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